Presented at the COMSOL Conference 2008 Hannover

Two-phase flow

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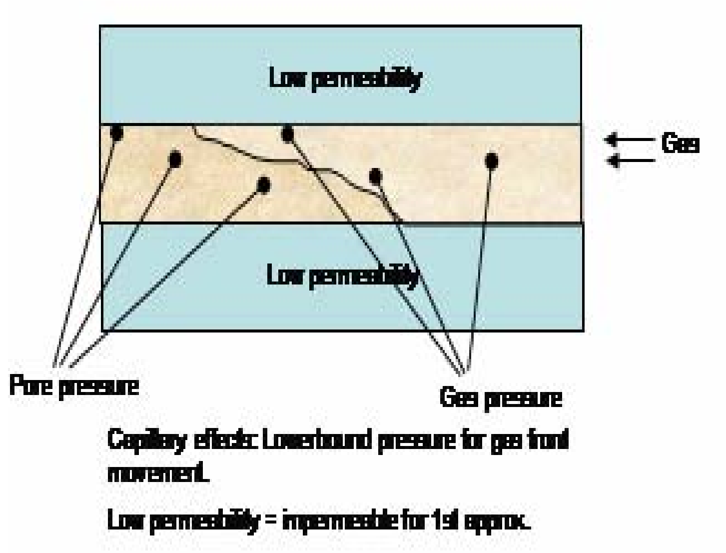
Two-phase flow

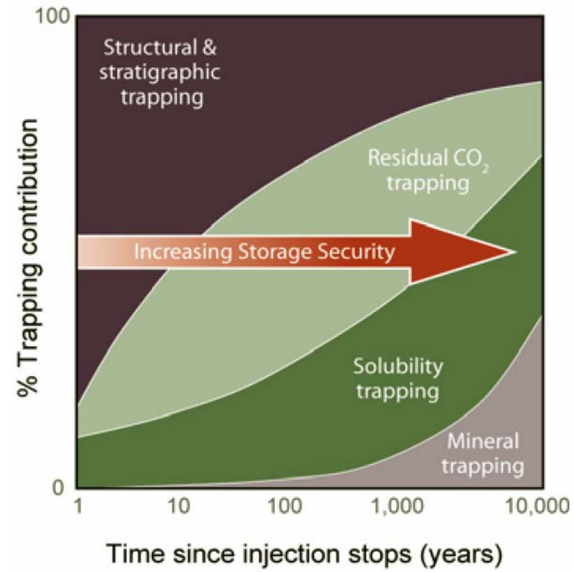
- Various implementations in COMSOL Multiphysics

Background and motivation

- Two main areas of interest:

