Finite Element Solution of Nonlinear Transient Rock Damage with Application in Geomechanics of Oil and Gas Reservoirs

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Abstract: The increasing energy demand calls for advances in technology which translate into more accurate and complex simulations of physical problems. We are trying to understand volumetric rock damage, which is essential to understanding the geomechanics of oil and gas reservoirs. The fragile microstructure of some rocks makes it difficult to predict the propagation of damage and fracture in these rocks, therefore a mathematical model is required to predict the fracture mechanisms in such materials. The governing equation of rock damage is a nonlinear parabolic partial differential equation (PDE). The physics of the problem imposes a number of complexities that should be handled numerically. In this paper, we present the results we obtained using COMSOL 3.5a and we show how a complicated problem can be solved using the finite element method incorporated in COMSOL. The results could be used in similar geomechanical and structural damage problems such as failure and rupture of Steel, Aluminum, Concrete, etc. Moreover, the pattern of rock damage in oil and gas reservoirs is of great significance in recovery of hydrocarbon in petroleum engineering.

Keywords: Volumetric rock damage, Damage diffusion, Reservoir geo-mechanics, Brittle fracture.

1. Introduction

Solid mechanics and strength of materials are two of the oldest engineering mechanics problems. The fundamental works of Galileo [1] and Griffith [2] were the early steps in predicting the fracture strength of materials using an energy balance approach. In the “Two New Sciences” (1638), Galileo asked the question how long an object under load can last before it fails due to damage. This question was much deeper and very different than those asked by Robert Hooke in 1660, when he discovered the laws of elasticity. In petroleum engineering, the problem of rock fracturing is one of the problems which have been of interest for the past few decades. However, rock fractures are more important now due to demand of production from low permeability reservoir rocks such as diatomite oilfield or gas shale [3]. Continuum damage mechanics (CDM) is a branch of solid mechanics which deals with the formation and coalescence of micro-fractures of various scales, called in general, micro-defects. Micro-defects are created by mechanical or environmental loads. These loads result in deterioration of material and bond brakeage mechanisms, leading to the loss of material stiffness. Damage mechanic looks into the formation of damage. As the damage propagates, the material body becomes discrete, however to preserve the continuity of material so that the continuum mechanics be applicable, the other branch of mechanics—fracture mechanics, is invoked. Fracture mechanics takes the effects of micro-cracks as discrete defects, into a continuum body of intact material. In this article we look at damage problem from a thermodynamic standpoint in which bond breakage mechanism leads to propagation of damage.

1.1 Definition of damage parameter

As a rock specimen undergoes external load, the chemical bonds in the microstructure of rock undergo excitation due to a thermodynamic process. This excitation makes the energy given to rock matrix go beyond the activation energy of bonds, therefore; bonds start to break. This breakage of bonds is called rock damage. In other words, damage at a point in rock can be physically interpreted as the properly averaged fraction of broken bonds inside microstructural elements of the body. Once the bonds start getting broken, their load carrying capacity becomes zero, hence the load transfer could only continue through intact bonds. This reduction of load transferring agents from cross sectional area S to S_r can be used to define damage parameter ω as;

$$\omega = \frac{S - S_r}{S} \tag{1}$$

S_r in Equation (1) is the portion of the total cross section S, which remains intact and can transfer load as shown in Figure 1. This is the definition used in damage mechanics.

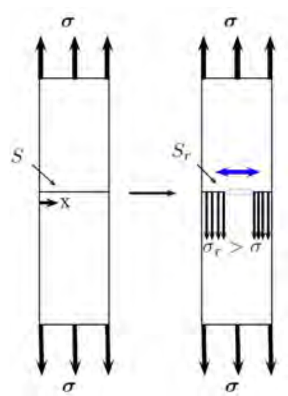

Figure 1. Macroscopic interpretation of damage

Alternatively, some damage mechanics books, define a continuity factor ψ as:

$$\psi = 1 - \omega \tag{2}$$

so that in original pristine material, damage parameter is 0 and continuity factor is 1. Fracture then corresponds to damage parameter equal to 1 or continuity factor of 0. In practice, damage parameter can never attain the value of 1.0 and failure occurs earlier, at lower values ω<1, which is obtained through analysis of localization [4, 5]. Therefore, the actual stress that can be carried across the partially damaged cross section is σ_r which is related to the bulk stress σ and damage parameter ω by Equation (3);

$$\sigma_r = \frac{\sigma}{(1-\omega)} \tag{3}$$

Figure 1 shows the increase in the value of stress σ_r at the damage zone in a bar under tension. The double-headed arrow along x direction indicates the direction of damage propagation which in the case shown here is perpendicular to the direction of load.

