Finite Element Analysis of Superconductive Tape by Using T-Ω Formulation

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Problem's Description:

- **PDE: Maxwell Equations**

$$\begin{cases} \nabla \times \mathbf{H} = \mathbf{J} = \sigma \mathbf{E} \\ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} = -\mu \frac{\partial \mathbf{H}}{\partial t} \\ \nabla \times \frac{1}{\sigma} \nabla \times \mathbf{H} = -\mu \frac{\partial \mathbf{H}}{\partial t} \end{cases}$$

$$\nabla \times \frac{1}{\sigma} \nabla \times \mathbf{H} + \mu \frac{\partial \mathbf{H}}{\partial t} = 0 \quad in\Omega_c$$
$$\nabla \times \mathbf{H} = \mathbf{J}_s \quad in\Omega_n$$

- **Boundary and interface conditions**

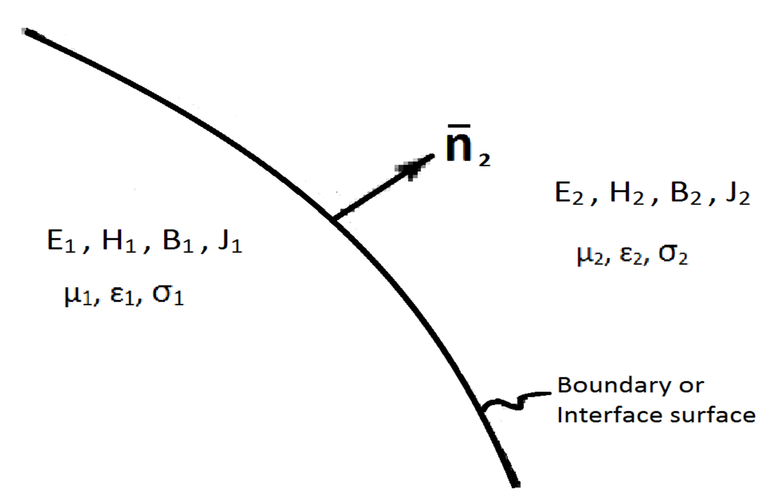


Table 1: Interface and boundary condition

Interfaces between two media	interface between a dielectric and a perfect conductor
$\bar{n}_2 \times (\mathbf{E}_1 - \mathbf{E}_2) = 0$	$\bar{n}_2 \times \mathbf{E}_2 = 0$
$\bar{n}_2 \times (\mathbf{H}_1 - \mathbf{H}_2) = J_s$	$\bar{n}_2 \cdot \mathbf{D}_2 = \rho_0$
$\bar{n}_2 \cdot (\mathbf{B}_1 - \mathbf{B}_2) = 0$	$\bar{n}_2 \times \mathbf{H}_2 = J_s$
$\bar{n}_2 \cdot (\mathbf{J}_1 - \mathbf{J}_2) = 0$	$\bar{n}_2 \cdot \mathbf{B}_2 = 0$

T-Ω formulation:

$$\begin{cases} \nabla \times \mathbf{T} = \mathbf{J} \quad and \quad \nabla \times \mathbf{H} = \mathbf{J} \rightarrow \mathbf{H} = \mathbf{T} - \nabla \Omega \\ \nabla \times \mathbf{E} = \nabla \times \frac{\mathbf{J}}{\sigma} = -\frac{\partial \mathbf{B}}{\partial t} = -\frac{\partial \mu \mathbf{H}}{\partial t} \\ \nabla \cdot \mathbf{B} = 0 \end{cases}$$

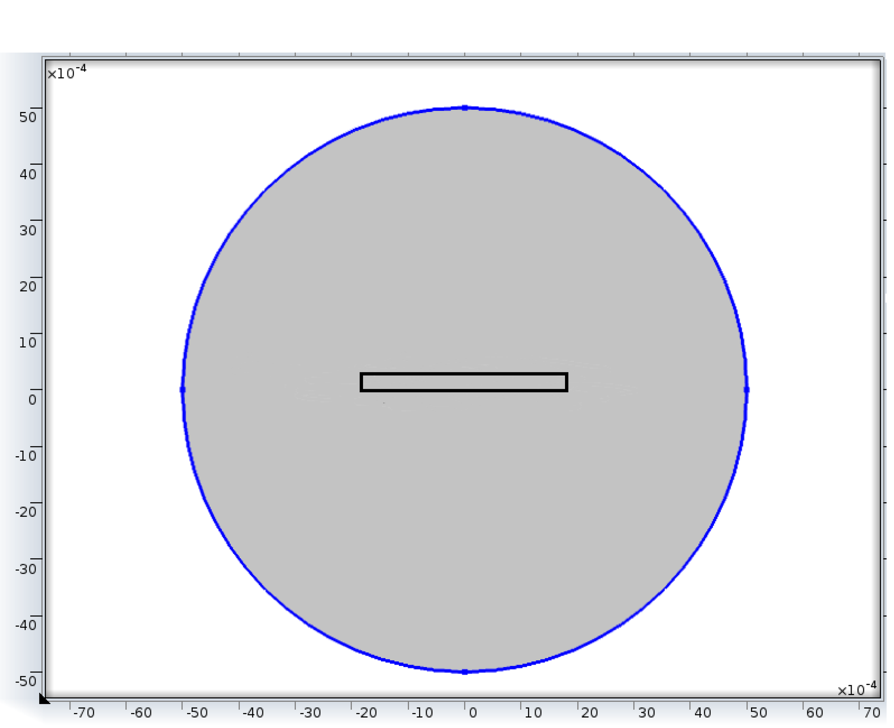
$$\nabla \times \frac{1}{\sigma} \nabla \times \mathbf{T} + \mu \frac{\partial}{\partial t}(\mathbf{T} - \nabla \Omega) = 0$$
$$\nabla . \mu (\mathbf{T} - \nabla \Omega) = 0$$