1. CO_2 Storage: short term, injection process
2. Gas flow into and out of well; shallow gas seepage



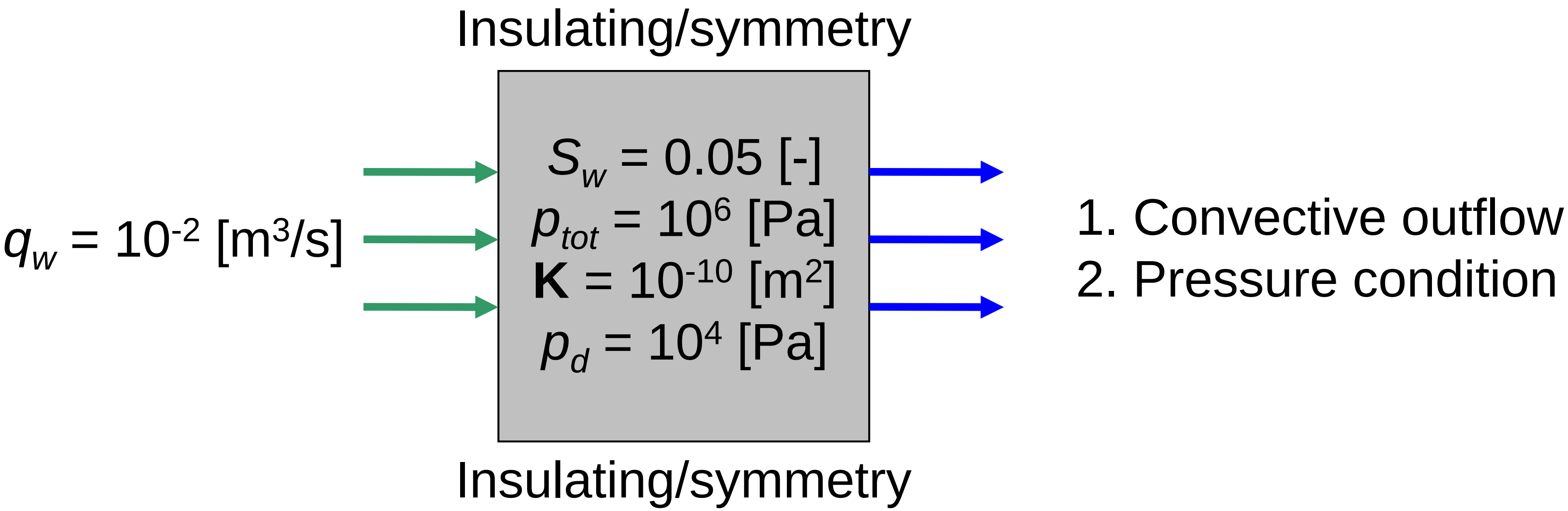


Purpose of this exercise

- Two phase flow can be a complicated set of equation to solve (depending on assumptions)
  - Consists of 2 PDEs and several auxiliary equations; enabling equation manipulation $\Rightarrow$ formulations
- Purpose of the investigation is to identify a preferred formulation that will be best suited for more complicated modelling:
  - Poroelasticity, energy balance, chemical reactions, dissolution of the phases, etc.
- Tailor-make our own simulator for two-phase flow and other physics (develop in-house know-how, no black box simulator)

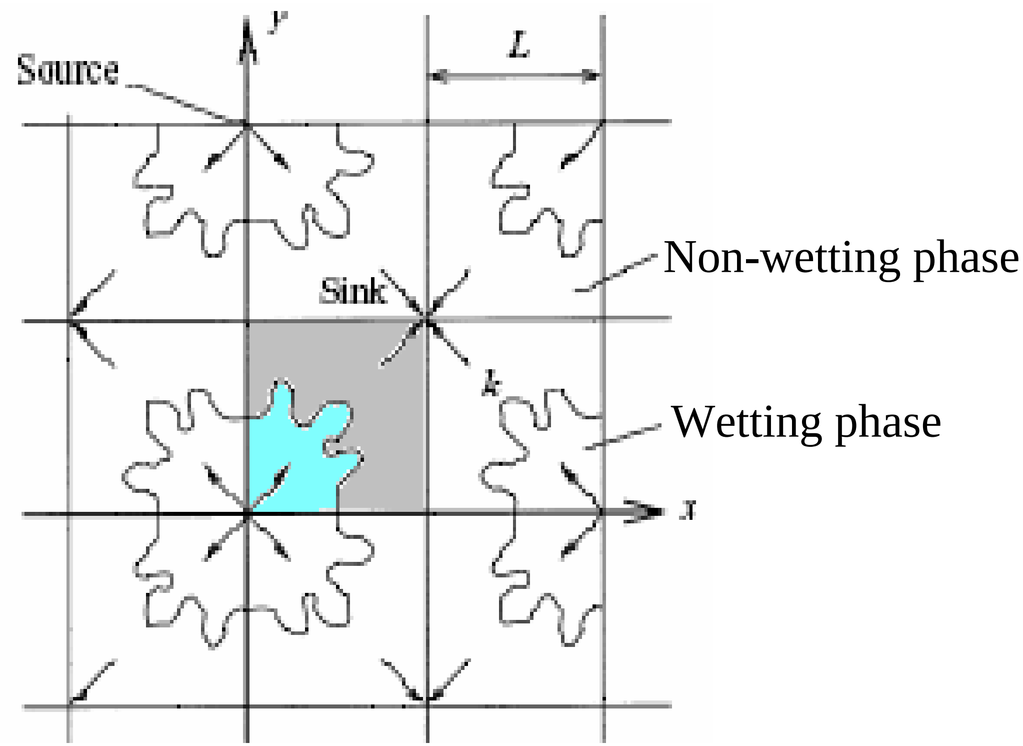
Two-phase flow modeling, case 1

- Formulations for modeling two-phase flow compared
- A 1D geometry, in 2D was developed
- Various cases simulated



