COMSOL MULTIPHYSICS®

Lossy Eigenvalue Calculations

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Lossy Eigenvalue Calculations

In mode analysis and eigenfrequency analysis, it is usually the primary goal to find a propagation constant or an eigenfrequency. These quantities are often real valued although it is not necessary. If the analysis involves some lossy part, like a nonzero conductivity or an open boundary, the eigenvalue is complex. In such situations, the eigenvalue is interpreted as two parts:

- The propagation constant or eigenfrequency
- The damping in space and time

Eigenfrequency Analysis

The eigenfrequency analysis solves for the eigenfrequency of a model. This analysis type is available for all application modes in the RF Module. The time-harmonic representation of the fields is more general and includes a complex parameter in the phase

$$\mathbf{E}(\mathbf{r}, t) = \mathrm{Re}(\tilde{\mathbf{E}}(\mathbf{r}_T)e^{j\omega t}) = \mathrm{Re}(\tilde{\mathbf{E}}(\mathbf{r})e^{-\lambda t})$$

where the eigenvalue, $(-\lambda) = \delta + j\omega$, has an imaginary part representing the eigenfrequency, and a real part responsible for the damping. It is often more common to use the *quality factor* or *Q-factor*, which is derived from the eigenfrequency and damping

$$Q_{\mathrm{fact}} = \frac{\omega}{2|\delta|}$$

VARIABLES

The following list shows the variables that the eigenvalues analysis affects:

NAME	EXPRESSION	CAN BE COMPLEX	DESCRIPTION
`omega`	`imag(-lambda)`	No	Angular frequency
`damp`	`real(lambda)`	No	Damping in time
`Qfact`	`0.5*omega/abs(damp)`	No	Quality factor
`nu`	`omega/(2*pi)`	No	Frequency

NONLINEAR EIGENFREQUENCY PROBLEMS

For some combinations of formulation, material parameters and boundary conditions, the eigenfrequency problem can be nonlinear, which means that the eigenvalue enters the equations in another form than the expected second-order polynomial form. The following table lists those combinations:

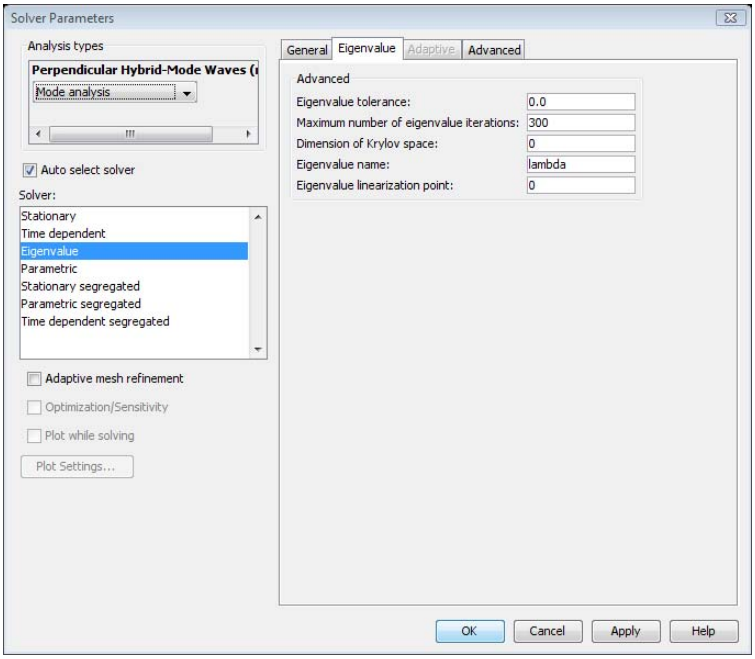
SOLVE FOR	CRITERION	BOUNDARY CONDITION
H	Nonzero conductivity	
H, E	Nonzero conductivity	Impedance boundary condition
H, E	Nonzero conductivity at adjacent subdomain	Scattering boundary condition
H, E	Nonzero conductivity at adjacent subdomain	Scattering boundary condition
H, E	Analytical ports	Port boundary condition

