COMSOL CONFERENCE
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Bloch Waves in an Infinite Periodically Perforated Sheet

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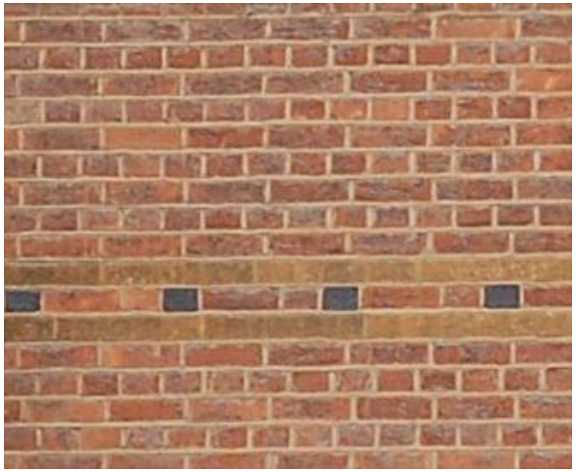
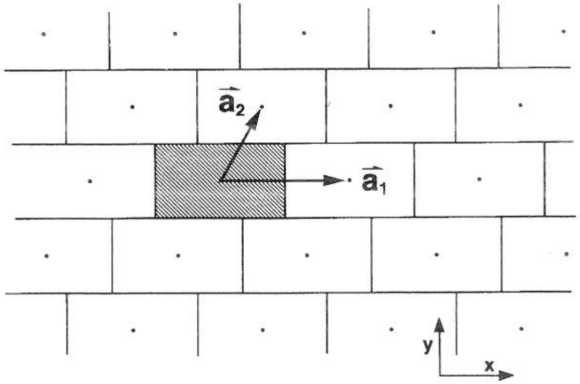
Background

Building Acoustics: Masonry Walls



Theoretical modelling (1988ff.):

- infinite 2D-periodic structures
- analytical approach
- structure-borne sound propagation
- transmission loss



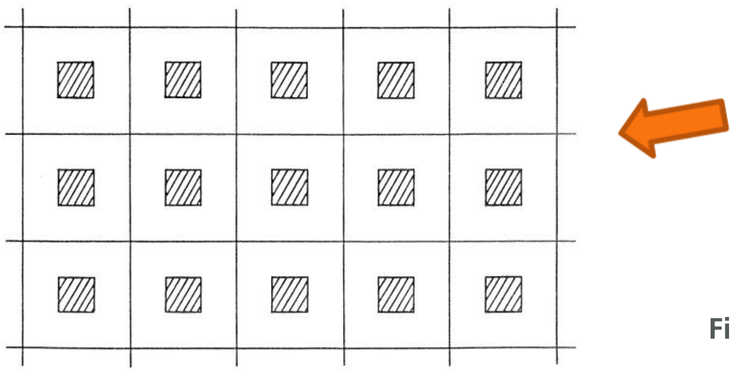
Focus on

- low frequencies
- homogenization
- structure-borne energy and intensity

Background

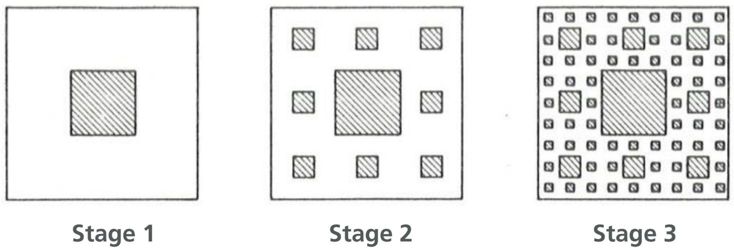
Back then: **Fractal structures in vogue**

2D periodic structure with Sierpinski Carpet unit cell:



simplest example

similar to masonry case



P. Sheng and R. Tao:
First-principles approach for effective elastic-moduli calculation:
Application to continuous fractal structure.
Physical Review B 31 (1985) 6131-6133

Back then: complicated analytical calculations
numerical evaluation of large linear systems of equations

Today: **COMSOL**

Outline

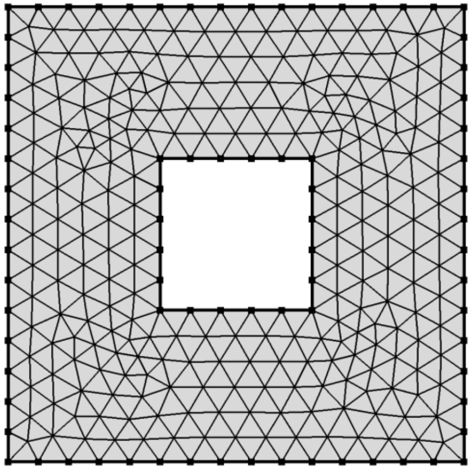
Fraunhofer IBP

1 COMSOL Model Setup

COMSOL Blog

'Modeling Phononic Band Gap Materials and Structures'

Nagi Elabbasi | February 10, 2016



Unit Cell

Essential feature

Floquet Periodicity in 2D

(orthogonal Bravais lattices only – so far)

Specify Bloch wave vector

→ Solve eigenvalue problem

→ Parameter study

→ Generate band structure

→ Analyze Bloch waves

2 COMSOL Results

2.0 Input Parameters

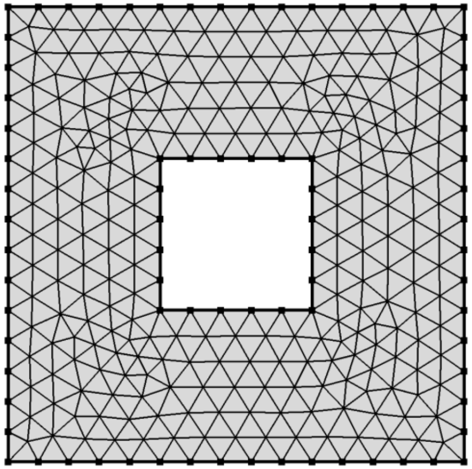
L_{uc} = 3 cm unit cell size

$L_{hole} = L_{uc}/3$ hole size

E = 10 MPa elastic material

$\nu = 1/6$

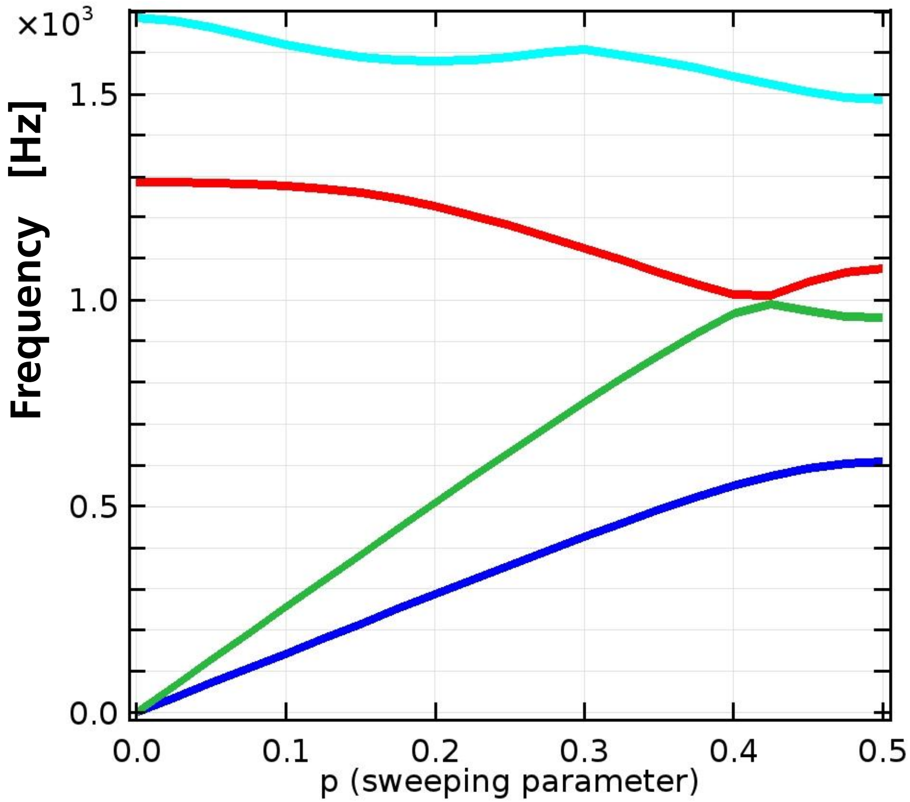
$\rho = 1500$ kg m^{-3}



Unit Cell

2 COMSOL Results

2.1 Band structure for Bloch wave vectors along x-direction (0°)



Normalized wavenumber

optical branches

longitudinal (mainly)

acoustic branches

transversal (mainly)