1.2 Governing equation of rock damage

Damage theory, originally developed by Kachanov [6, 7] and later extended to several areas of engineering and physics by many researchers including [8, 9] is based on simple ordinary differential equation of the form;

$$\frac{\partial \omega}{\partial t} = \tau^{-1} q(\omega, \sigma, T) \tag{4}$$

which governs the evolution of damage, where τ is the characteristic time and q is the damage accumulation term which is a dimensionless non-negative number specified for a given material. Once q is replaced with the appropriate kinetic law, the rate of damage shown on the left-hand side of the Equation (4) can be obtained as;

$$\frac{\partial \omega(X,t)}{\partial t} = [\nabla \bullet [\kappa \nabla (f(\omega))] + f(\omega)]_+ \tag{5}$$

In which $f(\omega)$ is an exponential function in the following form:

$$f(\omega) = (1-\omega)\exp(\frac{\mu_0}{1-\omega})$$

and κ is the damage diffusion parameter. The positive sign in the right hand side of the equation indicates that the rate of damage has to remain non-negative (>0 or =0) during the numerical analysis. This constraint is imposed by the physics of damage as a non-healing process. We access the solution vector and manipulate the vector such that the rate of damage is always positive. We will explain the steps we took to modify the solution to assure a positive damage rate in this article. μ_0 is a constant which has to do with the stress level applied to rock. In all analysis performed here, we use the constant value of 10 for μ_0.

Notice that, the partial differential equation (5) presented here, is nonlinear parabolic PDE and does not have an analytical solution, therefore; it has to be solved numerically. Here our focus is on the COMSOL features we used, therefore; we try to focus on the method of solution and demonstrate the technics which we came up with to solve this PDE using COMSOL3.5a and present the results we obtained. The derivation of this equation is beyond the scope of this paper and interested readers are recommended to see [10].

2. Description of the problem

The damage parameter and governing equation are now defined; therefore we can outline the problem we wish to solve. Given the initial distribution of damage in a domain Ω, we are interested in knowing how damage is propagated in rock. The damage parameter or state variable changes with time and space. Figure 2 shows the domain and boundary conditions of the problem. Vector n is the outward normal vector to the domain boundary at any point **X**. Here, **X** is a vector and we use a bold font for it. The damage parameter is a scalar as defined in Figure 2 and it is equal to zero on the boundary.

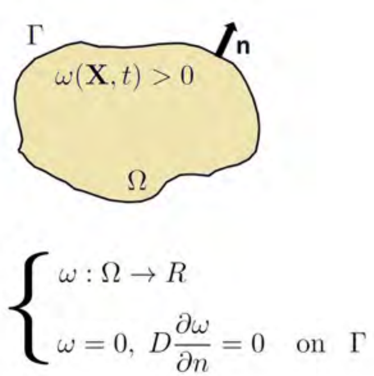

Figure 2. The time-dependent area Ω of damaged rock.

The flux of damage is also zero across the boundary. What we are interested to know is the distribution of damage over the domain as the time goes on. It should be noted that, when material undergoes damage and failure, it ruptures. The rupture or what is mathematically known as blow-up time is of great interest in our application. When blow-up occurs, damage parameter jumps to values greater than one and the solution to PDE ceases to exist. Studying the convergence of solution becomes significant in this problem and we are presenting convergence plot as well.

