Mutual Inductance and Induced Currents in a Coil Group

Introduction

The mutual inductance and induced currents between a single turn primary and twenty turn secondary coil in a concentric coplanar arrangement is computed using a frequency domain model. Each turn of the secondary coil is modeled explicitly. The results are compared against analytic predictions.

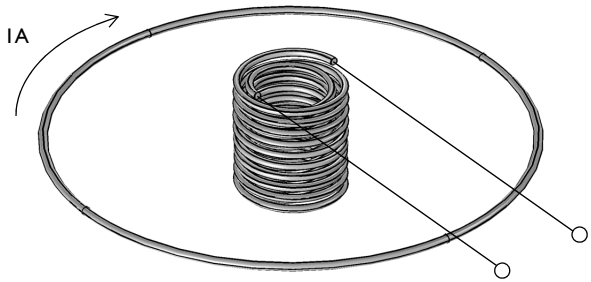


Figure 1: A twenty turn secondary coil inside of a single turn primary coil (not to scale).

Model Definition

The physical situation being modeled is shown in Figure 1. A prescribed current of 1 A is flowing through a single turn coil of radius R_1 = 100 mm, at a frequency of 1 kHz. A secondary coil composed of twenty turns, wound two deep, is concentric with the primary, and in the same plane. The radius of the centroid of the secondary coil is R_2 = 10 mm. The wire radii in both coils is r_0 = 1 mm. Although the coils are shown in 3D, they are modeled in the 2D axisymmetric space, assuming no physical variation around the center line. The objective of this model is to compute the voltage difference at the secondary coil for the open circuit case, and the induced currents for the closed circuit case.

For the case of a secondary multi-turn coil with N turns, and in the limit as $R_1 >> R_2 >> r_0$, the analytic expression for the mutual inductance between the two coils is:

$$M = N\mu_0\pi\frac{\pi R_2^{\ 2}}{2R_1} \tag{1}$$

where μ_0 is the permittivity of free space.

The two concentric coils are modeled in a 2D axisymmetric sense, as shown schematically in Figure 2. The modeling domain is surrounded by a region of Infinite Elements, which are a way to truncate a domain which stretches to infinity. Although the thickness of the Infinite Element domain is finite, it can be thought of as a domain of infinite extent.

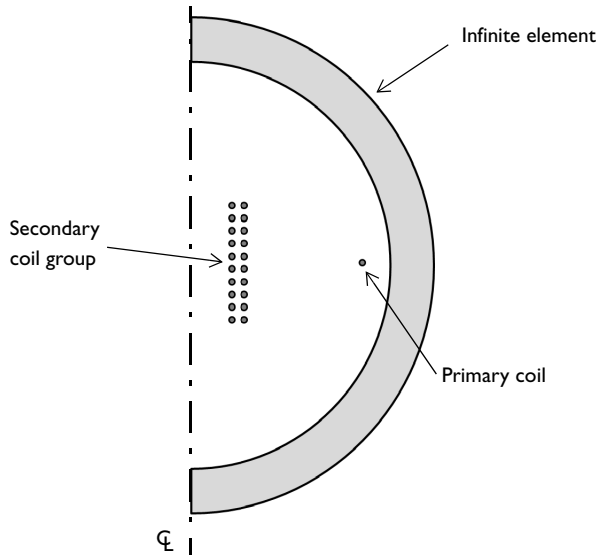


Figure 2: A schematic representation of the 2D axisymmetric model of the concentric coils.

The primary coil is modeled using the Single Turn Coil Domain feature, which can be thought of as introducing an infinitesimal slit in an otherwise continuous torus. The Single Turn Coil feature is used to excite the coil by specifying 1 A of current.

