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# Simulating Hodgkin-Huxley-like Excitation using Comsol Multiphysics

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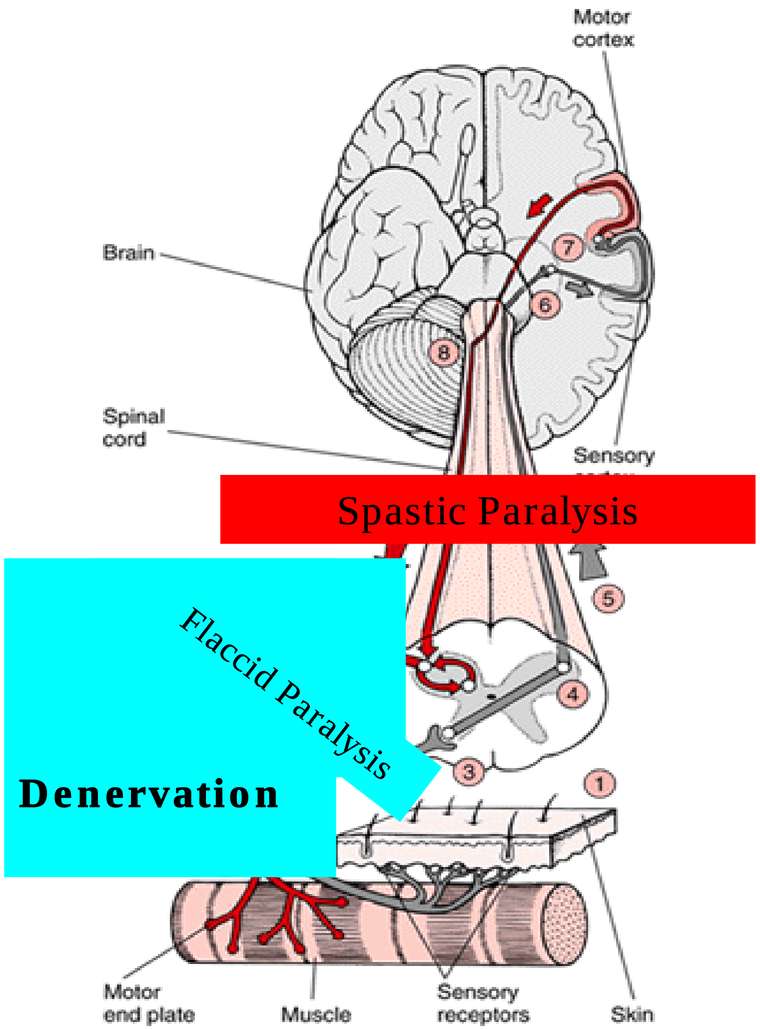
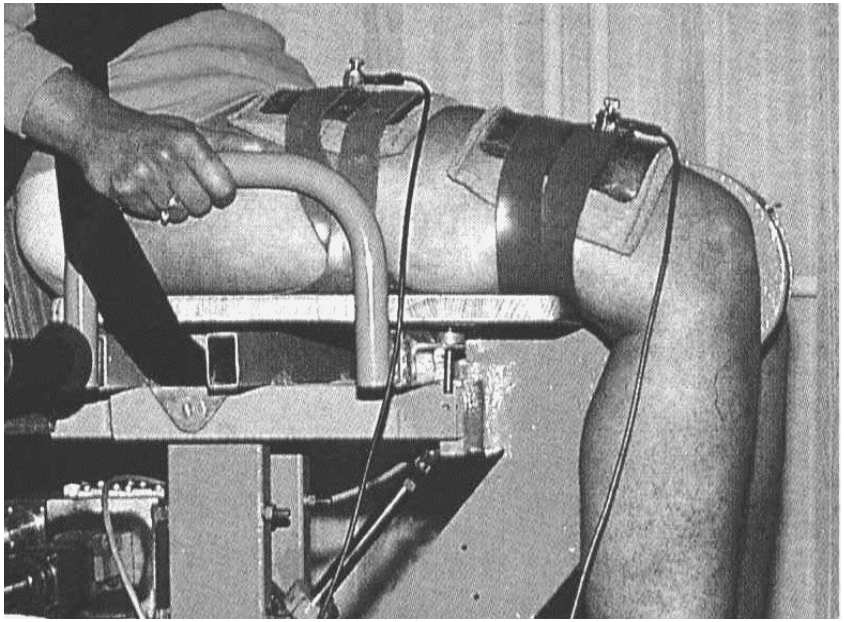
FACHHOCHSCHULE TECHNIKUM WIEN