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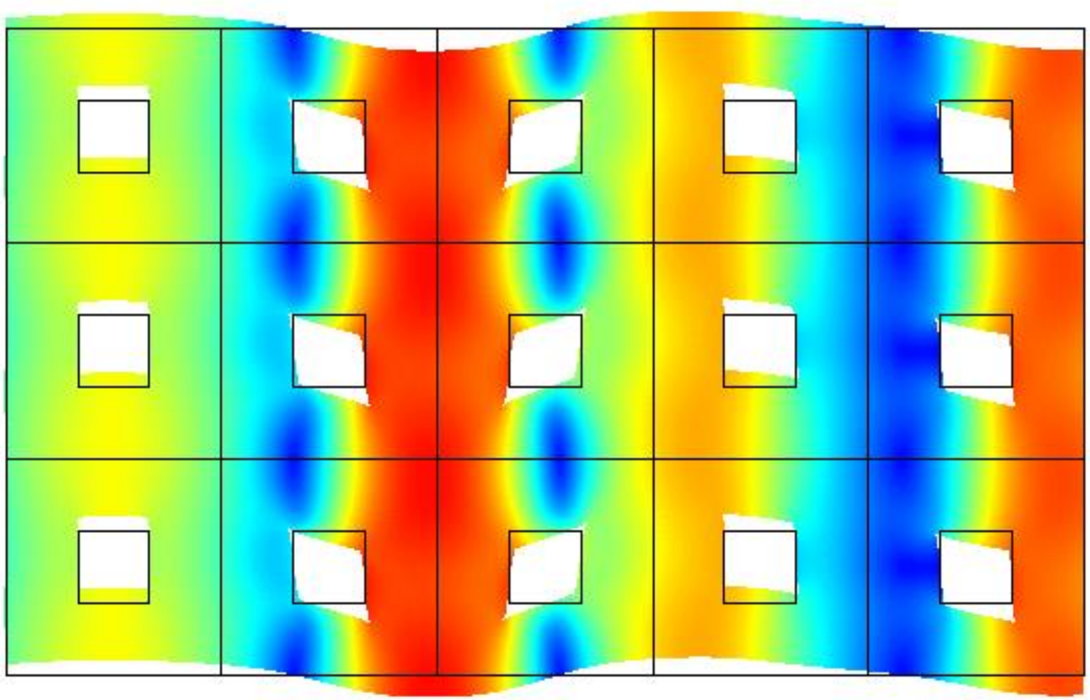
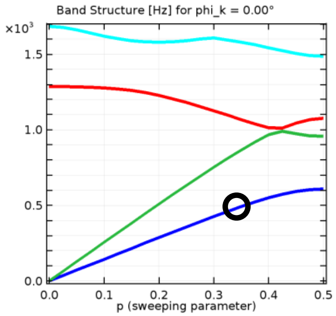
2 COMSOL Results