The vector potential is not unique and is fixed purely by means of its curl, the **Coulomb gauge** is a widely applied divergence condition

$$\nabla \times \frac{1}{\sigma} \nabla \times \mathbf{T} - \nabla \frac{1}{\sigma} \nabla \cdot \mathbf{T} + \mu \frac{\partial}{\partial t}(\mathbf{T} - \nabla \Omega) = 0 \quad \Omega_c$$
$$\nabla . \mu (\mathbf{T} - \nabla \Omega) = 0 \quad \Omega_c$$
$$\nabla . \mu (-\nabla \Omega) = 0 \quad \Omega_n$$

Application Examples:

The geometry consists of an superconductor tape that carry source current J_c, and surrounded by air



The nonlinearity of resistance given by:

$$\boldsymbol{E} = E_c \left(\frac{\boldsymbol{J}}{J_c}\right)^n$$

$$\rho(\boldsymbol{E}) = \frac{E_c}{J_c} \left(\frac{\boldsymbol{E}}{E_c}\right)^{\frac{n-1}{n}}$$

Results:

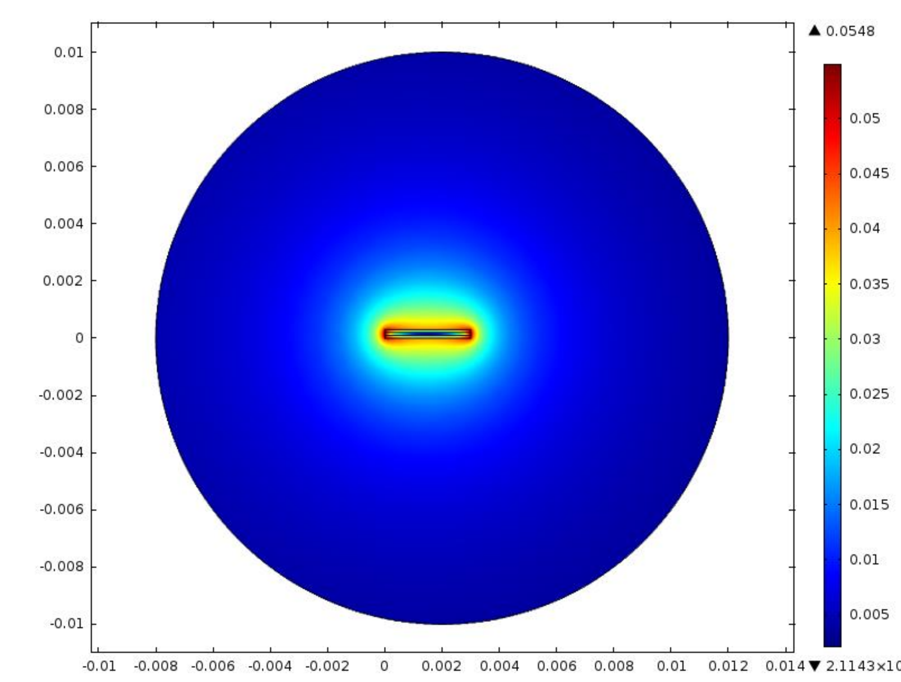