The secondary coil is modeled using the Coil Group Domain feature, which is similar to a Single Turn Coil Domain feature, but enforces that the same amount of current flow through each circular domain which represents one turn of the coil. This feature

 ©2013 COMSOL

can be used to model both the open circuit and the closed circuit case. To model the open circuit case, the current through the coil is specified to be 0 A, which specifies that there is no current flowing through the coil. The Coil Group Domain feature will compute the total coil voltage, V_{coil}, which can be used to compute inductance via:

$$M = \frac{V_{coil}}{i\omega I_p} \tag{2}$$

Where w is the angular frequency of excitation of the driving current in the primary, I_p. The inductance computed in this fashion is compared against the inductance predicted by taking the integral of the magnetic flux through the center of the coil.

To model the closed circuit case, the voltage drop across the coil is fixed at 0 V. Although this seems to imply a short circuit, the reactance of the copper coil is inherently included, so the case being modeled is analogous to a closed continuous loop of wire. The Coil Group Domain condition enforces that the same current flows through each turn.

Results and Discussion

The magnetic flux lines are plotted in Figure 3 for the open circuit case. The Coil Group Domain computes the voltage across the secondary, which can be used to evaluate the mutual inductance, 39.863 – 0.574i nH. This agrees well with the mutual inductance predicted by taking the integral of the magnetic flux passing through the surface defining the center of the coil, 39.500 – 0.607i nH. The analytic mutual inductance, assuming no capacitive effects, in 39.478 nH.

The induced currents of the secondary coil is plotted in Figure 4 for the closed circuit case. The skin effect is apparent, the current is being driven to the boundaries of the domains. The induced current through the secondary coil is –10.936 – 3.473i mA, the imaginary component implies a reactive current.

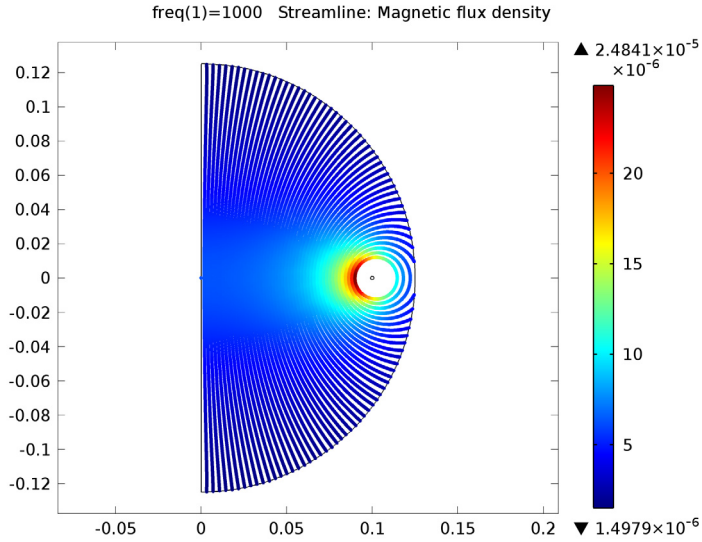


Figure 3: Magnetic flux lines for the open circuit case.

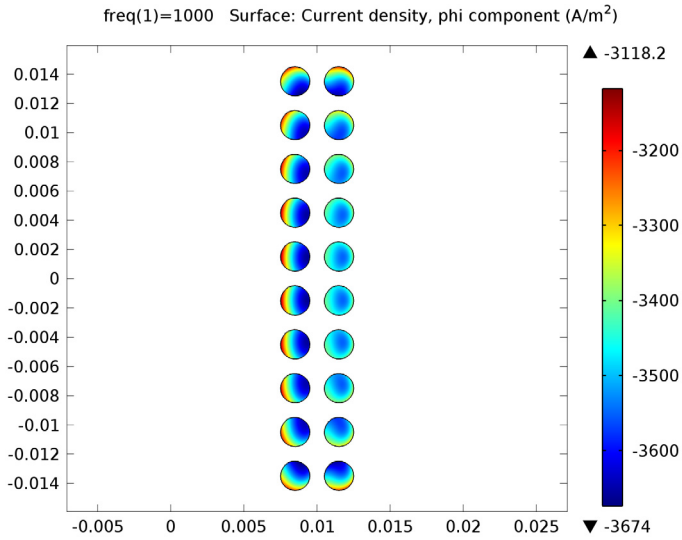


Figure 4: Induced currents in the coil for the closed circuit case.