2.2 Bloch wave (0°): blue branch ≈ transversal

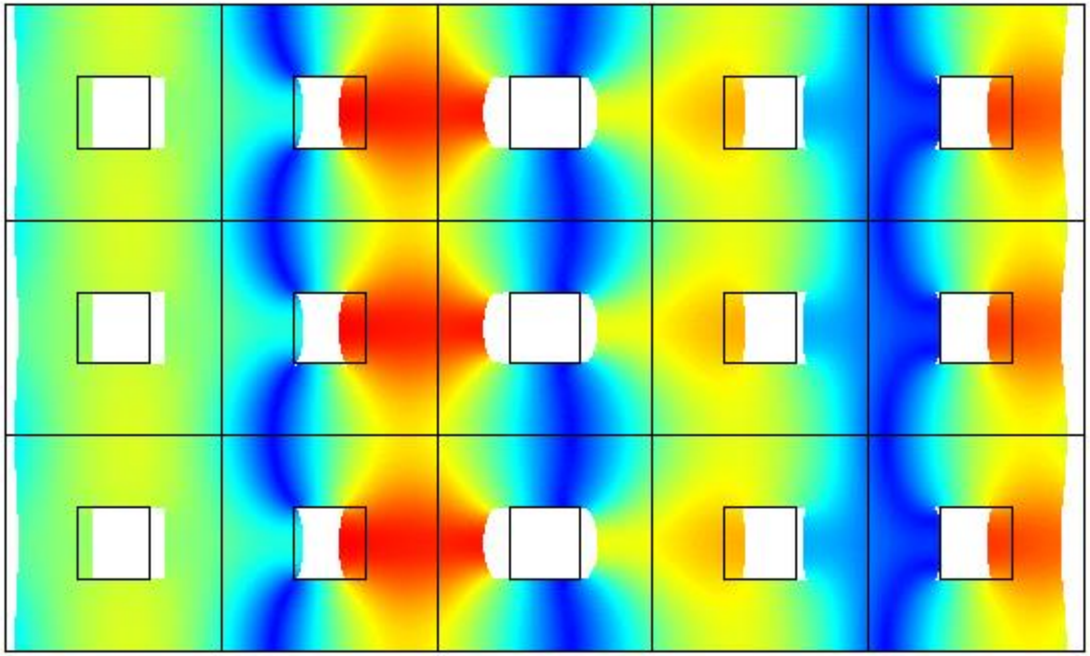


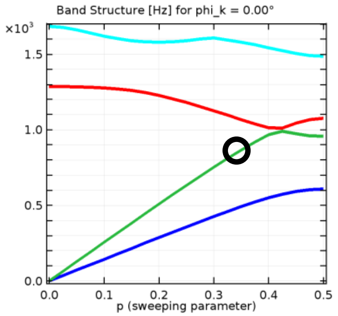


p = 0.35 , $\lambda_{Bloch} \approx 2.9\ L_{uc}$, 491 Hz

2 COMSOL Results

2.2 Bloch wave (0°): green branch ≈ longitudinal





p = 0.35 , $\lambda_{Bloch} \approx 2.9\ L_{uc}$, 866 Hz

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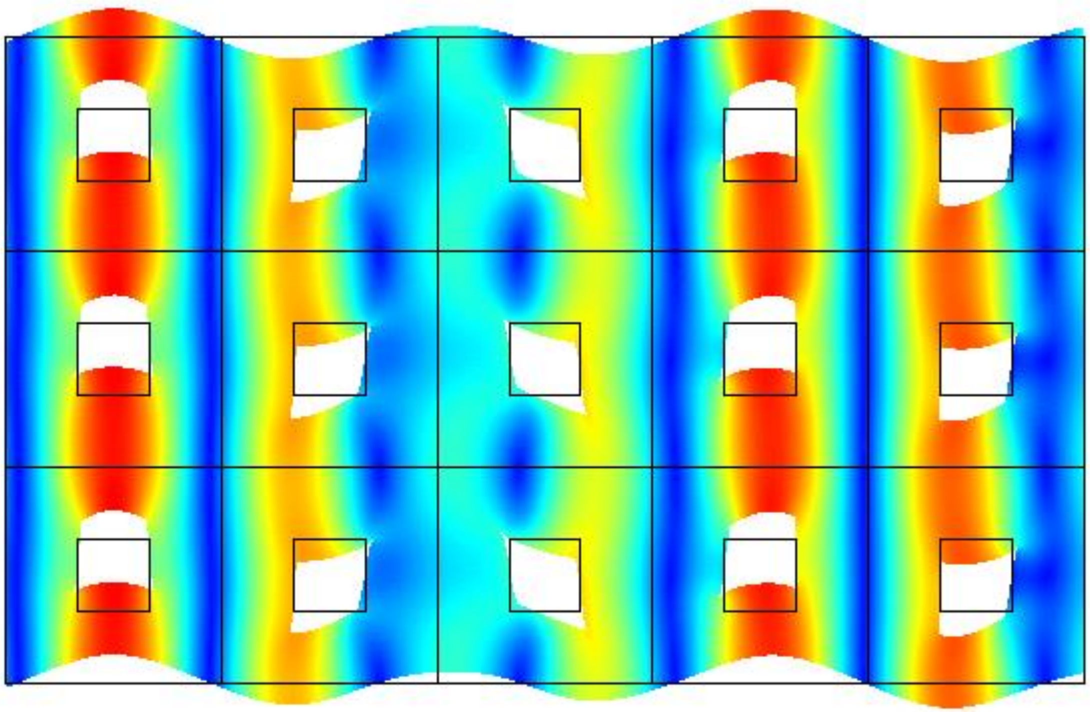
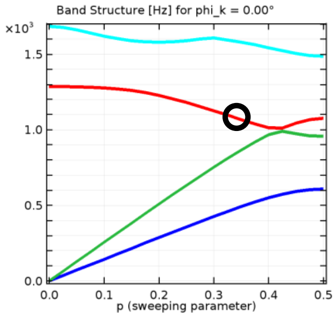
2 COMSOL Results

2.2 Bloch wave (0°): red branch





p = 0.35 , $\lambda_{Bloch} \approx 2.9\ L_{uc}$, 1068 Hz

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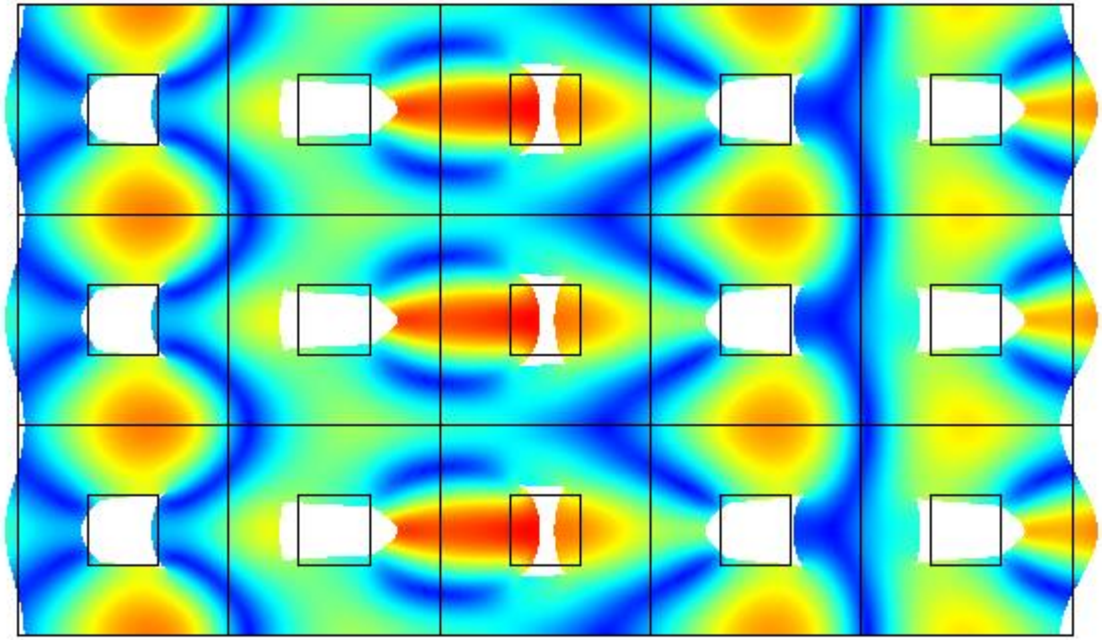
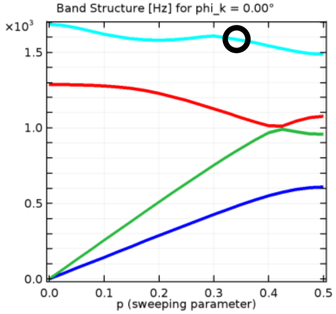
2 COMSOL Results