Two-phase flow modeling, case 2

- Various setups for two-phase flow simulated
- A standard benchmark model, five-spot model,



Model implementations – various equations

1. Partial pressure formulation
2. Flooding formulation
3. Phase formulation
4. Fractional flow formulation
5. Weighted formulation
6. Buckely-Leverett (1D)

(1–2) Pressure based

(3–6) Pressure/saturation based

$$\left.\begin{array}{l} \frac{\partial}{\partial t}(\phi\rho_\alpha S_\alpha) - \nabla \cdot \left[\rho_\alpha \lambda_\alpha \mathbf{K}(\nabla p_\alpha + \gamma_\alpha \nabla z)\right] = \rho_\alpha q_\alpha \\ \sum S_\alpha = 1, \quad p_c = p_n - p_w, \quad S_{e\alpha} = f(p_c) \end{array}\right\}$$

General mass balances and auxiliary equations

Partial pressure formulation (p_n, p_w)

$$C_w\left[\frac{\partial p_n}{\partial t}-\frac{\partial p_w}{\partial t}\right]-\nabla\cdot\left[\lambda_w\mathbf{K}\nabla p_w\right]=q_w$$

$$C_n\left[\frac{\partial p_n}{\partial t}-\frac{\partial p_w}{\partial t}\right]-\nabla\cdot\left[\lambda_n\mathbf{K}\nabla p_n\right]=q_n$$

Flooding formulation (p_s, p_c)

$$\frac{\partial}{\partial x}\left(\Lambda_s \frac{\partial p_s}{\partial x} + \Lambda_c \frac{\partial p_c}{\partial x}\right) + \frac{\partial}{\partial y}\left(\Lambda_s \frac{\partial p_s}{\partial y} + \Lambda_c \frac{\partial p_c}{\partial y}\right) = q_{ps}$$

$$2\phi \frac{\partial S_w}{\partial p_c}\frac{\partial p_c}{\partial t} + \frac{\partial}{\partial x}\left(\Lambda_s \frac{\partial p_c}{\partial x} + \Lambda_c \frac{\partial p_s}{\partial x}\right) + \frac{\partial}{\partial y}\left(\Lambda_s \frac{\partial p_c}{\partial y} + \Lambda_c \frac{\partial p_s}{\partial y}\right) = q_{pc}$$

Phase formulation (p_n, S_w) (p_w, S_n)

$$\nabla \cdot \left(\lambda_w p_c^{'} K \nabla S_w - \lambda K \nabla p_n \right) = q_w + q_n$$

$$\phi \frac{\partial S_w}{\partial t} + \nabla \cdot \left(- \lambda_w K \nabla p_n + \lambda_w p_c^{'} K \nabla S_w \right) = q_w$$

Fractional flow formulation (p_s, S_w) (p_s, S_n)

$$\nabla \cdot \mathbf{u} = q_w + q_n$$

$$\phi \frac{\partial (S_\alpha)}{\partial t} + \nabla \cdot \mathbf{u}_\alpha = q_\alpha$$

$$\mathbf{u}_w = f_w \mathbf{u} + \lambda_n f_w K \nabla p_c$$

$$\mathbf{u}_n = f_n \mathbf{u} - \lambda_w f_n K \nabla p_c$$

$$\mathbf{u} = -\mathbf{K} \lambda \nabla p$$

Weighted formulation (p_s, S_w) (p_s, S_n)

$$\nabla \cdot \mathbf{u} = q_w + q_n$$

$$\phi \frac{\partial (S_\alpha)}{\partial t} + \nabla \cdot \mathbf{u}_\alpha = q_\alpha$$

$$\mathbf{u}_w = f_w \mathbf{u} + \lambda_n f_w K \nabla p_c$$

$$\mathbf{u}_n = f_n \mathbf{u} - \lambda_w f_n K \nabla p_c$$

$$\mathbf{u} = -\mathbf{K}\left(\lambda \nabla p + \left(S_w \lambda - \lambda_w\right) \nabla p_c + \lambda p_c \nabla S_w\right)$$