 ©2013 COMSOL

Model Library path: `ACDC_Module/Inductive_Devices_and_Coils/mutual_inductance_coil_group`

Modeling Instructions

MODEL WIZARD

1 Go to the **Model Wizard** window.

2 Click the **2D axisymmetric** button.

3 Click **Next**.

4 In the **Add physics** tree, select **AC/DC>Magnetic Fields (mf)**.

5 Click **Add Selected**.

6 Click **Next**.

7 Find the **Studies** subsection. In the tree, select **Preset Studies>Frequency Domain**.

8 Click **Finish**.

GLOBAL DEFINITIONS

Parameters

1 In the **Model Builder** window, right-click **Global Definitions** and choose **Parameters**.

2 In the **Parameters** settings window, locate the **Parameters** section.

3 In the table, enter the following settings:

Name	Expression	Description
r_wire	1[mm]	Radius, wire
R1	100[mm]	Radius, outer coil
R2	10[mm]	Radius, inner coil
M	(mu0_const*pi*R2^2)/(2*R1)	Analytic mutual inductance

Here, `mu0_const` a predefined COMSOL constant for the permeability in vacuum.

GEOMETRY 1

Create a circle for the simulation domain. Define layer in the circle where you will assign the Infinite Element Domain.

Circle 1

1 In the **Model Builder** window, under **Model 1** right-click **Geometry 1** and choose **Circle**.

2 In the **Circle** settings window, locate the **Size and Shape** section.

3 In the **Sector angle** edit field, type `180`.

4 In the **Radius** edit field, type `1.75*R1`.

5 Locate the **Rotation Angle** section. In the **Rotation** edit field, type `-90`.

6 Click to expand the **Layers** section. In the table, enter the following settings:

Layer name	Thickness (m)
`Layer 1`	`50[mm]`

Circle 2

Create a circle for the outer coil.

1 In the **Model Builder** window, right-click **Geometry 1** and choose **Circle**.

2 In the **Model Builder** window, click **Circle 2**.

3 In the **Circle** settings window, locate the **Size and Shape** section.

4 In the **Radius** edit field, type `r_wire`.

5 Locate the **Position** section. In the ***r*** edit field, type `R1`.

Circle 3

Then, create a circle for the inner coil.

1 In the **Model Builder** window, right-click **Geometry 1** and choose **Circle**.

2 In the **Circle** settings window, locate the **Size and Shape** section.

3 In the **Radius** edit field, type `r_wire`.

4 Locate the **Position** section. In the ***r*** edit field, type `R2-1.5*r_wire`.

5 In the ***z*** edit field, type `-13.5*r_wire`.

Array 1

1 Right-click **Geometry 1** and choose **Transforms>Array**.

2 Select the object **c3** only.

3 In the **Array** settings window, locate the **Size** section.

4 In the ***r* size** edit field, type `2`.

5 In the ***z* size** edit field, type `10`.

6 Locate the **Displacement** section. In the ***r*** edit field, type `3*r_wire`.

 ©2013 COMSOL

7 In the **z** edit field, type `3*r_wire`.

Polygon 1

Add a line for computing the mutual inductance as a line integral of the magnetic flux.