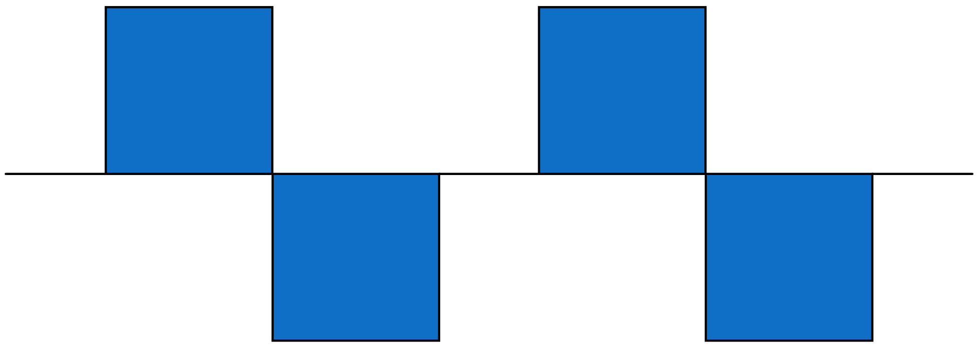
TU VIENNA

# Goal of Simulation – Clinical View




Large Surface Electrodes





Long Impulses

# Goal of Simulation - Modeling View

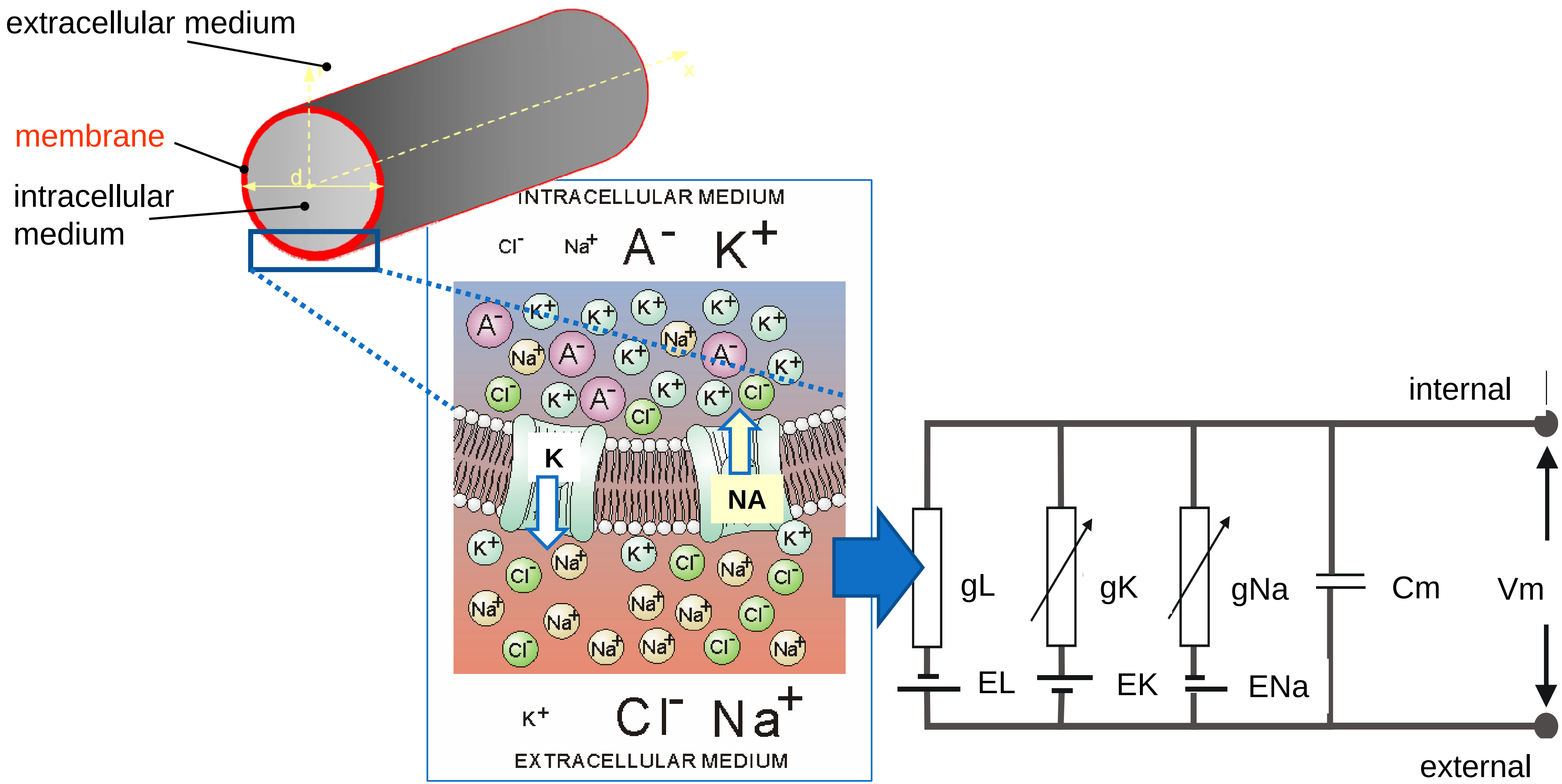
- Model of Single Fiber
  - **Hodgkin-Huxley**
- Electrical Field Distribution
- Coupling
  - Reaction of fiber in external Electrical Field

# History of Hodgkin-Huxley Model

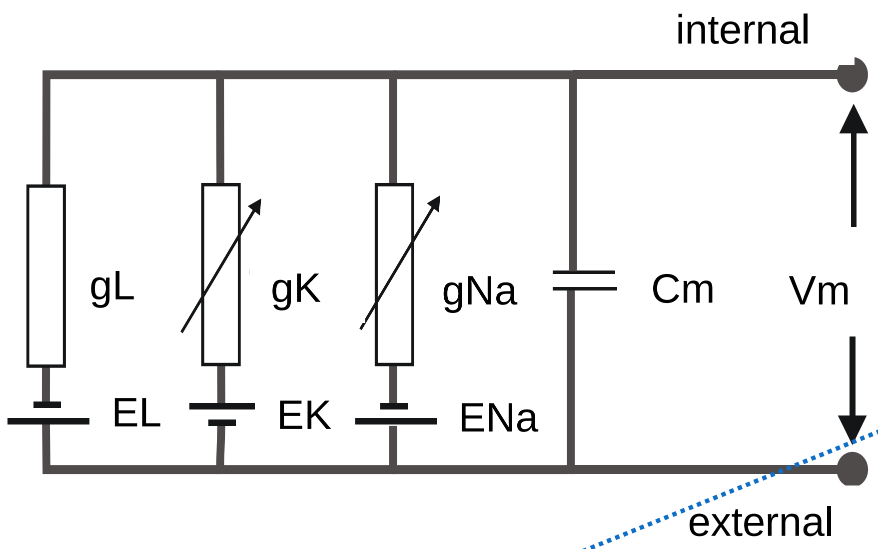
- Hodgkin and Huxley experimented on squid giant axon and explored how the Action Potential is produced in the (nerve) fiber (Journal of Physiology, 1952)

- Hodgkin and Huxley awarded the 1963 Nobel Prize for the model

# The muscle fiber - membrane



# Hodgkin-Huxley PDE




$$C_m \frac{dV}{dt} = I_{ext} + I_{NA} + I_K + I_L$$

$C_m$ …capacitance of membrane

$r$…radius of axon

$\rho$…resistance of intracellular space

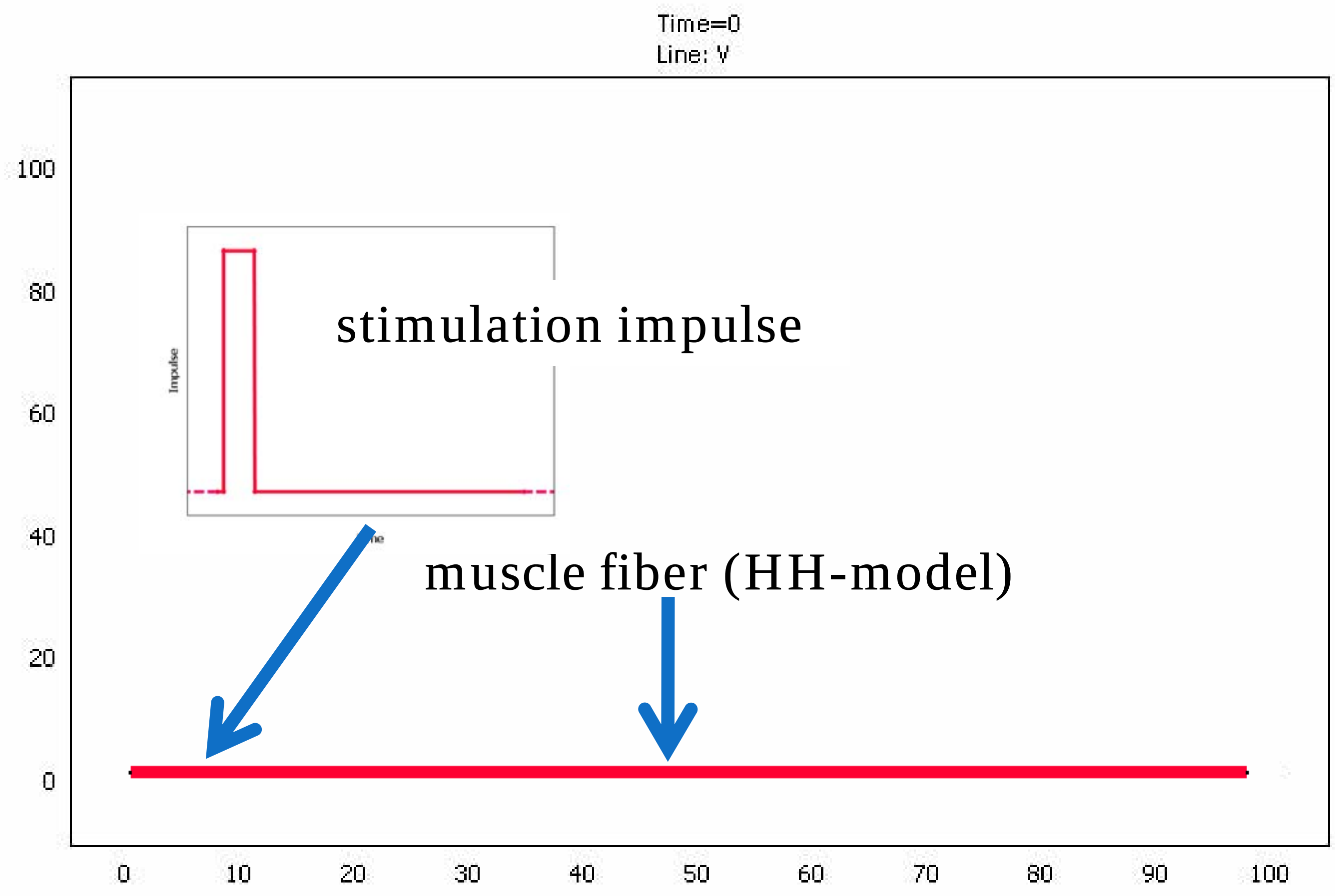
$g_{Na}, g_K, g_L$,…conductance of Natrium, Potassium and Leakage

$$C_m \frac{dV}{dt} = \frac{r}{2\rho}\frac{\partial^2 V}{\partial x^2} - g_{Na} m^3 h (V - V_{Na}) - g_K n^4 (V - V_K) - g_L (V - V_L)$$

$$\frac{\mathrm{dw}}{\mathrm{dt}} = \alpha_w (1 - w) - \beta_w w \text{ (w is m, n or h)}$$