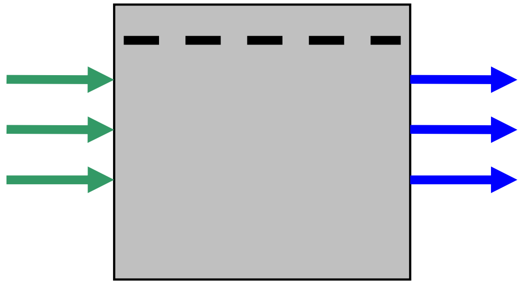
Buckley-Leverett (S_w) (S_n)

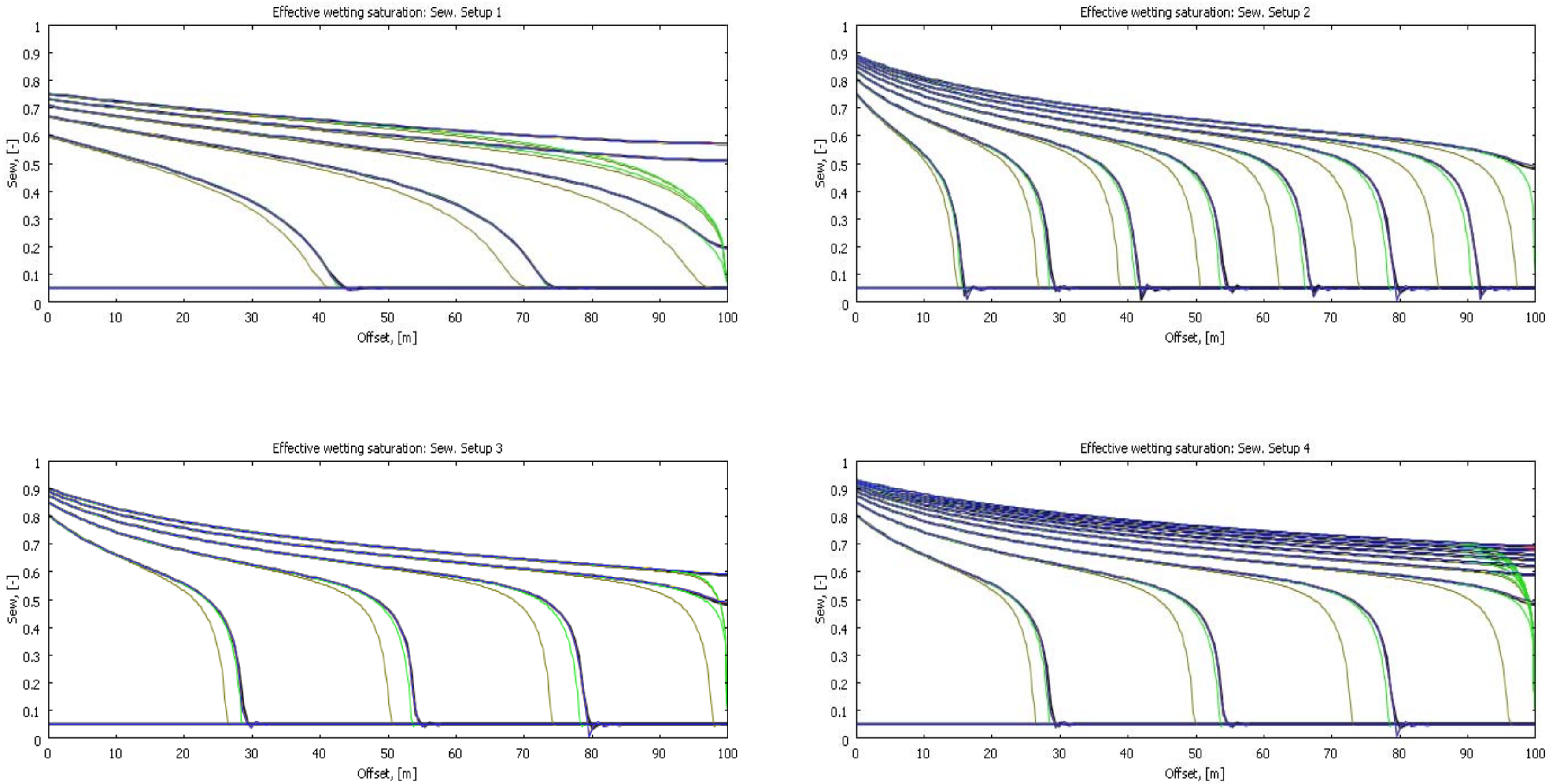
$$\phi \frac{\partial S_w}{\partial t} + \left(f_w q_t - D_w \frac{\partial S_w}{\partial x} \right) \frac{\partial}{\partial x} = 0$$