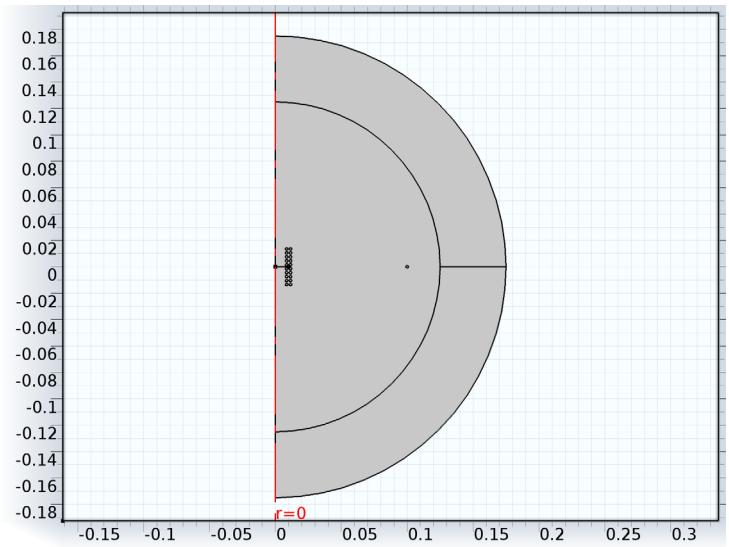
1 Right-click **Geometry 1** and choose **Polygon**.

2 In the **Polygon** settings window, locate the **Coordinates** section.

3 In the **r** edit field, type `0 R2`.

4 In the **z** edit field, type `0 0`.

5 Click the **Build All** button.



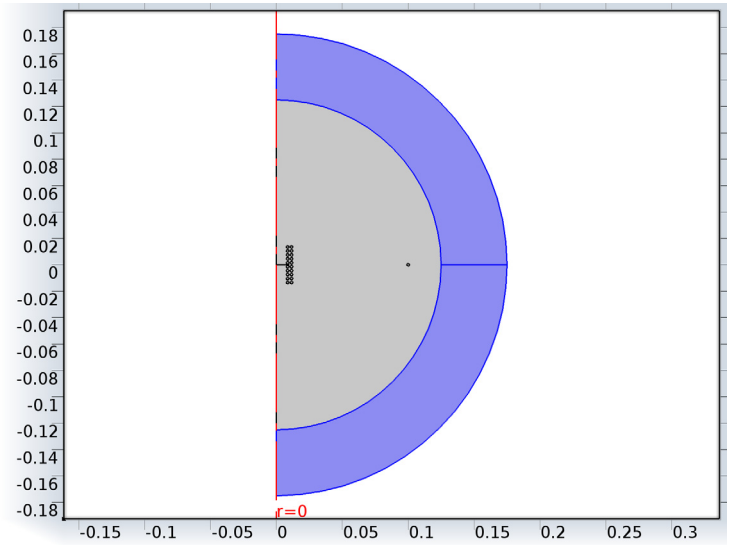
DEFINITIONS

Define the Infinite Element Domain where you truncate the domain which stretches to infinity.

Infinite Element Domain 1

1 In the **Model Builder** window, under **Model 1** right-click **Definitions** and choose **Infinite Element Domain**.

2 Select Domains 1 and 3 only.



MAGNETIC FIELDS

Now, set up the physics. First, set the Single-Turn Coil Domain boundary condition on the outer coil.

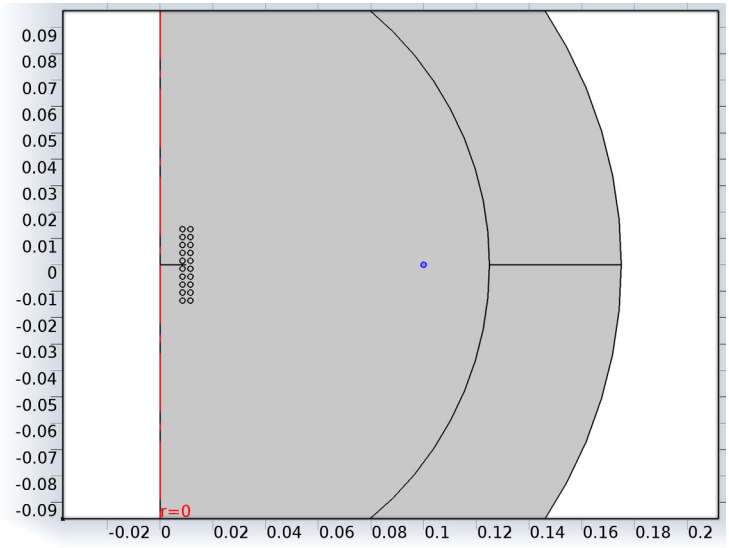
1 Click the **Zoom In** button on the Graphics toolbar.