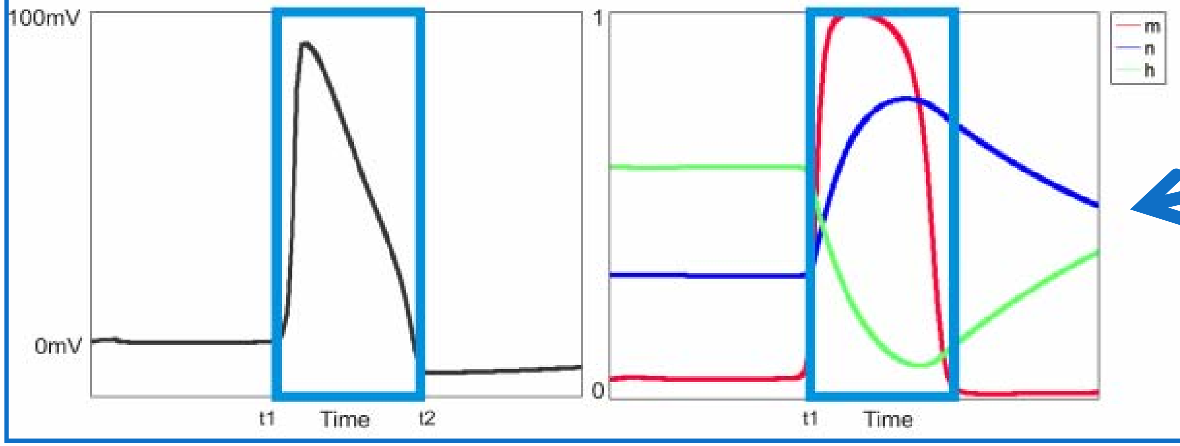
System of
**1 non-linear** and
**3 linear differential** equations

# 1D muscle fiber model



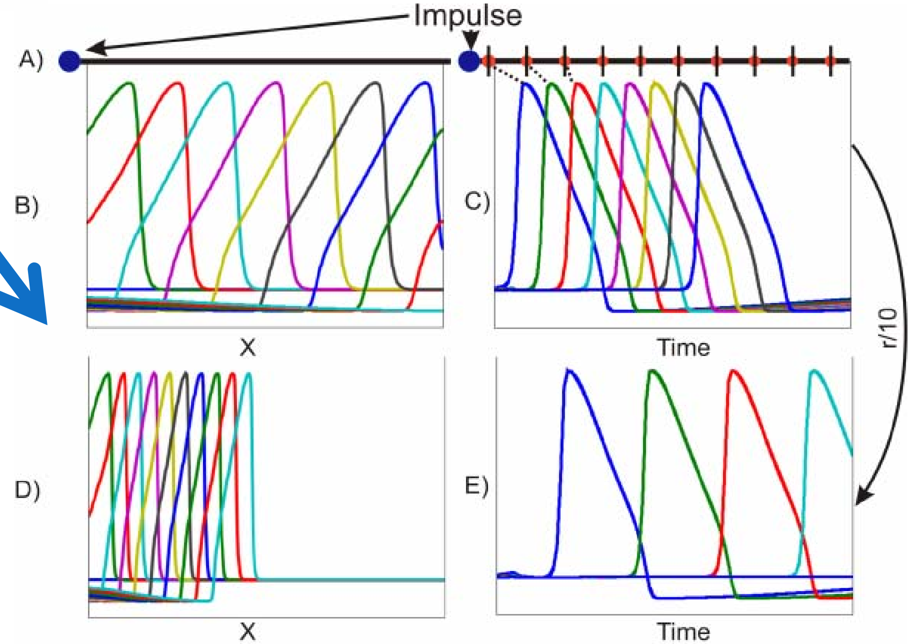

| | |
|---|---|
| App. Mode | PDE – Coeff. Form |
| Dimension | 1D |
| Dep. Variables | 4: V, m, n, h |
| N. of Elements | 200 |
| Solver | Direct (UMFPACK) Time dependent (0:0.1:10) |
| rel. Tolerance | 10e-5 |
| abs. T0lerance | 10e-7 |
| Solution Time | <30 sec |

# 1D muscle fiber



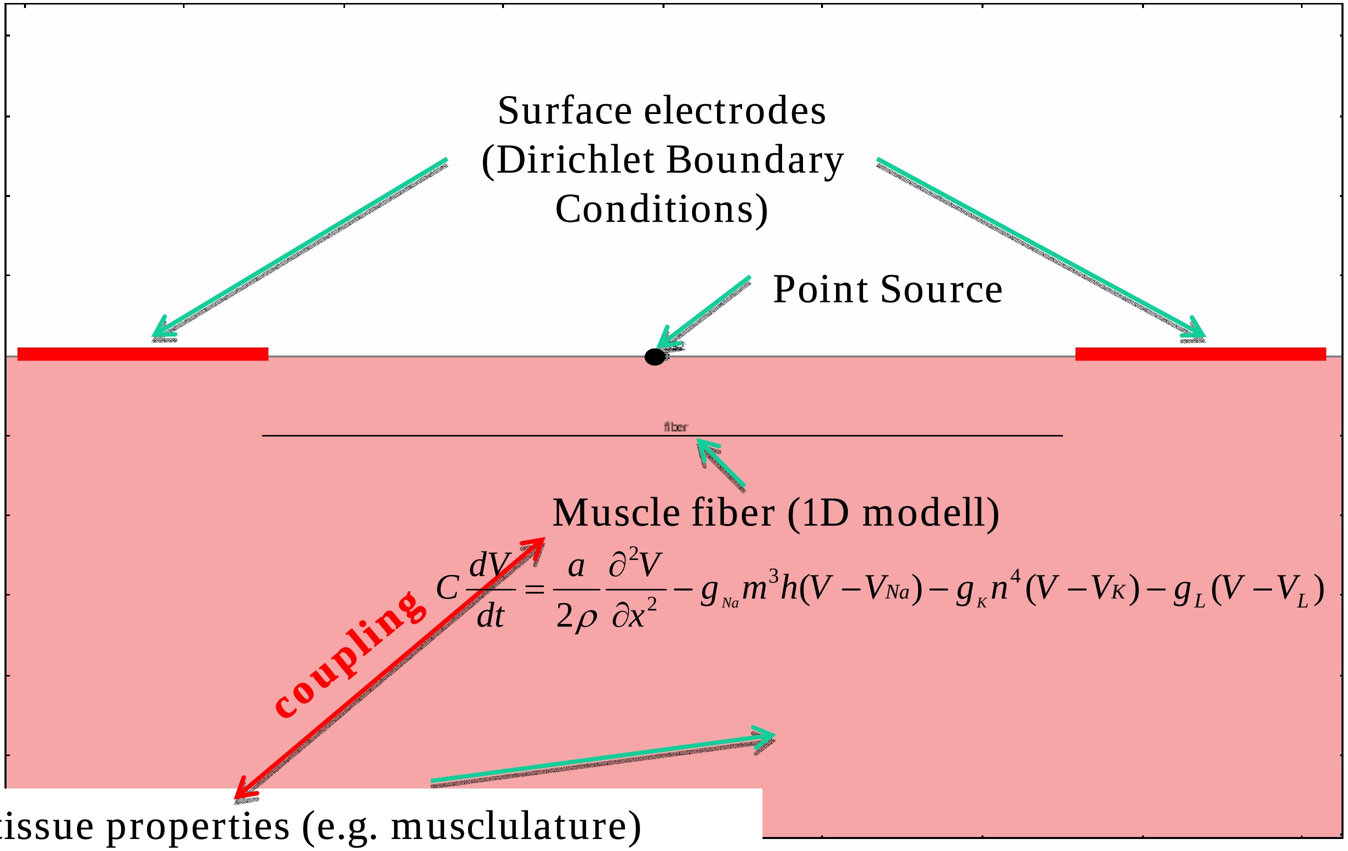

action potential and gating variables

change of radius




Results are comparable to e.g. „Hodgkin AL, Huxley AF, **A quantitative description of membrane current and its application to conduction and excitation in nerve**, J Physiol, **117**, 500-544 (1952)"