These situations require special treatment, especially since it can lead to singular matrix messages if not treated correctly. The complication is not only the nonlinearity itself, it is also the way it enters the equations. For example the impedance boundary conditions with nonzero boundary conductivity has the term

$$-(-\lambda)\frac{\sqrt{\varepsilon_0\mu_0}\sqrt{\mu_{\mathrm{rbnd}}}}{\sqrt{\varepsilon_{\mathrm{rbnd}}+\dfrac{\sigma_{\mathrm{bnd}}}{(-\lambda)\varepsilon_0}}}(\mathbf{n}\times(\mathbf{n}\times\mathbf{H}))$$

where $(-\lambda) = \delta + j\omega$. When the solver starts to solve the eigenfrequency problem it linearizes the entire formulation with respect to the eigenvalue around a certain linearization point (see the "The Eigenvalue Solver" on page 402 of the *COMSOL Multiphysics User's Guide* for more information). By default this linearization point is zero, which leads to a division by zero in the expression above. To avoid this problem and also to give a suitable initial guess for the nonlinear eigenvalue problem, it is necessary to provide a "good" linearization point for the eigenvalue solver. You can do this in the **Solver Parameters** dialog box, under the **Eigenfrequency** tab, by entering a new complex value in the **Eigenvalue linearization point** edit field. For example, if you

know that the eigenfrequency is close to 1 GHz, enter the eigenvalue `-i*2*pi*1e9` in the edit field.



In many cases it is enough to specify a good linearization point and then solve the problem once. If you need a more accurate eigenvalue, an iterative scheme is necessary:

1. Specify that the eigenvalue solver only search for one eigenvalue.
2. Solve the problem with a “good” linearization point. As the eigenvalues shift, use the same value with the real part removed.
3. Extract the eigenvalue from the solution and update the linearization point and the shift.
4. Repeat until the eigenvalue does not change more than a desired tolerance.

Mode Analysis

In mode analysis COMSOL Multiphysics solves for the propagation constant, which is possible for the Perpendicular Waves and Boundary-Mode Analysis application modes. The time-harmonic representation is almost the same as for the eigenfrequency analysis, but with a know propagation in the out-of-plane direction

$$\mathbf{E}(\mathbf{r},t) = \mathrm{Re}(\tilde{\mathbf{E}}(\mathbf{r}_T)e^{j\omega t - j\beta z}) = \mathrm{Re}(\tilde{\mathbf{E}}(\mathbf{r})e^{j\omega t - \alpha z})$$

The spatial parameter, $\alpha = \delta_z + j\beta = -\lambda$, can have a real part and an imaginary part. The propagation constant is equal to the imaginary part, and the real part, δ_z, represents the damping along the propagation direction.

VARIABLES

The following table lists the variables that are influenced by the eigenfrequency analysis:

NAME	EXPRESSION	CAN BE COMPLEX	DESCRIPTION
`beta`	`imag(-lambda)`	No	Propagation constant
`dampz`	`real(-lambda)`	No	Attenuation constant
`dampzdB`	`20*log10(exp(1))*dampz`	No	Attenuation per meter in dB
`neff`	`j*lambda/k0`	Yes	Effective mode index

Example Model—Circular Waveguide with Lossy Walls

This model finds the modes of a circular waveguide with walls made of a nonperfect conductor, which is copper in this case. The losses in the walls lead to attenuation of the propagating wave. The attenuation in dB scale is calculated using the formula

$$\Delta_{\mathrm{dB}} = 20\delta_z \log e$$

which is available as the variable `dampzdB_rfwv`. The first six eigenvalues show the following attenuation per km for a propagating wave with a frequency of 1 GHz.