Single-Turn Coil 1

1 In the **Model Builder** window, under **Model 1** right-click **Magnetic Fields** and choose **Single-Turn Coil**.

 ©2013 COMSOL

2 Select Domain 24 only.



Single-Turn Coil 2

Create another Single Turn Coil Domain and specify that it should behave as a coil group. Specify zero voltage to model the short circuit case.

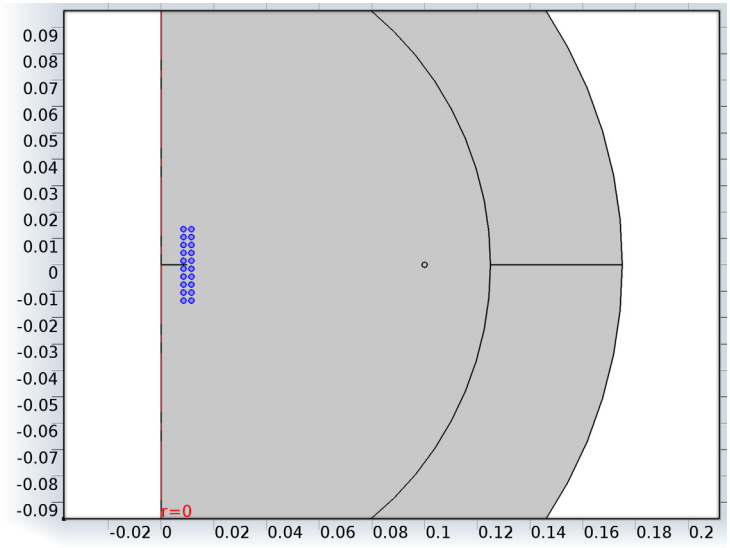
1 In the **Model Builder** window, right-click **Magnetic Fields** and choose **Single-Turn Coil**.

2 In the **Single-Turn Coil** settings window, locate the **Single-Turn Coil** section.

3 Select the **Coil group** check box.

4 Select Domains 4–23 only.

5 In the I_{coil} edit field, type `0[A]`.



MATERIALS

Next, assign material properties. Use air for all domains.

Material Browser

1 In the **Model Builder** window, under **Model 1** right-click **Materials** and choose **Open Material Browser**.

2 In the **Material Browser** settings window, In the tree, select **Built-In>Air**.

3 Click **Add Material to Model**.

Then, override the coil domains with copper.

4 In the **Model Builder** window, right-click **Materials** and choose **Open Material Browser**.

5 In the **Material Browser** settings window, In the tree, select **AC/DC>Copper**.

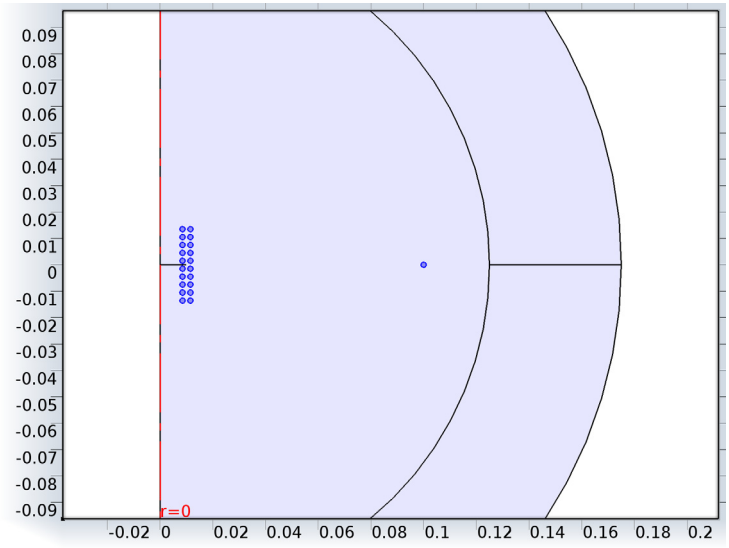
6 Click **Add Material to Model**.