# 2D model



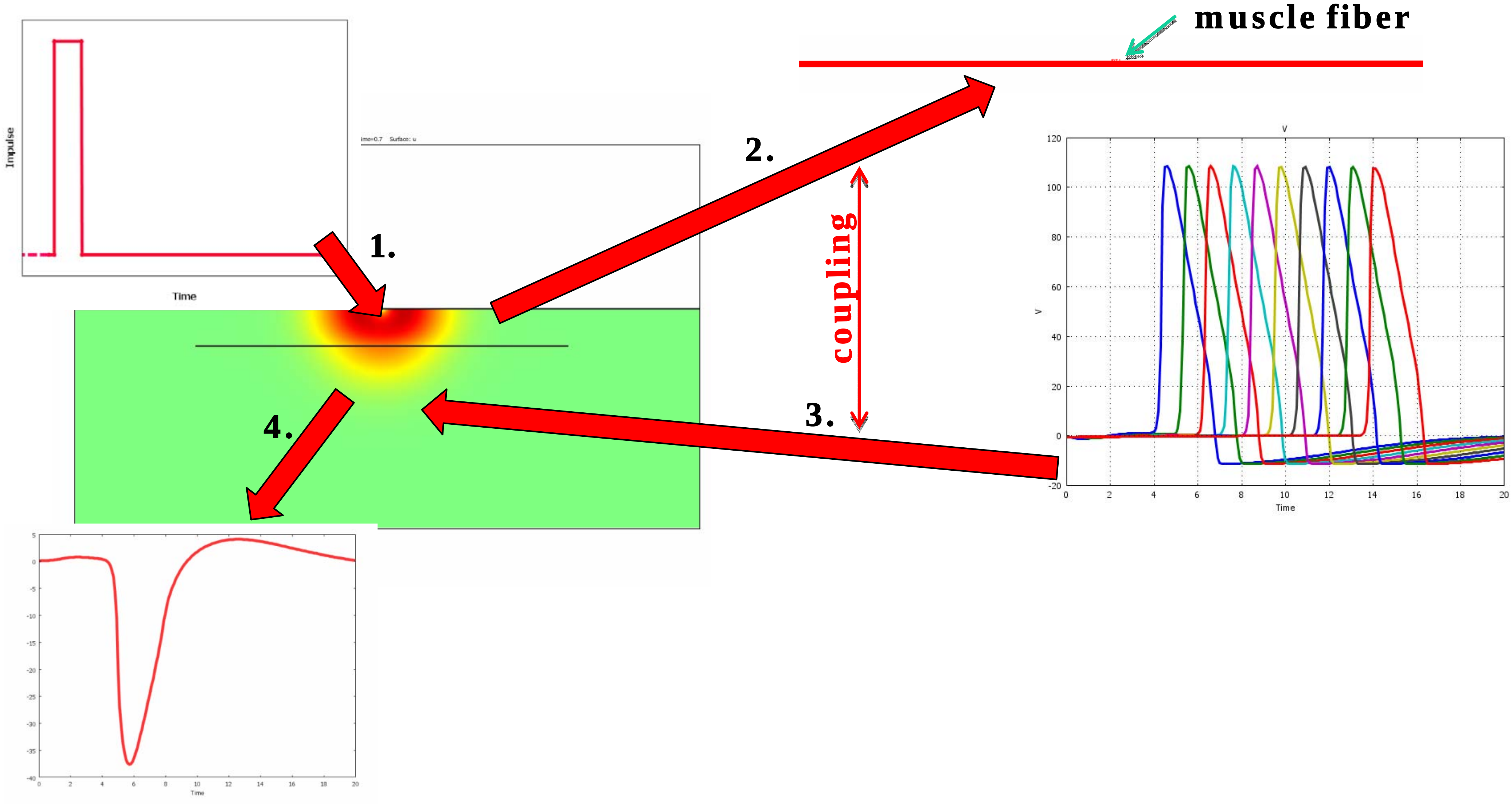

$$C\frac{dV}{dt}=\frac{a}{2\rho}\frac{\partial^2 V}{\partial x^2}-g_{Na}m^3h(V-V_{Na})-g_K n^4(V-V_K)-g_L(V-V_L)$$

tissue properties (e.g. musculature)

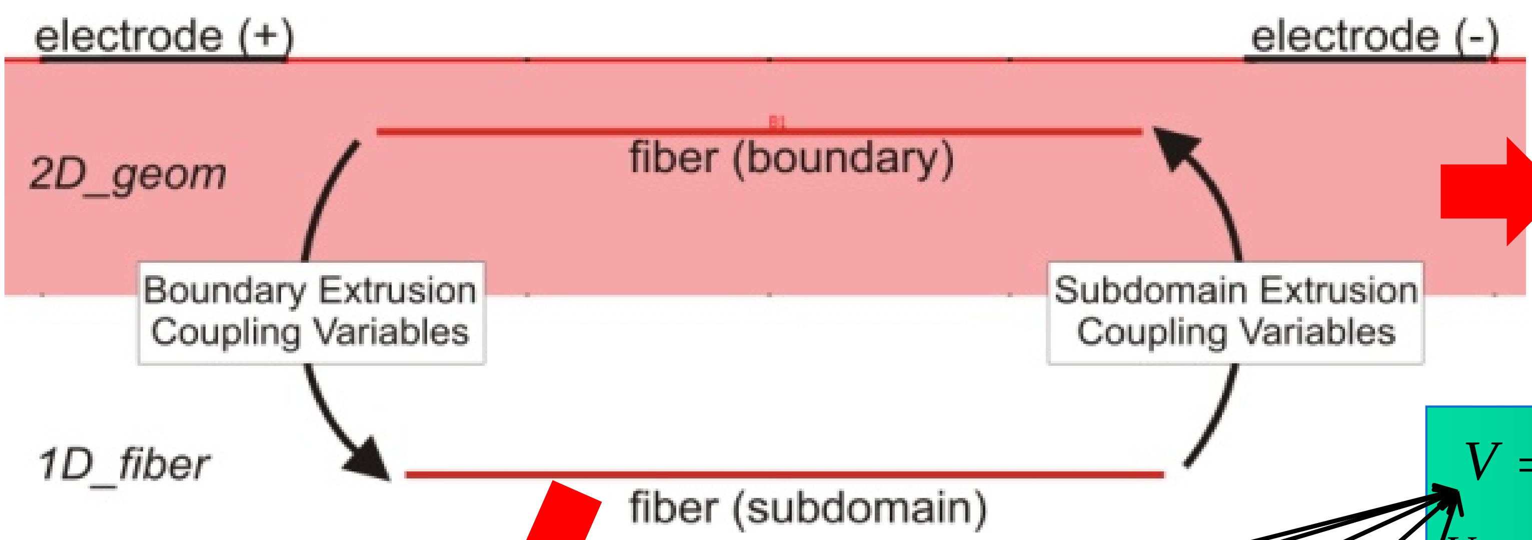
$$-\nabla(\sigma\nabla V_e)-\nabla\left(\varepsilon\nabla\frac{\partial V_e}{\partial t}\right)=0$$

| | |
|---|---|
| App. Mode | PDE – Coeff. Form |
| Dimension | 2D & 1D |
| Dep. Variables | 4 (1D): Vi, m, n, h<br>1 (2D): Ve |
| N. of Elements | 1D: 200<br>2D: ca. 12000 |
| Solver | Direct (UMFPACK)<br>Time dependent (0:0.1:10) |
| rel. Tolerance | 10e-5 |
| abs. T0lerance | 10e-7 |
| Solution Time | <600 sec |