Simulations, setups

Parameter	Setup			
	1	2	3	4
Intrinsic permeability, [m^2], **K**	1e-10	**1e-11**	1e-10	1e-10
Entry pressure, [Pa], p_d	1e4	1e4	**1e3**	1e4
Influx wetting phase, [m^3/s], q_w	1e-2	1e-2	1e-2	**1e-1**

Results: Time step plots





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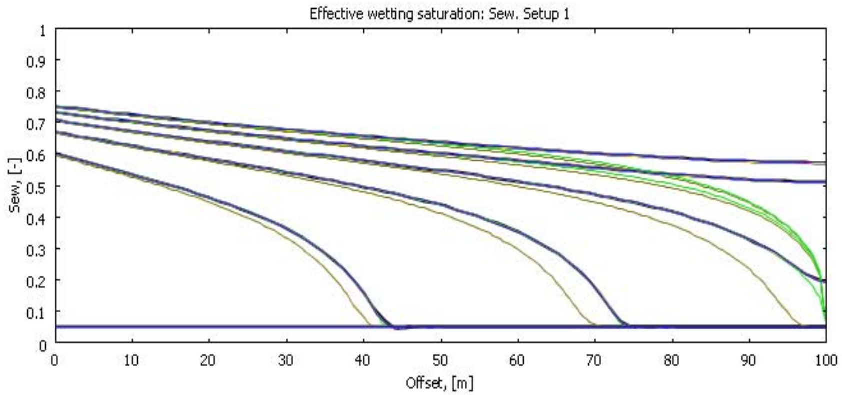