Figure 1. Electric field

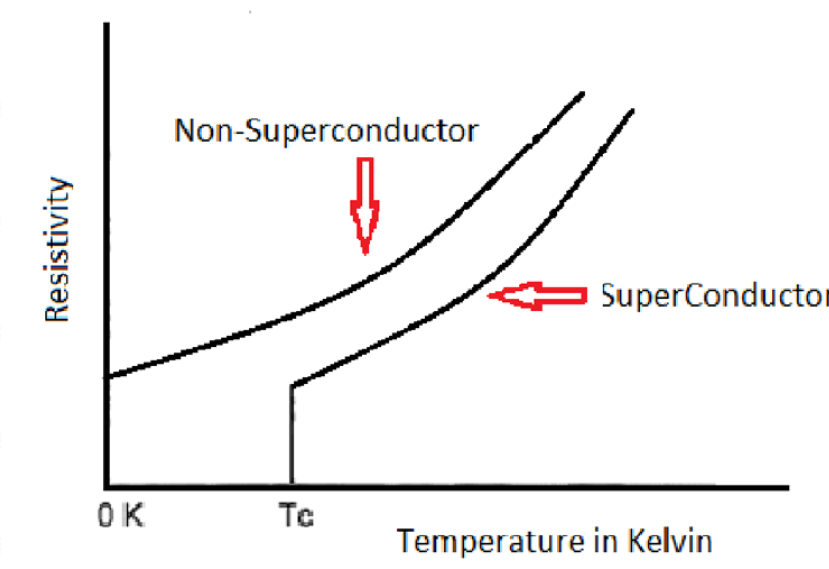


Figure 2. Resistance versus temperature

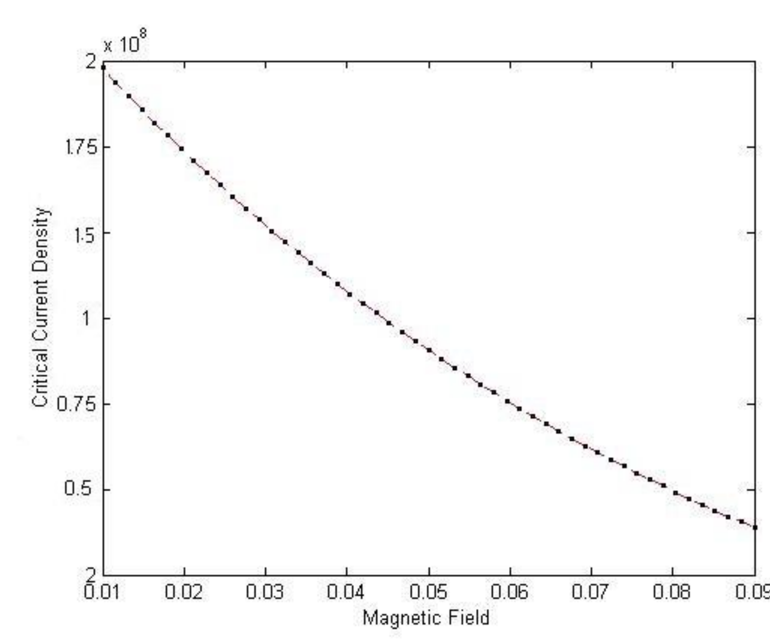


Figure 3. Transverse magnetic field versus J_c

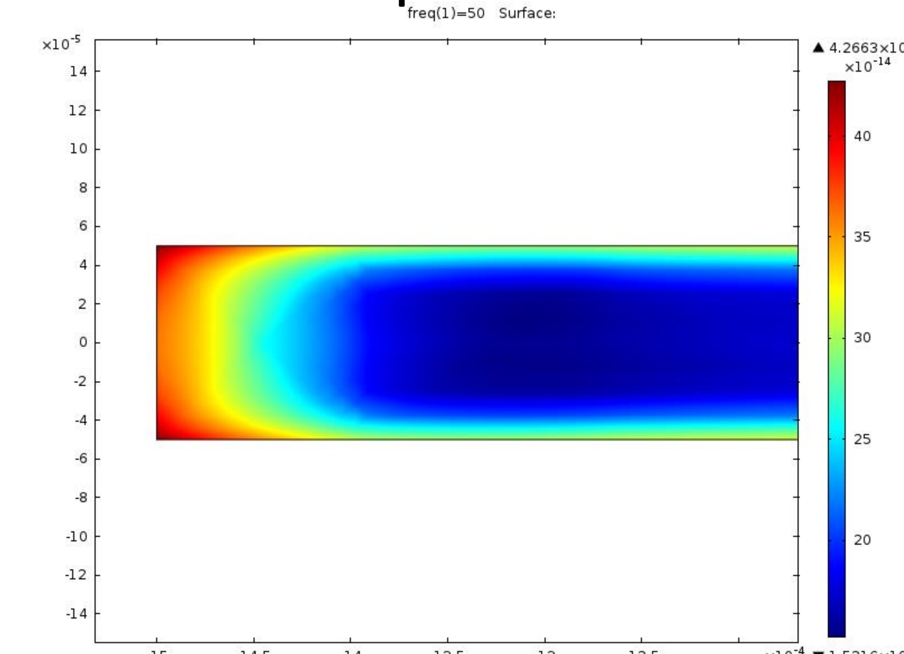


Figure 4. Current Density

Conclusions:

This paper deals with a numerical modelling technique based on finite elements method for computing magnetic field and current density distributions in high temperature Superconducting (HTS) tapes. The model is developed using the T-Ω formulation for which the degree of freedom (DOF) and the CPU time decreased considerably in AC losses analysis, and it is also observe that T-Ω formulation give better convergence results with iteration methods than the other formulation.

References:

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