2.2 Bloch wave (0°): light blue branch

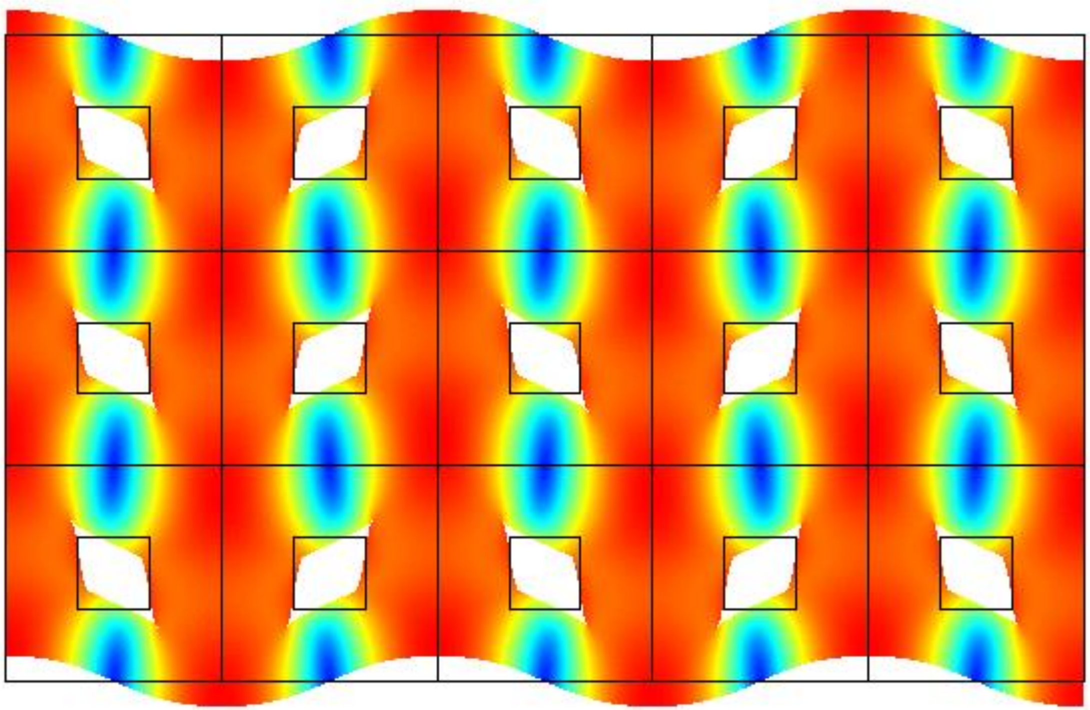


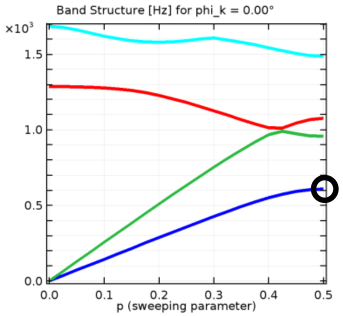


p = 0.35 , $\lambda_{Bloch} \approx 2.9\ L_{uc}$, 1579 Hz

2 COMSOL Results

2.2 Standing Bloch wave (0°): blue branch

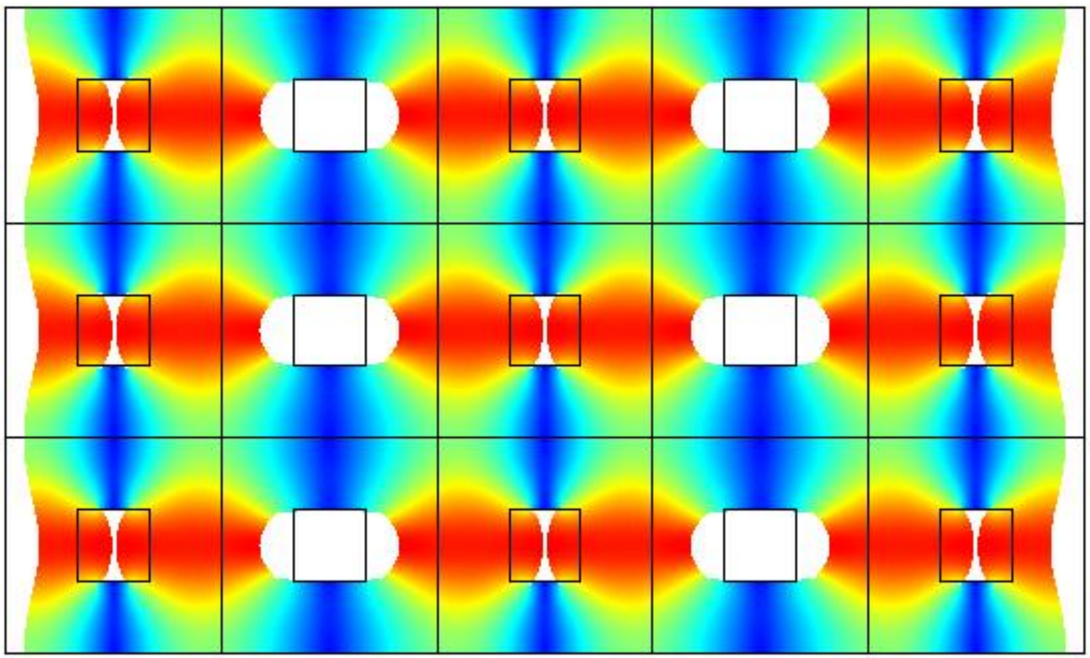


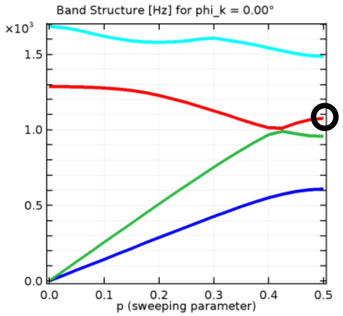


p = 0.5 , $\lambda_{Bloch} = 2\ L_{uc}$, 609 Hz

2 COMSOL Results

2.2 Standing Bloch wave (0°): red ? green! branch

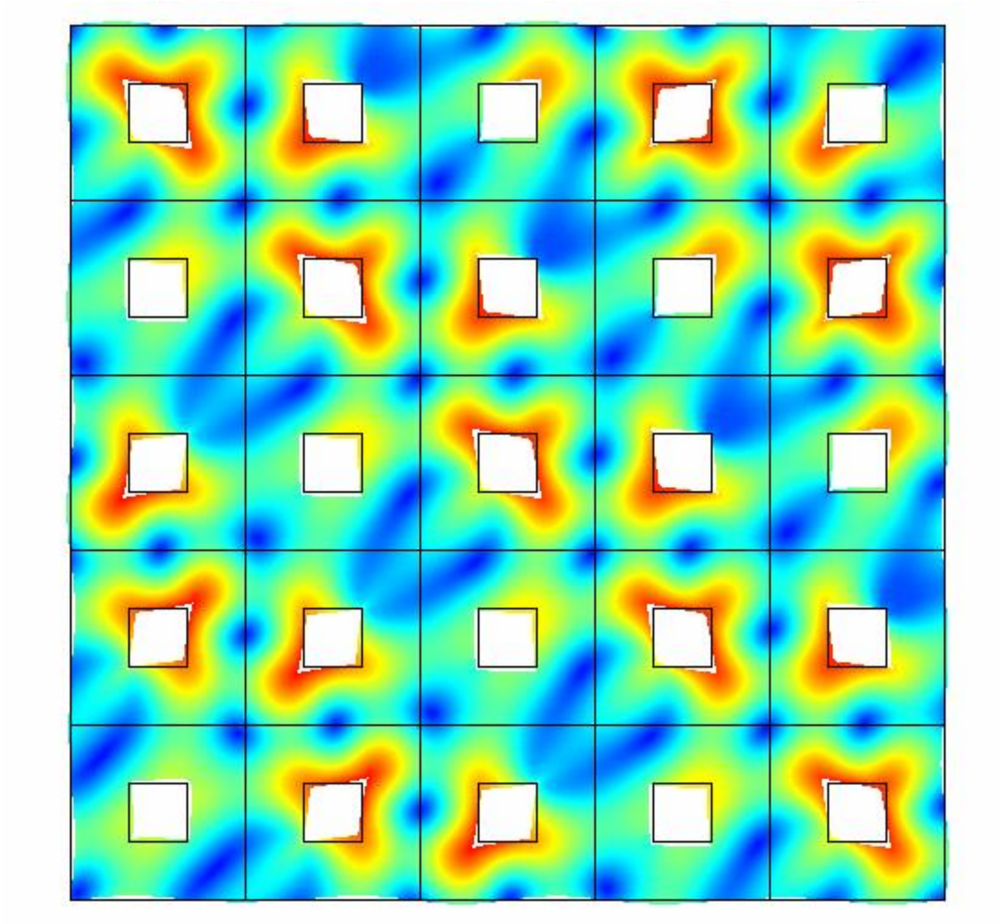




p = 0.5 , $\lambda_{Bloch} = 2\ L_{uc}$, 1076 Hz

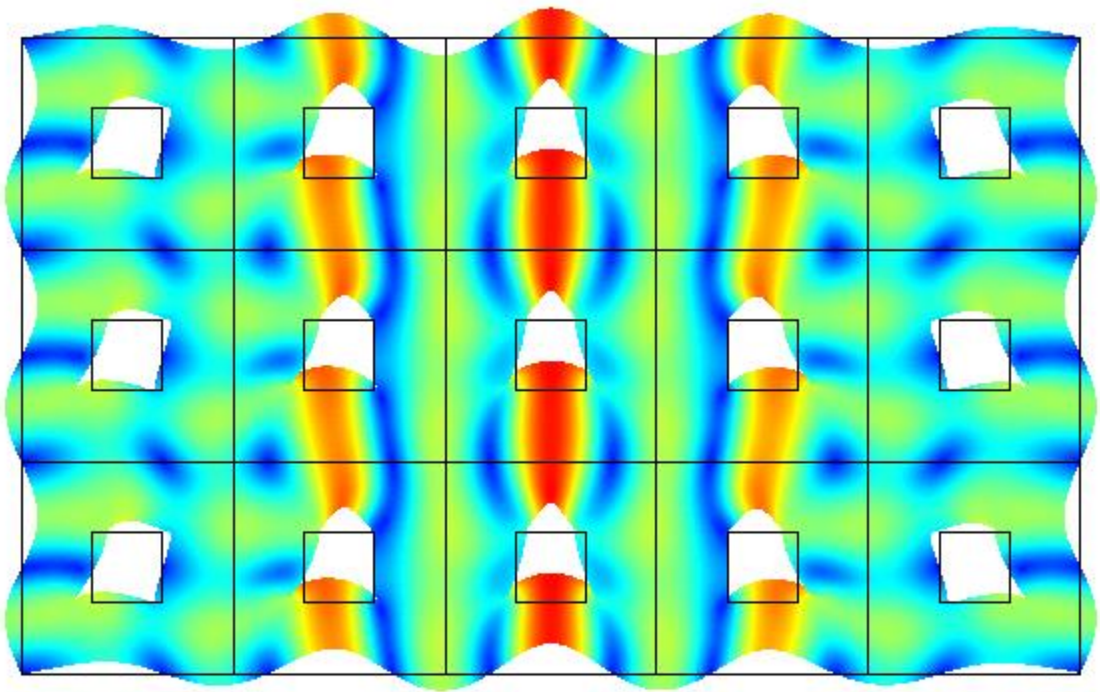
2 COMSOL Results

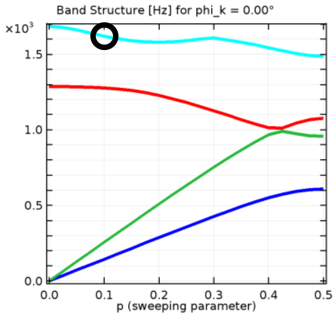
2.2 Bloch wave (45°)