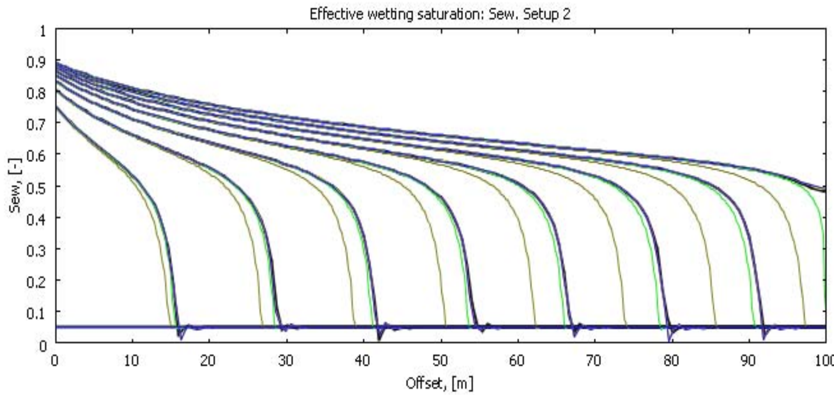

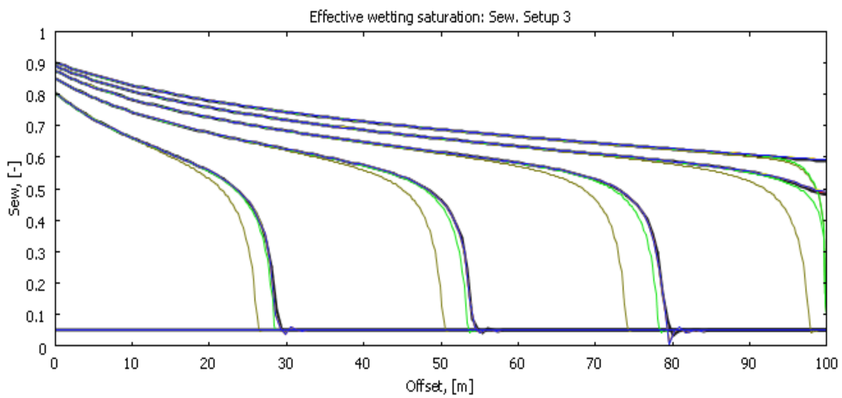

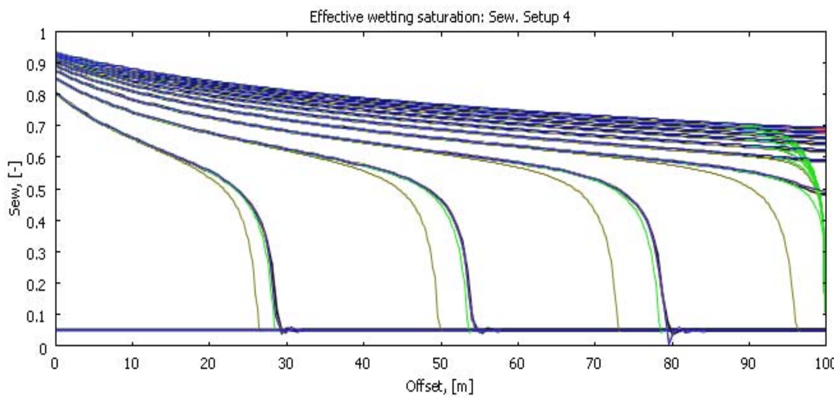

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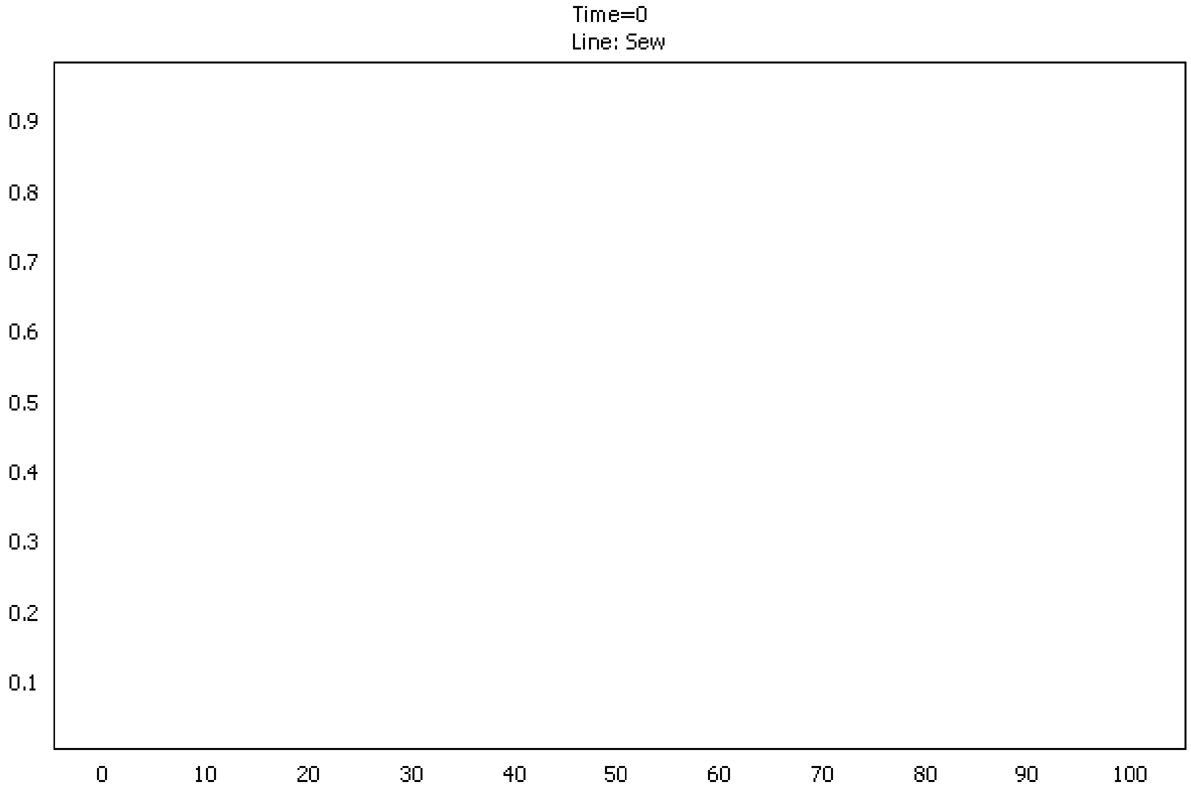
Results: In numbers