2.1 Set up the problem in COMSOL

In order to solve Equation (5), we are utilizing the coefficient form of PDE in COMSOL. The coefficient form is used to model a physics problem using a system of one or more time-dependent partial differential equations and is in the form of Equation (6).

$$e_a \frac{\partial^2 u}{\partial t^2} + d_a \frac{\partial u}{\partial t} + \nabla \bullet (-c \nabla u - \alpha u + \gamma) + \beta \bullet \nabla u + \alpha u = f \tag{6}$$

We assign coefficients of Equation (6) so it models Equation (5). Here is the path we took to perform this:

COMSOL 3.5a >Model Navigator>COMSOL Multiphysics>PDE Modes> PDE, Coefficient Form> Time-dependent analysis

To assign the coefficients in Equation (6), we use zero for d_a, α, γ, β and Equations (7) and (8) for f and c.

$$f = (1-u)\exp\left(\frac{\mu_0}{1-u}\right) \tag{7}$$

and

$$c = \kappa\left(\frac{\mu_0 + u - 1}{1-u}\right)\exp\left(\frac{\mu_0}{1-u}\right) \tag{8}$$

Once the coefficient form is created, we can solve the transient problem. The following sections give the details of analysis. To have a better control on problem variables and post processing features, we use Livelink for MATLAB and part of the script which demonstrate our method is presented here.

2.2 Blow-up time

As the solution time goes on, the onset of rupture is reached. This time is when the solution ceases to exist and is called the life time of material also known as blow-up time in mathematics. This is a known phenomenon in parabolic problems and occurs when the rate of input into the system is larger than that of output. Here we first obtain the blow-up time for the case of κ=0, numerically, and call it the blow-up time for no damage diffusion case denoted by t_{bu0}. This is used as a reference time in all our analysis and it shows how long it takes for a rock sample under tensile load to fracture, if damage is accumulated in one point. This is similar to the case of brittle material undergoing rupture. In other words, when the tensile load is applied to a brittle rock, the damage is accumulated at one point and may not diffuse through rock because of brittle nature of material. It goes without saying that, if the same load is applied to a ductile material, the life time or the time required to rupture is larger.

Figure 3 shows the onset of blow-up for κ=0. It can be seen that the damage increases around the mid-point of the 1D bar under tension.

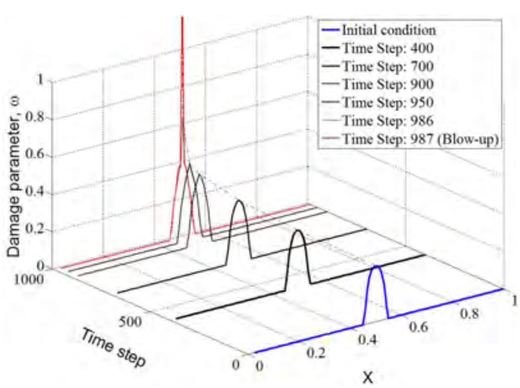

Figure 3. Solution at the onset of blow-up to obtain the t_{bu0}.

The solution time for time steps 986 and 987 are 8.36x10^{-11} and 8.37x10^{-11} respectively. These are dimensionless times relevant to the physics of this problem. It can be seen that a minute change in time is required for the solution to blow-up. In other words, to get the exact time of rupture for material or to obtain the exact values of damage distribution right before the rupture, extremely small time steps are required. In engineering applications, however; the level of accuracy that we have considered here is not required.

2.3 Solution of PDE

Figure 4 shows the distribution of damage with time for κ=0.06. A quadratic function is used for the initial distribution of damage which is plotted in blue in Figure 4.

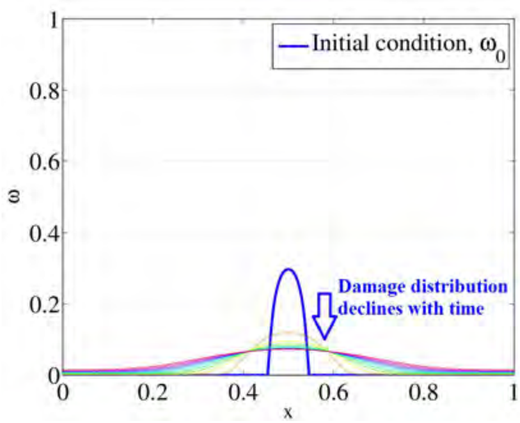

Figure 4. Solution without taking into accounts the non-healing effect of damage.

Since damage at any points in the domain of problem remains either constant or increases due to the non-healing nature of damage process, the solution has to be either constant or ever increasing. Therefore; to honor the physics of the problem, the solution vector has to be manipulated such that the rate of damage remains non-negative. This is achieved by accessing the structure of solution and making modifications through scripting in MATLAB.