2 COMSOL Results

2.2 Bloch wave (0°): light blue branch





p = 0.1 , $\lambda_{Bloch} = 10\ L_{uc}$, 1619 Hz

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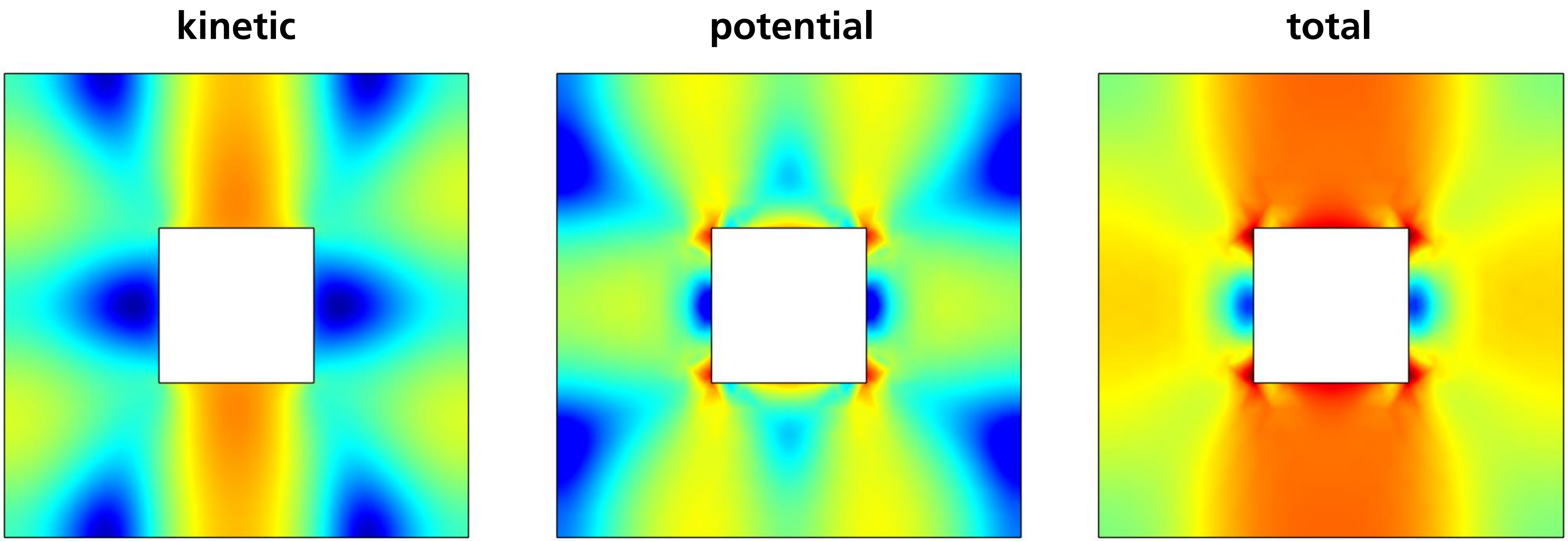
2 COMSOL Results

2.3 Energy Densities (time average: log10(solid.Wk+solid.Wh))

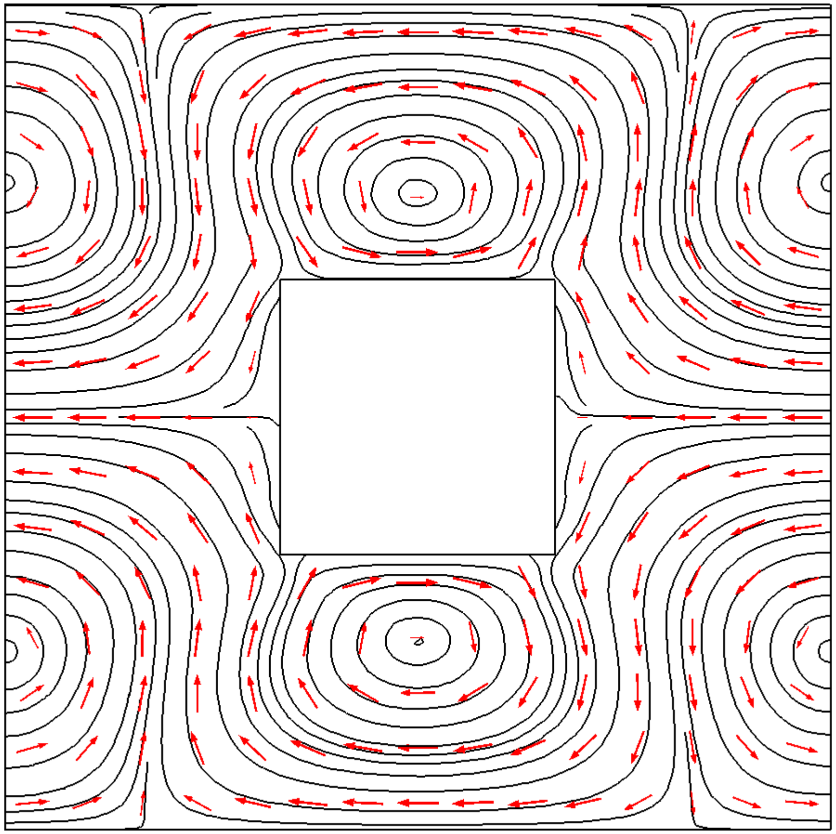
Bloch wave (0°): light blue branch

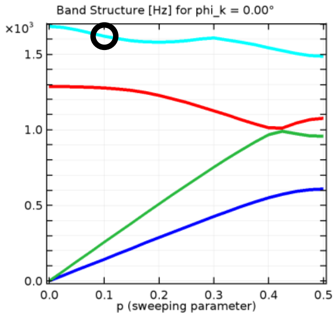


p = 0.1 , λ_{Bloch} = 10 L_{uc} , 1619 Hz

2 COMSOL Results

2.3 Intensity





p = 0.1 , λ_{Bloch} = 10 L_{uc} , 1619 Hz

3 Analytical Results

3.1 Two theorems for Bloch waves in periodic media

Rayleigh's principle

$$\left\langle\left\langle e_{\mathrm{kin}}\right\rangle\right\rangle = \left\langle\left\langle e_{\mathrm{pot}}\right\rangle\right\rangle$$

Equivalence of group velocity and energy velocity

$$\nabla_{\vec{k}_{\mathrm{Bloch}}}\,\omega \equiv \vec{C} = \frac{\left\langle\left\langle \vec{S}\right\rangle\right\rangle}{\left\langle\left\langle e_{\mathrm{tot}}\right\rangle\right\rangle} \equiv \frac{\left\langle \vec{I}\right\rangle}{\left\langle w_{\mathrm{tot}}\right\rangle}$$