Equation formulation	Setup, dofs/sec			
	1	2	3	4
Buck	6	4,4,5	5,3,3	6,4,5
Frac	**96**	**70,55,57**	**56,61,68**	**33,56,55**
Part	69,88	42,16,12	8,18,22	8,8,12
Flod	59,49	42,19,13	13,19,28	10,10,13
Phas	90	62,52,54	50,60,62	31,49,49
Weig	94	49[1)],50,52	50,53[1)],59[1)]	32,47,48

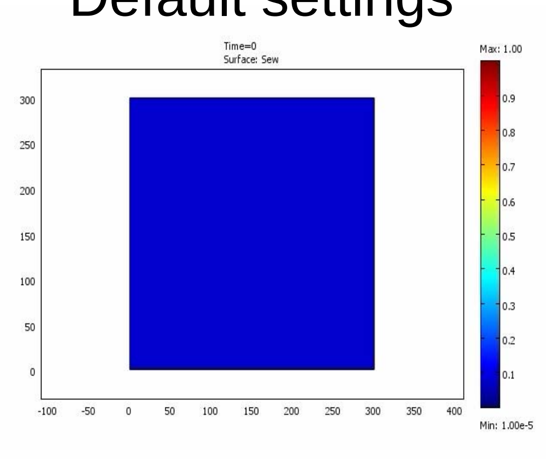
1) Needed a denser mesh than the other formulations