Copper

1 In the **Model Builder** window, under **Model 1>Materials** click **Copper**.

 ©2013 COMSOL

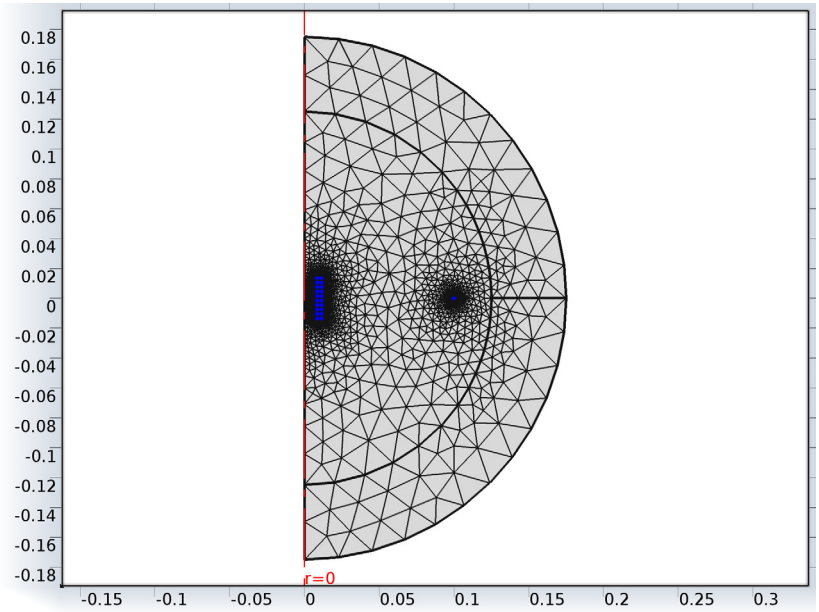
2 Select Domains 4–24 only.



3 Click the **Zoom Extents** button on the Graphics toolbar.

MESH 1

1 In the **Model Builder** window, under **Model 1** right-click **Mesh 1** and choose **Build All**.



STUDY 1

Step 1: Frequency Domain

1 In the **Model Builder** window, under **Study 1** click **Step 1: Frequency Domain**.

2 In the **Frequency Domain** settings window, locate the **Study Settings** section.

3 In the **Frequencies** edit field, type `1[kHz]`.

4 In the **Model Builder** window, click **Study 1**.

5 In the **Study** settings window, locate the **Study Settings** section.

6 Clear the **Generate default plots** check box.

7 Click the **Compute** button.

RESULTS

Data Sets

1 In the **Model Builder** window, expand the **Results>Data Sets** node.

2 Right-click **Solution 1** and choose **Add Selection**.

 ©2013 COMSOL

3 In the **Selection** settings window, locate the **Geometric Entity Selection** section.

4 From the **Geometric entity level** list, choose **Domain**.

5 Click the **Select Box** button on the Graphics toolbar.

6 Select Domains 2 and 4–24 only.

2D Plot Group 1

1 In the **Model Builder** window, right-click **Results** and choose **2D Plot Group**.

2 Right-click **2D Plot Group 1** and choose **Streamline**.

3 In the **Streamline** settings window, locate the **Streamline Positioning** section.

4 From the **Positioning** list, choose **Start point controlled**.

5 From the **Entry method** list, choose **Coordinates**.

6 In the ***r*** edit field, type `range(0,0.9*R1/49,0.9*R1)`.

7 In the **z** edit field, type `0`.

8 Locate the **Coloring and Style** section. From the **Line type** list, choose **Tube**.

9 Right-click **Results>2D Plot Group 1>Streamline 1** and choose **Color Expression**.

10 Click the **Zoom Extents** button on the Graphics toolbar.

This reproduces Figure 3.