W. Maysenhölder, Acustica 78 (1993) 246-249

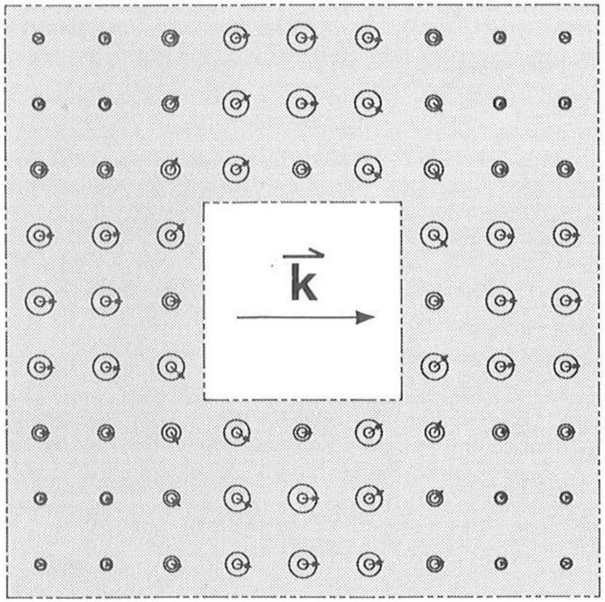
3 Analytical Results

3.2 Low-frequency Approximation

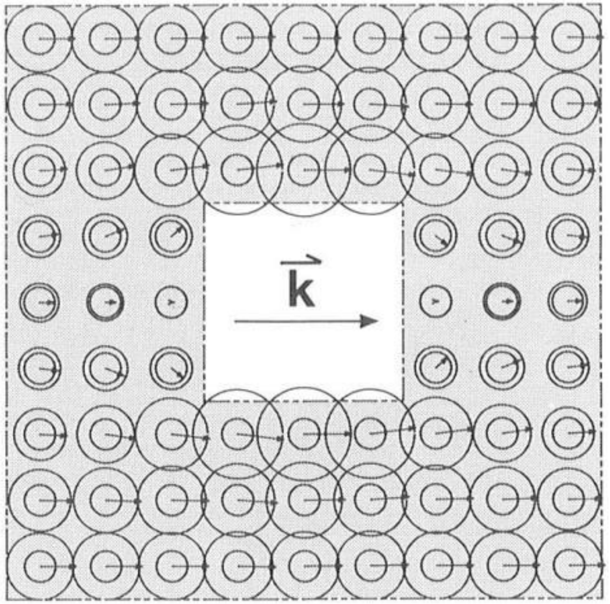
Energy densities: Circles

Intensity: Arrows

local violation of Rayleigh's principle !



blue branch ≈ transversal



green branch ≈ longitudinal

W. Maysenhölder: Körperschallenergie. Hirzel, Stuttgart, 1994

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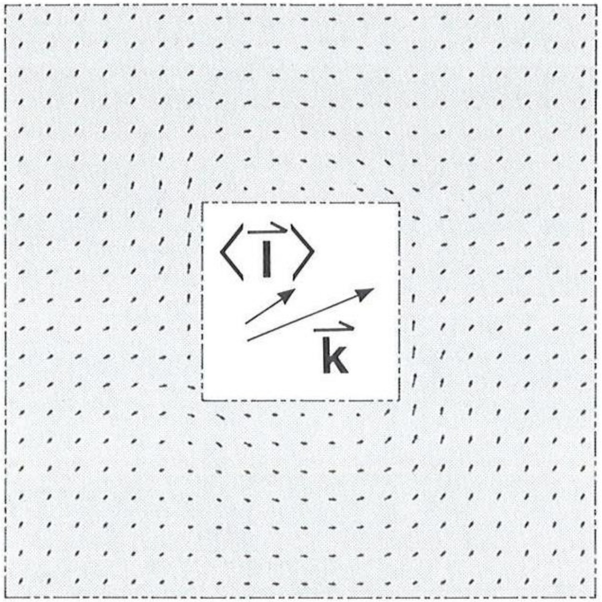
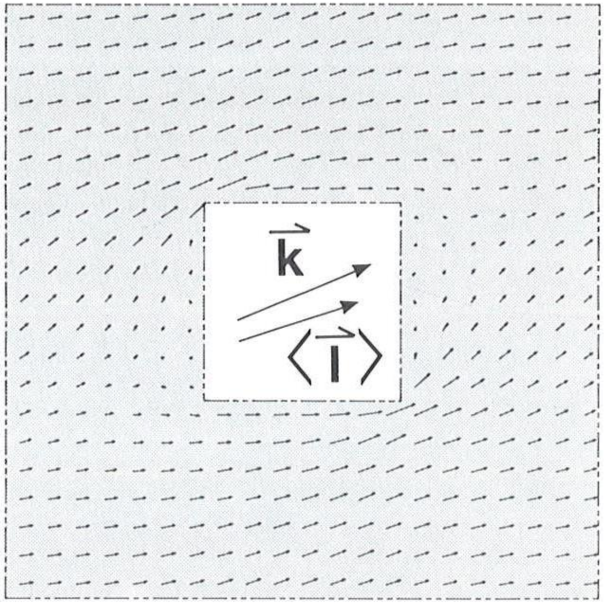
3 Analytical Results

3.2 Low-frequency Approximation

average intensity direction ≠ propagation direction



blue branch ≈ transversal



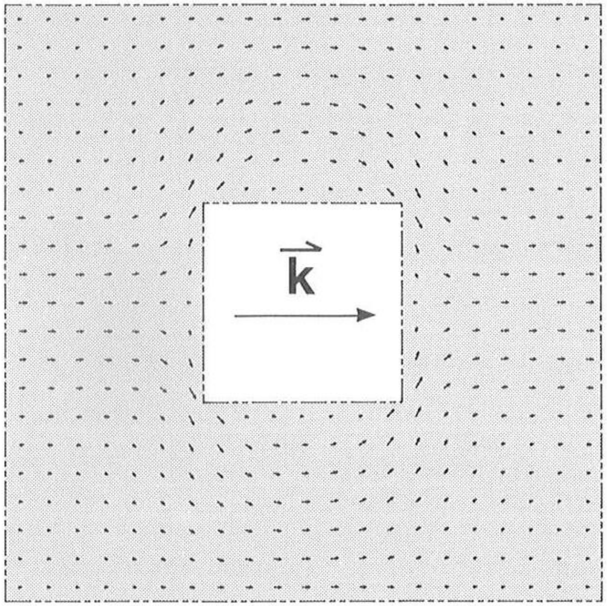
green branch ≈ longitudinal

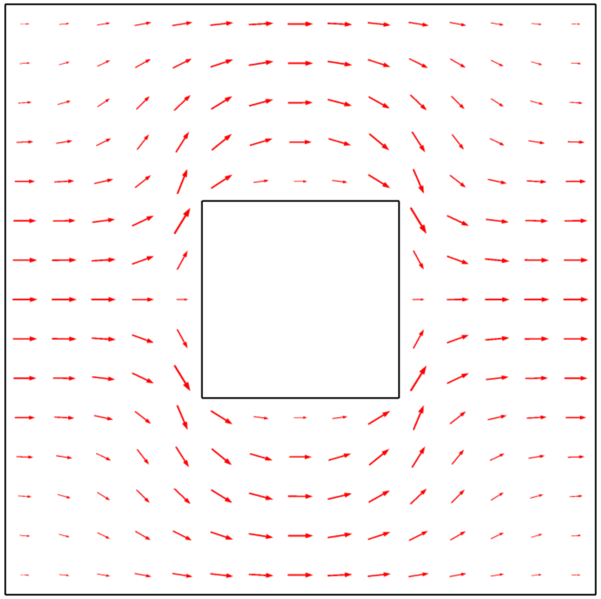
W. Maysenhölder: Körperschallenergie. Hirzel, Stuttgart, 1994