# 2D model – Coupling („real life“)



# 2D model - Coupling („model“)




**2D Area**
**PDE Coeff. Form**

$$-\nabla(\sigma\nabla V_e)-\nabla\left(\varepsilon\nabla\frac{\partial V_e}{\partial t}\right)=0$$

$$V = V_i - V_e - V_{Rrest}$$

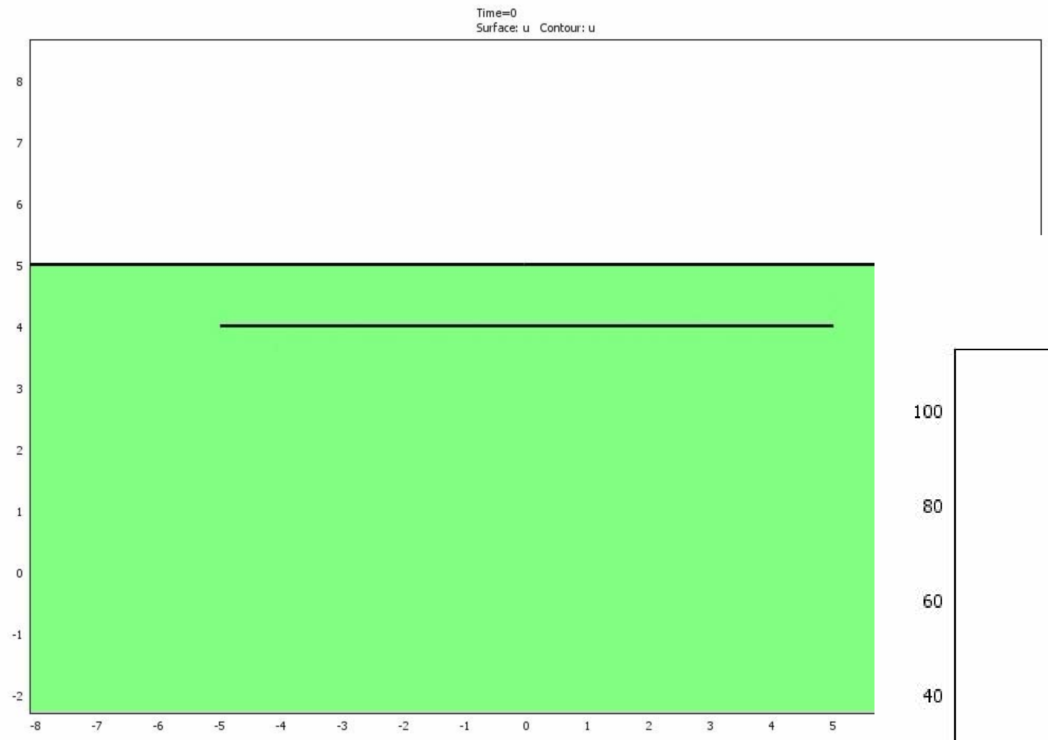
$V_i$.... intracellular Potential

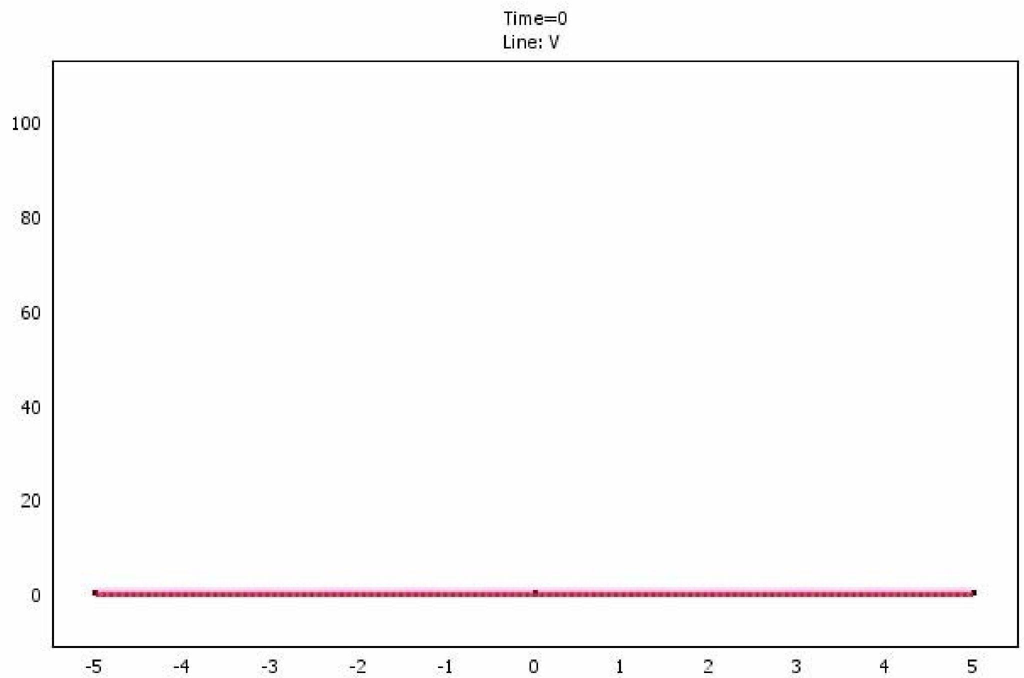
$V_{Rrest}$.... Resting Potential ($\approx -90mV$)

$$C\frac{dV}{dt}=\frac{a}{2\rho}\frac{\partial^2 V}{\partial x^2}-g_{Na}m^3h(V-V_{Na})-g_K n^4(V-V_K)-g_L(V-V_L)$$