2.3 Accessing the structure of solution through MATLAB

Once the solution is completed successfully, nodal values and degrees of freedom are saved in “nodes” and “dofs” variables. These can be accessed using the following commands (Lines 1-4).

```
1  nodes = xmeshinfo(fem,'out','nodes');
2  dofs=nodes.dofs;
3  coords=nodes.coords;
4  X=fem.sol.u;
```

Line (1), retrieves the nodal information from the finite element solution. Line (2), retrieves the degree of freedom of nodes. Line (3), gives the coordinates of the degrees of freedom obtained in line (2). Line (4), saves the solution vector in variable "X" for modifications.

To eliminate the declining values of damage, the following lines (5-9) are used. We use the values of the earlier step if the later step has lower values.

```
5  for i=1:length(dofs)
6      if (X(i,2))<(X(i,1))
7          X(i,2)=X(i,1);
8      end
9  end
```

The result obtained after making these changes, is shown in Figure 5.

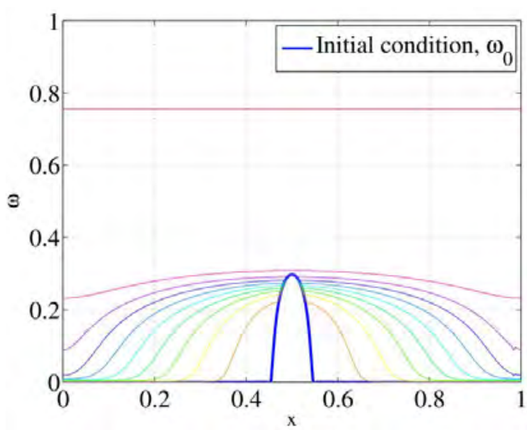

Figure 5. Solution corrected for non-healing effect of damage.

It can be seen that the damage distribution increases from the initial condition to about a uniform value of 0.72 as the time elapses. Besides, as we increased the damage diffusion parameter from κ=0 to κ=0.06, we noticed two modes of damage diffusion in rock. The former is a brittle failure as shown in Figure (3) in which damage accrues locally until failure at mid-point of the bar under tension and the latter is a ductile failure in which, damage initially diffuses toward boundaries and once the damage attains a uniform value along the bar, it starts increasing uniformly as shown in Figure (5).

2.4 Convergence of solution

Obtaining a solution using numerical methods, does not guarantee the accuracy of solution. One more thing which should be done to make sure the results are correct, is the analysis of convergence. Detailed convergence of solution in numerical methods can be studied in many books in numerical methods including [11].
We performed convergence studies in this problem and results are presented in Figure (6). This result exhibits the convergence of solution at mid-point of the bar where the maximum damage parameter is observed, takes place beyond 10,000 time steps. This could not be predicted and the result obtained, should be incorporated in solution process.

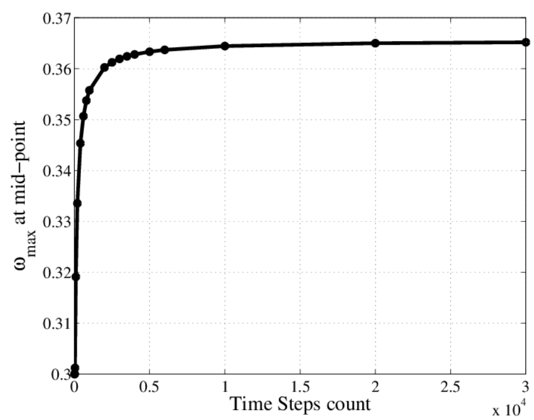

Figure 6. Convergence analysis.

3. Conclusions

1- In this paper we have used COMSOL Multiphysics and COMSOL Script to solve the transient rock damage problem. We have analyzed the non-healing process and incorporated the positive rate of damage in the finite element solution we obtained from COMSOL.
2- Numerical results indicate that there are two regimes of propagation depending on damage diffusion parameter κ. These are shown in Figures (3) and (5).
3- Due to the nonlinearity of damage problem, to obtain an accurate converged solution, time steps have to be very small. Our numerical results indicate that for the number of time steps

beyond 10,000, the solution gets converged.

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