Results: Animations, 1D, case 1

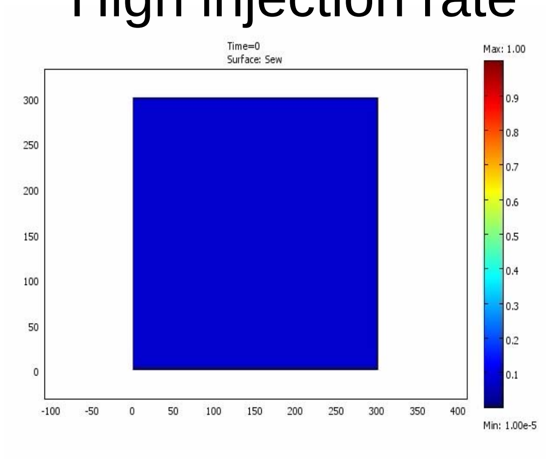


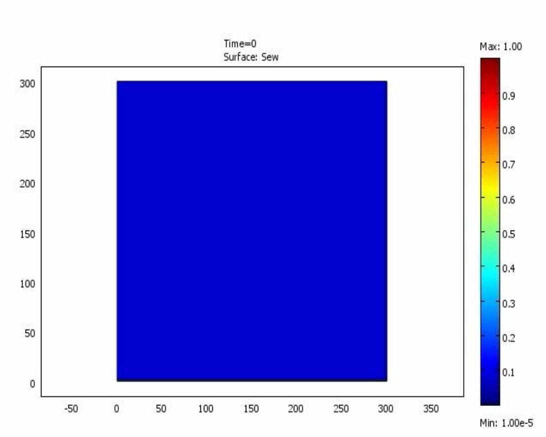
Results: Animations, 2D, case 2

Default settings

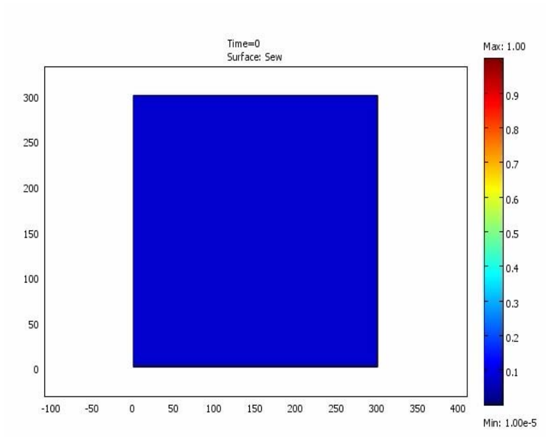


High injection rate





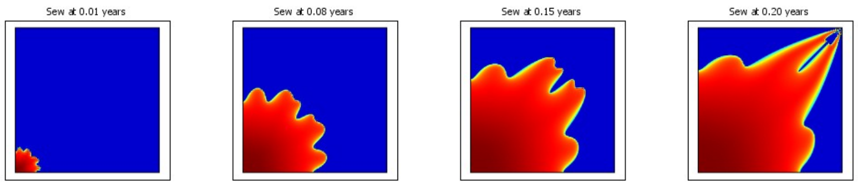
Low entry pressure



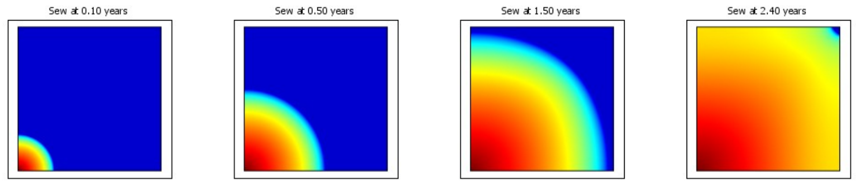
Low permeability

Results: Plots, case 2, various setups

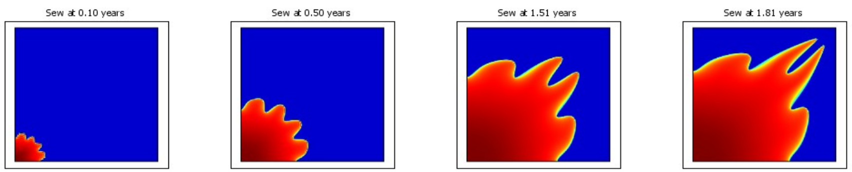
High injection rate



High entry pressure



Low entry pressure



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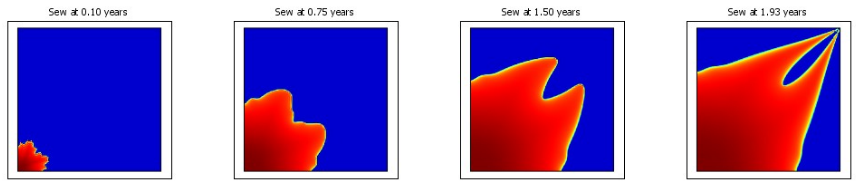

(high and low values are relative to default/common model parameters)

Conclusion

- Big difference in numerical performance speed/dofs, as much as a factor of 7
- Pressure and phase saturation-based formulations are preferred (especially fractional flow formulation)
  - Quicker and more stable
- Partial pressure (and flooding equation) needs more work and attention

- Acknowledgements: Financial support from the Research Council of Norway and NGI is gratefully acknowledged

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