3 Analytical Results

3.2 Low-frequency Approximation

"analytical" versus COMSOL result





blue branch ≈ transversal

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3 Analytical Results

3.3 Exact Homogenization: Equivalent Anisotropic Medium

Anisotropic elastic moduli

from phase velocities

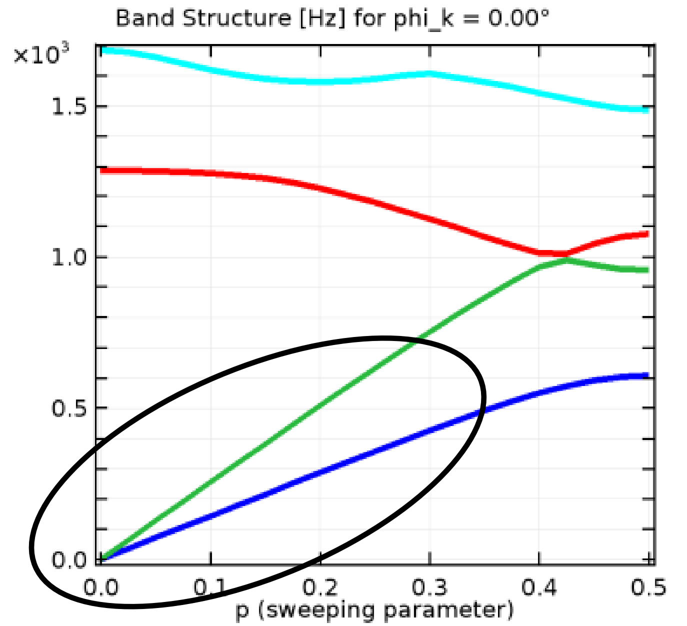
of Bloch waves

at low frequencies

Slowness diagram

Polarization

Intensity

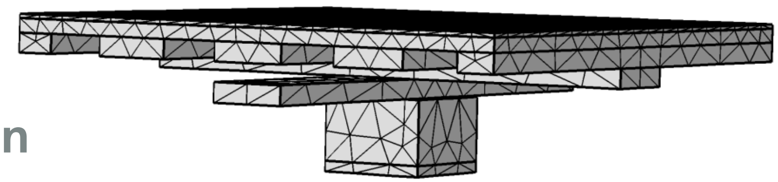


A. N. Norris: Q. J. Mech. Appl. Math. 42 (1989) 413-426

W. Maysenhölder: Körperschallenergie. Hirzel, Stuttgart, 1994

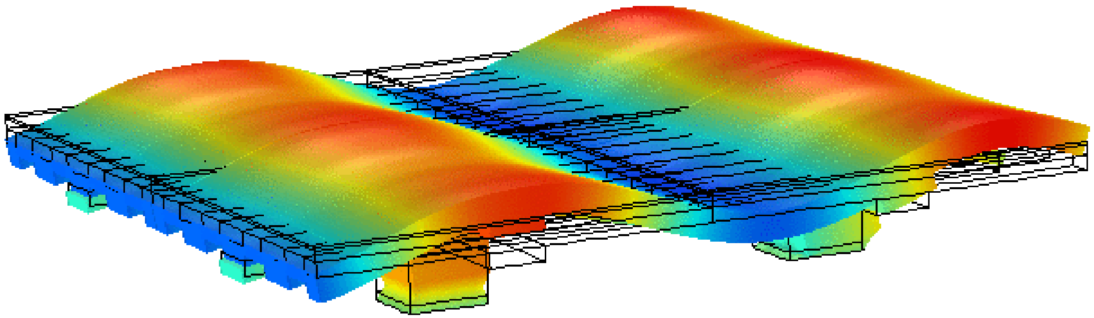
4 Applications

- **Periodic metamaterials**



  - band structure (gap) optimization

- **Floors of sports halls**



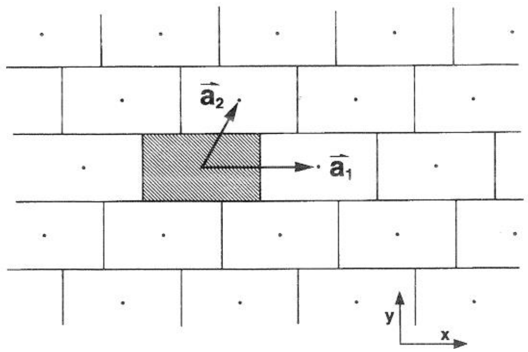
- ...

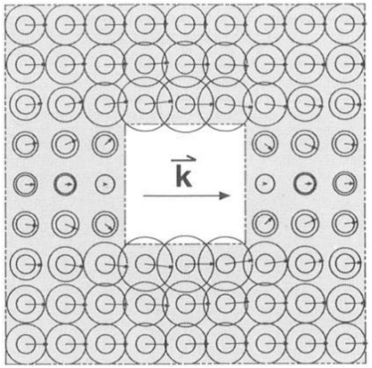
- **Sound transmission loss !**

Conclusion

- **COMSOL: useful, convenient, powerful tool for academic and practical problems**

- **Suggestion: Implementation of**
  - **non-orthogonal Bravais lattices**
  - **"Circle Surface" graphics**





Conclusion

- **COMSOL:** useful, convenient, powerful tool
  for academic and practical problems

- **Suggestion:** Implementation of
  - non-orthogonal Bravais lattices
  - "Circle Surface" graphics

Many thanks to COMSOL Support !

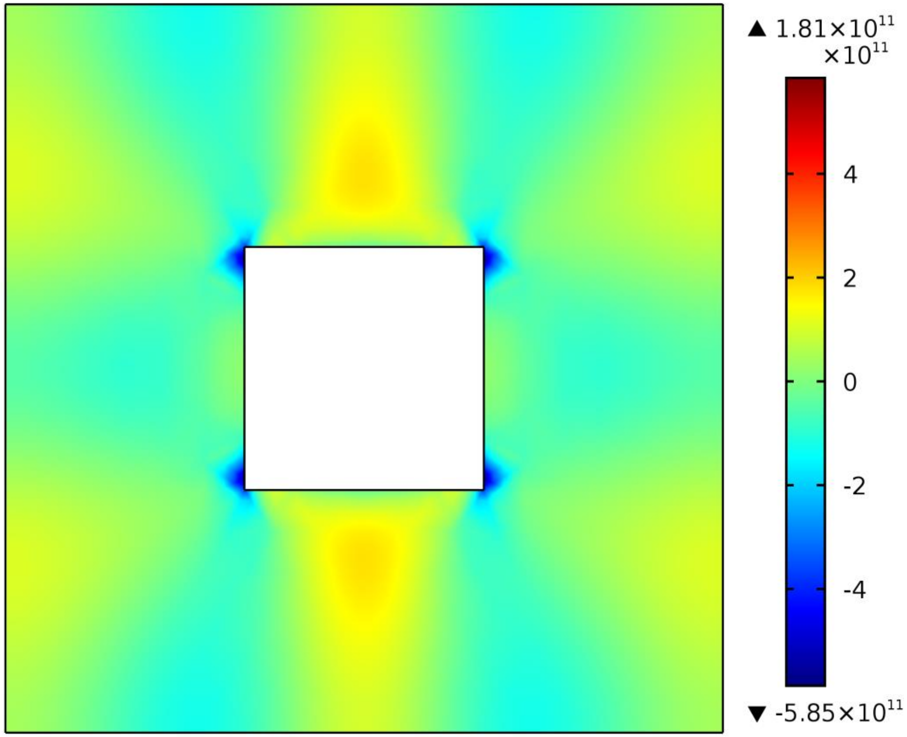
Linus Andersson, Gilles Pigasse, Dennis Cronbach,
Rune Westin, Maria Iuga-Römer, Hinrich Arnoldt
et al.