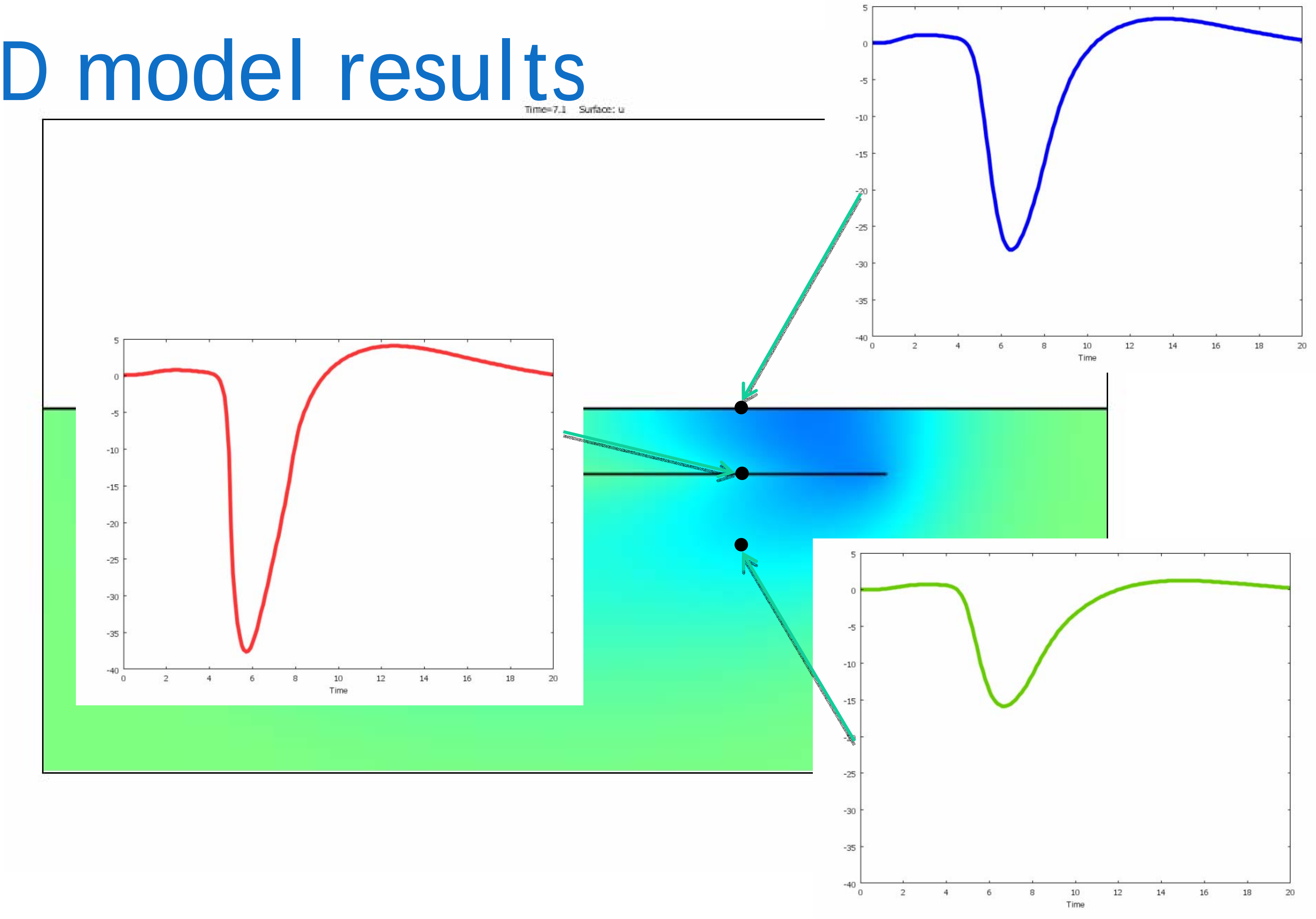
**1D Fiber**
**HH-PDE**

# 2D model – Animation

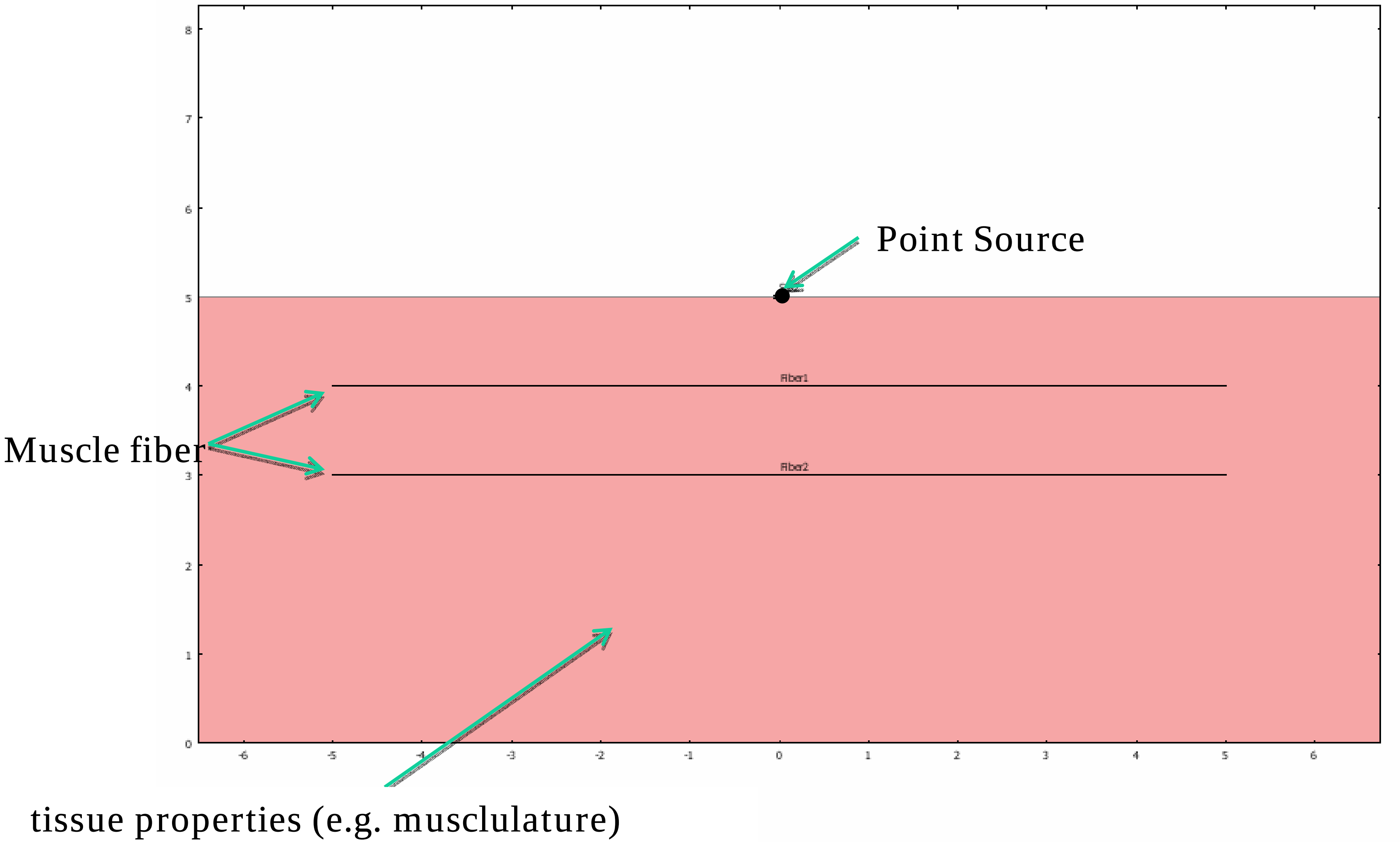


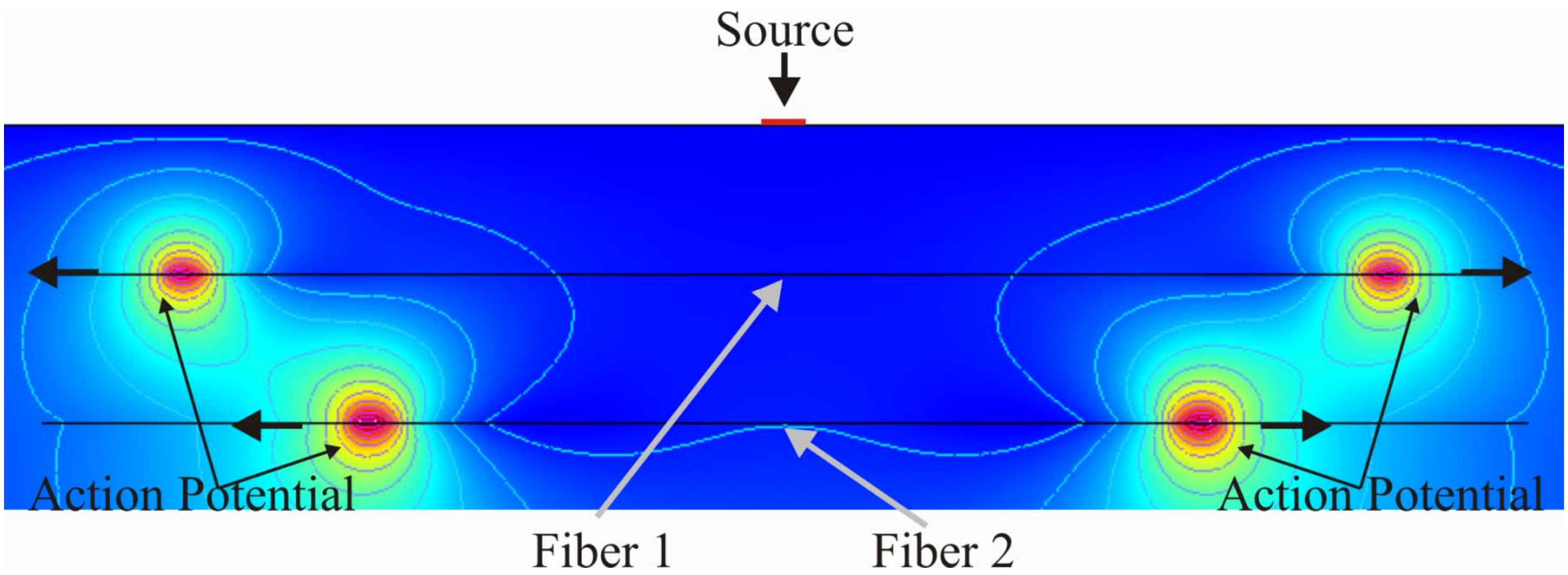
# 2D model results



# 2D model – 2 fibers

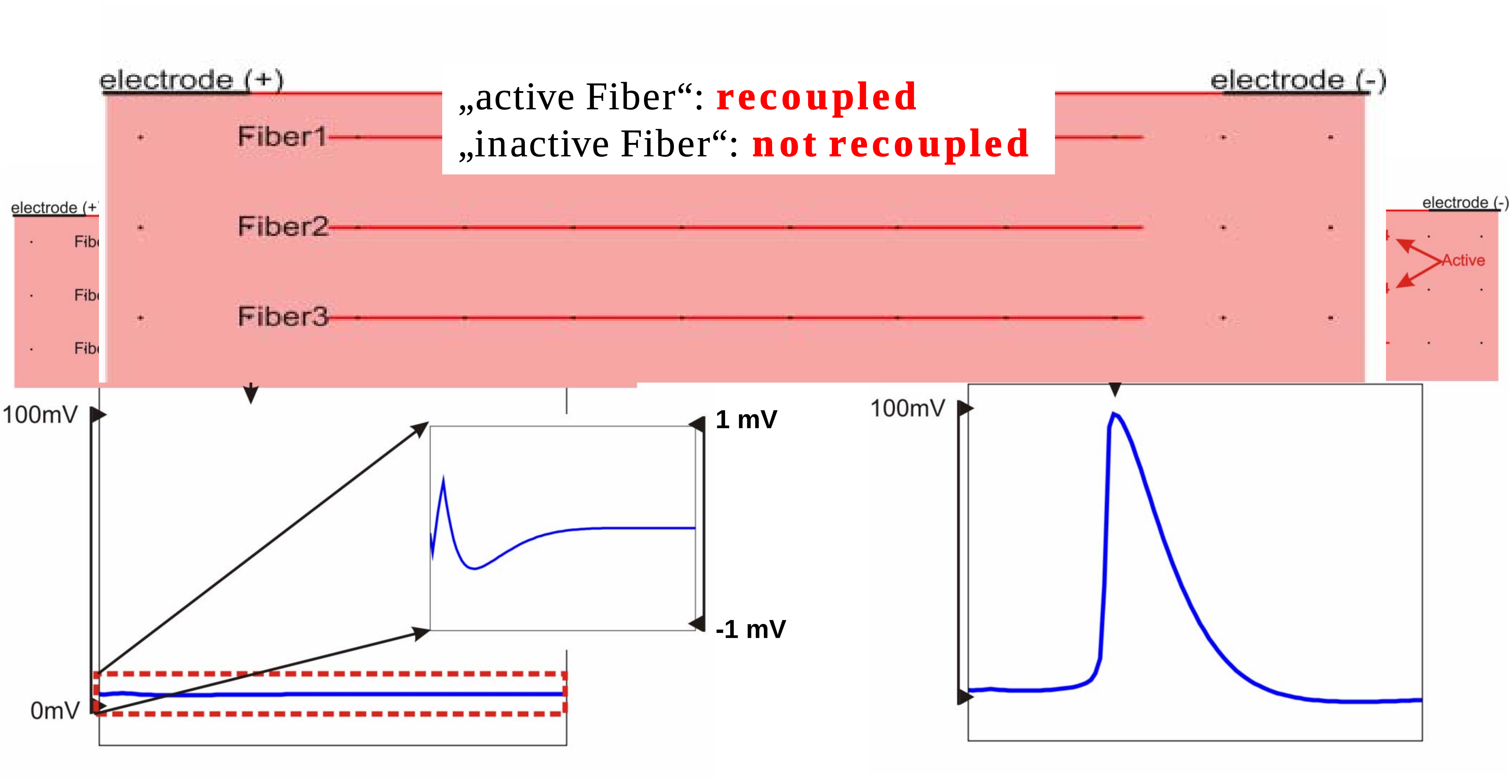


# 2D model – 2 fibers

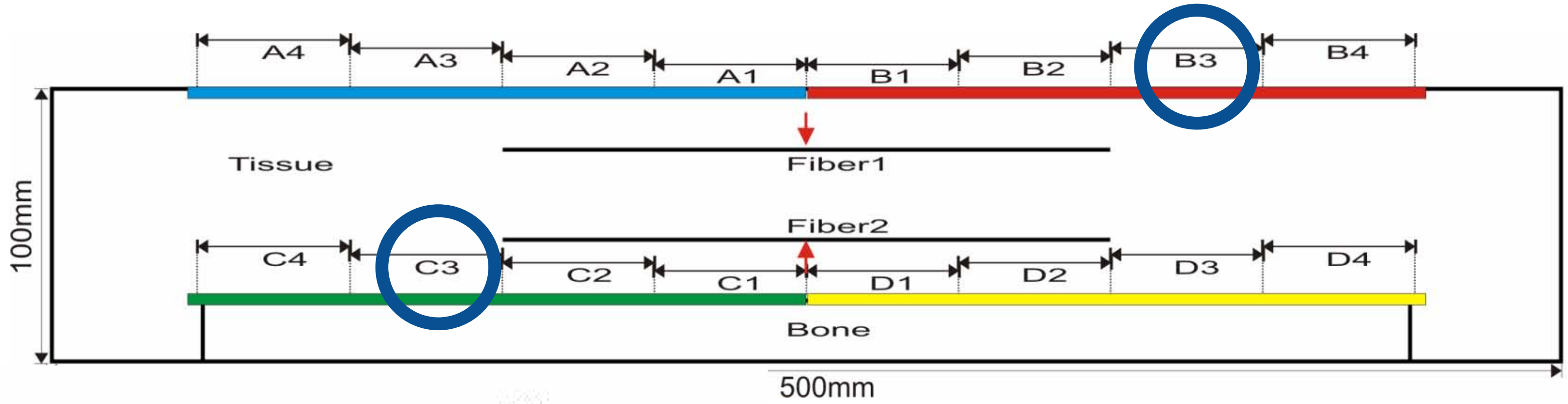


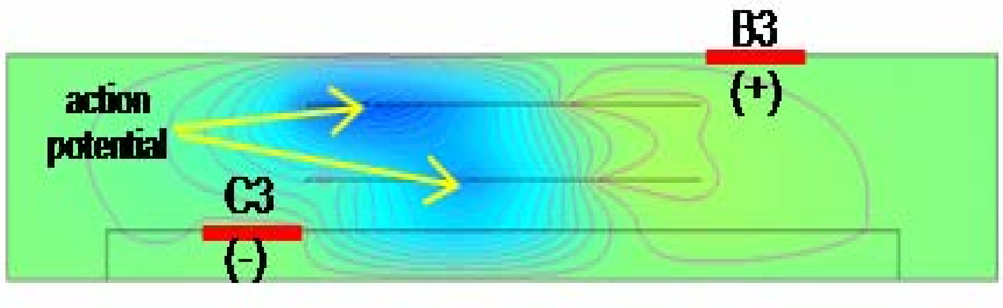
# 2D model – Influence of fibers

„active Fiber“: **recoupled**
„inactive Fiber“: **not recoupled**

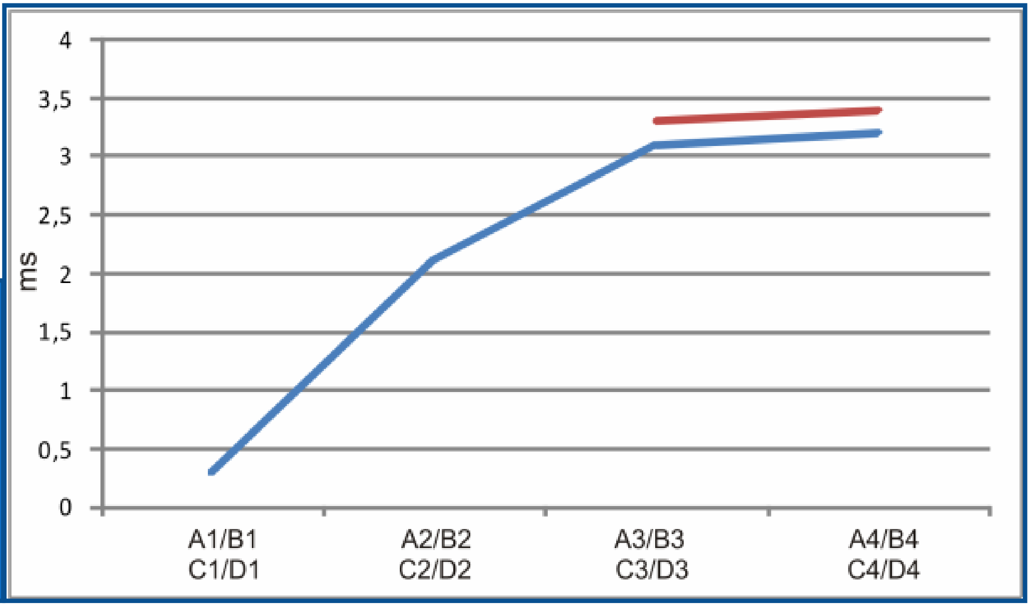


# Influence of electrodes position

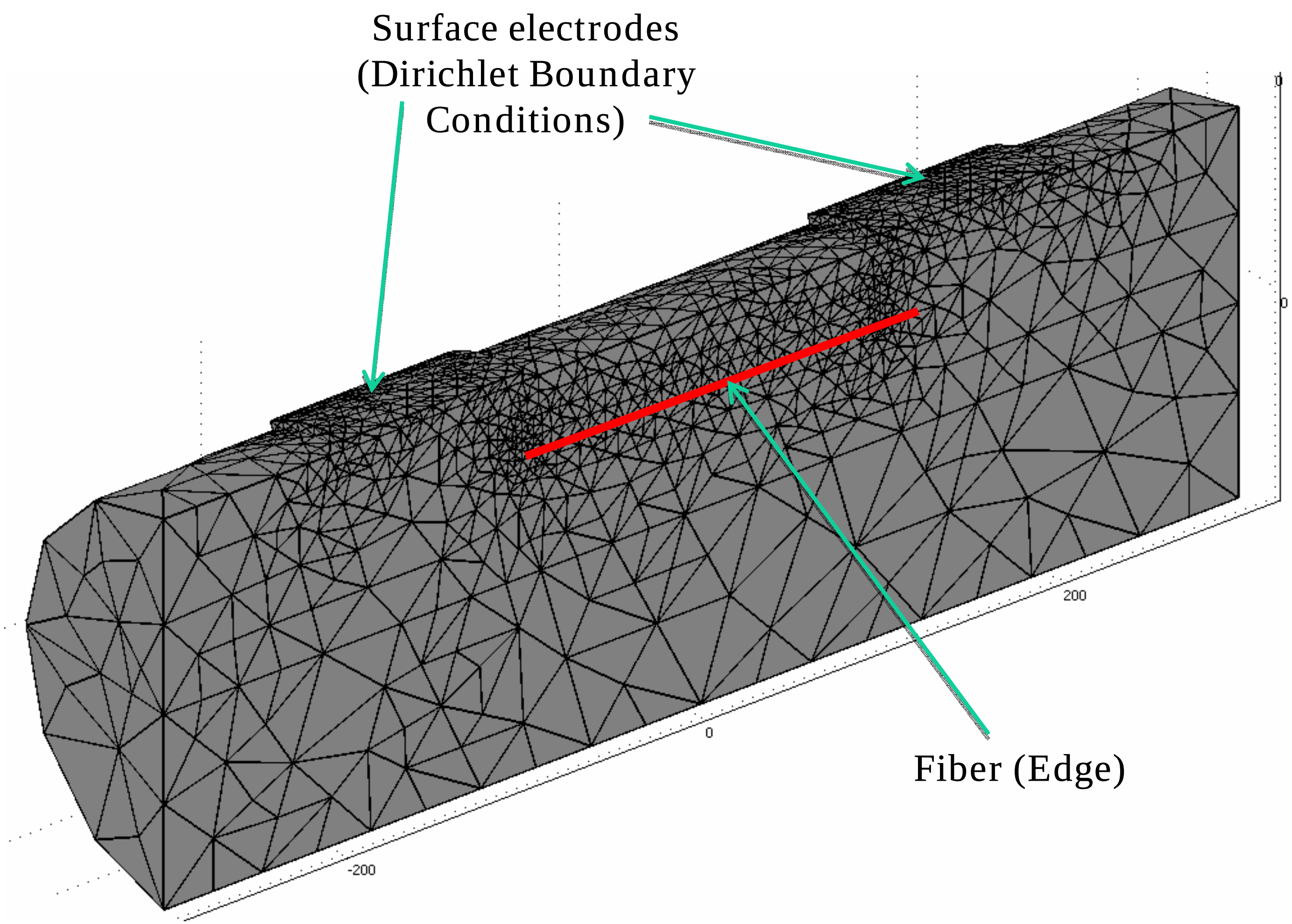



Time from stimulation impulse till an action potential is initiated. The diagram shows the excitation time of different electrode positions at the surface and implanted electrodes. The red line represents the fiber placed closer to the electrodes. The blue line is representing the more distant fiber.



# Outlook – 3D model




| | |
|---|---|
| App. Mode | PDE – Coeff. Form |
| Dimension | 3D & 1D |
| Dep. Variables | 4 (1D): Vi, m, n, h<br>1 (3D): Ve |
| N. of Elements | 1D: 200<br>3D: ca. 30000 |
| Solver | Different solvers<br>Time dependent (0:0.1:10) |
| rel. Tolerance | 10e-5 |
| abs. T0lerance | 10e-7 |
| Solution Time | **Very long** ☹ |

# Future Work

- Adaption of model parameters (denervated muscle, ....)
- Implementation of musclepaths ?
- Creation of 3D geometry (CT / MRI based ?)
- ......

# Conclusion / Discussion

- **HH PDE & Comsol Multiphysics are a great combination to**
  - Easily create models with different geometries
  - Coupling of extracellular and intracellular potential in **both** directions
  - Simulation of effects of stimulation and generated action potentials

# Questions ?!?

# Thank you for your attention !!!