EFFECTIVE MODE INDEX	PROPAGATION CONSTANT	ATTENUATION
0.931 + 1.9e-6i	19.51 rad/m	0.34 dB/km
0.957 + 1.1e-6i	20.05 rad/m	0.20 dB/km
0.957 + 1.2e-6i	20.05 rad/m	0.22 dB/km
0.973 + 1.3e-6i	20.40 rad/m	0.24 dB/km
0.984 + 4.4e-7i	20.63 rad/m	0.08 dB/km
0.984 + 7.6e-007i	20.63 rad/m	0.14 dB/km

Model Library path: `RF_Module/Tutorial_Models/lossy_circular_waveguide`

Modeling Using the Graphical User Interface

MODEL NAVIGATOR

1. In the **Model Navigator** select **2D** from the **Space dimension** list.
2. Open the **RF Module** folder, then select **Perpendicular Waves> Hybrid-Mode Waves>Mode analysis**.
3. Click **OK**.

GEOMETRY MODELING

Use **Specify Objects>Circle** under the **Draw** menu to create a circle with radius `0.5`.

PHYSICS SETTINGS

Subdomain Settings

Use the default settings for the subdomain.

Boundary Conditions

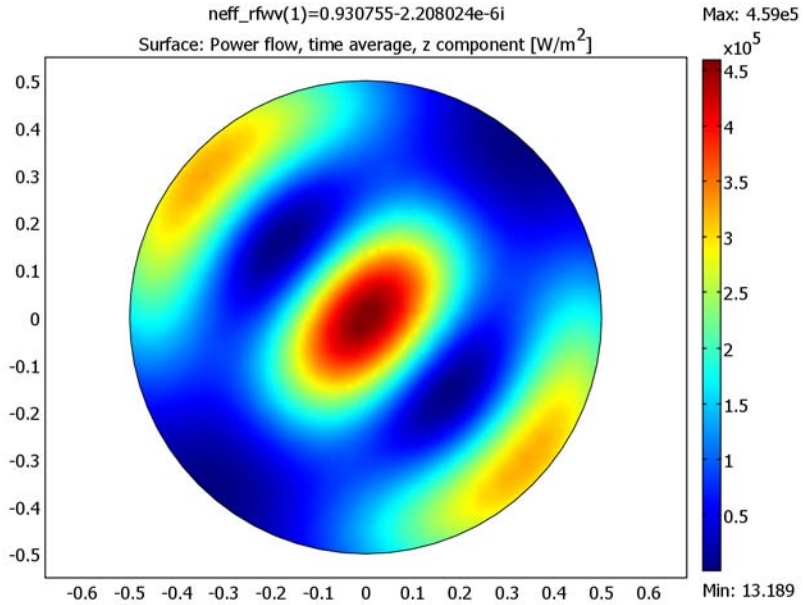
1. Open the **Boundary Settings** dialog box from the **Physics** menu.
2. Select all boundaries and choose the **Impedance boundary condition** from the **Boundary condition** list.
3. Click the **Material Properties** tab, click the **Load** button, and select **Copper** from the list of materials in the dialog box that appears.
4. Click **OK** twice to close the **Materials/Coefficients Library** dialog box and the **Boundary Settings** dialog box in that order.

COMPUTING THE SOLUTION

Click the **Solve** button on the Main toolbar to solve the problem with the default mesh and solver settings.

POSTPROCESSING AND VISUALIZATION

The default plot shows the *z*-component of the time-averaged power flow for the first eigenmode in the table on page 4.



To calculate the attenuation values in the third column of the table, do as follows:

1. Open the **Global Data Display** dialog box by choosing **Data Display>Global** from the **Postprocessing** menu.
2. Enter `1000*dampzdB_rfwv` in the **Expression** edit field and click **Apply**.

   This produces the first value for the third column in the table on page 4.
3. Go through the other entries in the **Effective mode index** list to get the full table.