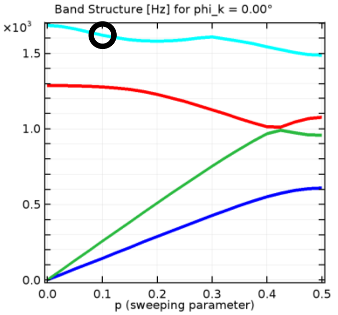
Appendix

Appendix

Lagrange Density (time average $\langle L \rangle_T$: solid.Wk - solid.Wh)

$$\langle\langle L \rangle\rangle = 0$$





p = 0.1 , λ_{Bloch} = 10 L_{uc} , 1619 Hz

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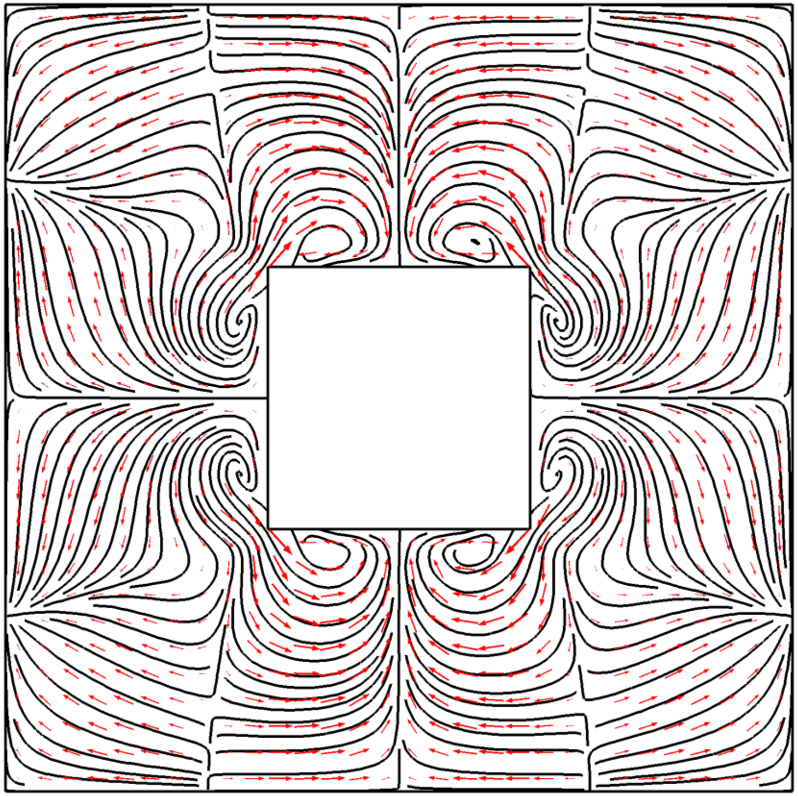
Appendix

Reactive Intensity $\vec{Q}$ (imag(solid.IcomplexX, imag(solid.IcomplexY))

$$\nabla \cdot \vec{Q} =$$

$$2\omega\left(\langle e_{\mathrm{kin}}\rangle_T - \langle e_{\mathrm{pot}}\rangle_T\right)$$

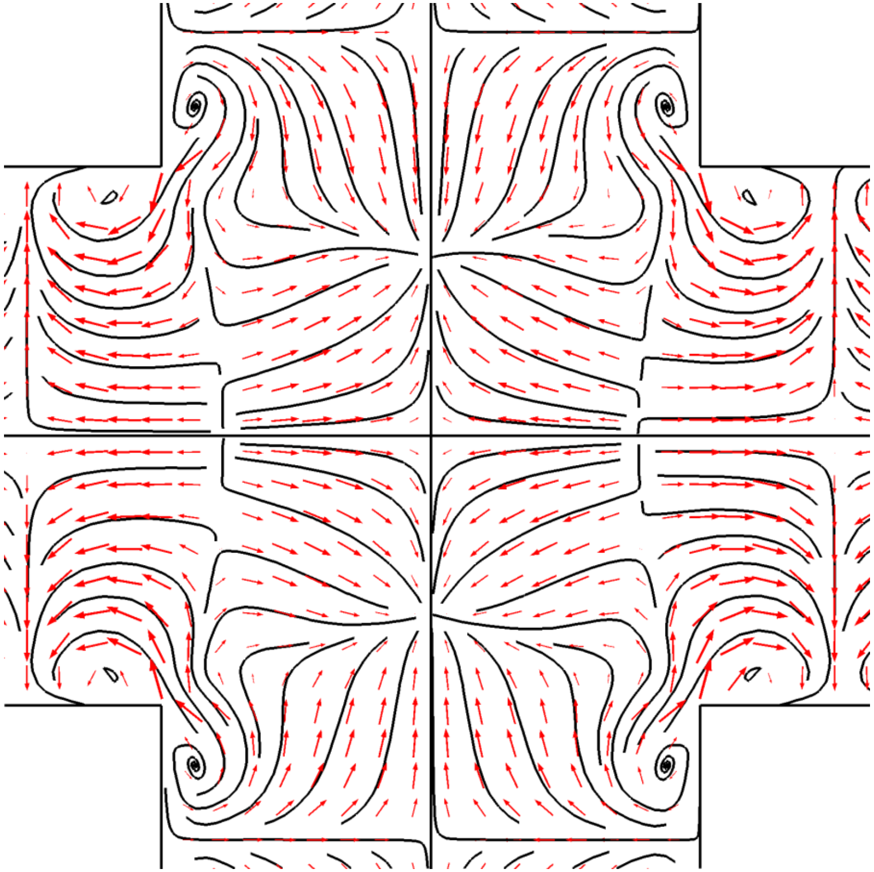
$$= 2\omega\langle L\rangle_T$$



p = 0.1 , λ_{Bloch} = 10 L_{uc} , 1619 Hz

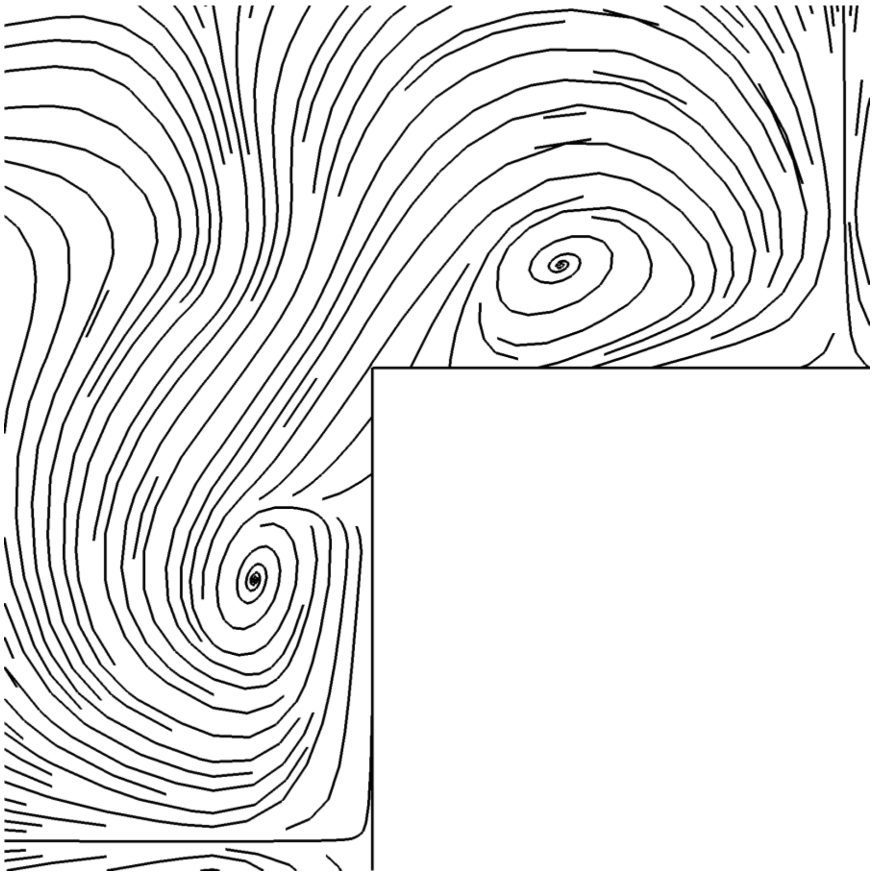
Appendix

Due to symmetry the reactive intensity does not leave the unit cell !



Appendix

Close-up of Vortices



Fraunhofer IBP