Derived Values

Evaluate the mutual inductance using the line integral of the magnetic flux density.

1 In the **Model Builder** window, under **Results** right-click **Derived Values** and choose **Integration>Line Integration**.

2 Select Boundary 4 only.

3 In the **Line Integration** settings window, locate the **Expression** section.

4 In the **Expression** edit field, type `20*mf.Bz/1[A]`.

5 Click to expand the **Integration Settings** section. Select the **Compute surface integral** check box.

6 Locate the **Expression** section. From the **Unit** list, choose **nH**.

7 Click the **Evaluate** button.

Evaluate the mutual inductance using Equation 2.

8 In the **Model Builder** window, right-click **Derived Values** and choose **Global Evaluation**.

9 In the **Global Evaluation** settings window, locate the **Expression** section.

10 In the **Expression** edit field, type `mf.VCoil_2/1[A]/mf.iomega`.

11 From the **Unit** list, choose **nH**.

12 Click the **Evaluate** button.

MAGNETIC FIELDS

Specify zero voltage for the Coil assigned on the inner coil group to model the closed circuit case.

Single-Turn Coil 2

1 In the **Model Builder** window, under **Model 1>Magnetic Fields** click **Single-Turn Coil 2**.

2 In the **Single-Turn Coil** settings window, locate the **Single-Turn Coil** section.

3 From the **Coil excitation** list, choose **Voltage**.

4 In the V_{coil} edit field, type `0`.

ROOT

In the **Model Builder** window, right-click the root node and choose **Add Study**.

MODEL WIZARD

1 Go to the **Model Wizard** window.

2 Find the **Studies** subsection. In the tree, select **Preset Studies>Frequency Domain**.

3 Click **Finish**.

STUDY 2

Step 1: Frequency Domain

1 In the **Model Builder** window, under **Study 2** click **Step 1: Frequency Domain**.

2 In the **Frequency Domain** settings window, locate the **Study Settings** section.

3 In the **Frequencies** edit field, type `1[kHz]`.

4 In the **Model Builder** window, click **Study 2**.

5 In the **Study** settings window, locate the **Study Settings** section.

6 Clear the **Generate default plots** check box.

7 Click the **Compute** button.

RESULTS

Data Sets

Select the inner coil group domain.

1 In the **Model Builder** window, expand the **Results>Data Sets** node.

 ©2013 COMSOL

2 Right-click **Solution 2** and choose **Add Selection**.

3 In the **Selection** settings window, locate the **Geometric Entity Selection** section.

4 From the **Geometric entity level** list, choose **Domain**.

5 Select Domains 4–23 only.

2D Plot Group 2

1 In the **Model Builder** window, right-click **Results** and choose **2D Plot Group**.

2 In the **2D Plot Group** settings window, locate the **Data** section.

3 From the **Data set** list, choose **Solution 2**.

4 Right-click **Results>2D Plot Group 2** and choose **Surface**.

5 In the **Surface** settings window, click **Replace Expression** in the upper-right corner of the **Expression** section. From the menu, choose **Magnetic Fields>Currents and charge>Current density>Current density, phi component (mf.Jphi)**.

6 Click the **Plot** button.

7 Click the **Zoom Extents** button on the Graphics toolbar.

Compare the plot with Figure 4 describing the induced currents in the coil for the closed circuit case.

Derived Values

Evaluate the total induced current on the inner (secondary) coil group.

1 In the **Model Builder** window, under **Results** right-click **Derived Values** and choose **Global Evaluation**.

2 In the **Global Evaluation** settings window, locate the **Data** section.

3 From the **Data set** list, choose **Solution 2**.

4 Click **Replace Expression** in the upper-right corner of the **Expression** section. From the menu, choose **Magnetic Fields>Coil parameters>Coil current (mf.ICoil_2)**.

5 Locate the **Expression** section. From the **Unit** list, choose **mA**.

6 Click the **Evaluate** button.

 ©2013 COMSOL