Loudspeaker Response Optimization with the aid of Impulse Response

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Abstract: COMSOL Multiphysics® simulation software is receiving a lot of attention in the loudspeaker industry thanks to the quality of the results it provides making it an essential tool to optimize geometries so that the best reproduction capabilities of the loudspeaker system are achieved.

Setting up a loudspeaker simulation requires defining a geometry with moving diaphragm. This can be done simply by specifying diaphragm's velocity within the Acoustics Module; or by including full definitions of the Electromagnetic domain, Mechanical domain and Acoustical domain in a Multiphysics setup. Alternatively, it can be done with just the AC/DC and Acoustics Modules with the aid of lumped circuit equivalents. Whatever the case, the analysis is typically done in the frequency domain on a series of values for frequencies in order to profile a resulting frequency response. This process of iterations is rather time consuming for an average workstation.

With the latest 5.3 update, improvements were done in the Acoustics Module on time domain that allow for an easier setup of acoustic simulations. It is now practical to use of an impulse response analysis in time domain. Results with a Fourier Transform can give an idea of the behavior in the frequency response for a quick optimization. A more detailed inspection in frequency domain is possible by mapping results from time domain so that the far field can be calculated. Additional analysis time is saved by focusing only on frequencies of interest within the same model which, when thinking of iterative changes, speeds up optimization of the geometries for the intended design target. This paper illustrates such an approach to loudspeaker design.

Introduction

In acoustic measurements, it has been extensively proved essential the use of the impulse response formulation in determining the frequency response of a loudspeakers.

For acoustic room measurements, examples of a good approximation of an acoustic impulse might be the popping of a balloon or the firing of a blank. A complete time domain recording of the impulse, including its decay, provides most of the information necessary for that study.

For the measurements of electrical transducer systems such a stimulus is not as practical due to several limitations [1]. The first limitation is the high crest factor as loudspeakers could be driven out of their linearity range. An additional limitation is poor signal to noise ratio because all the energy is emitted in a brief fraction of time. Measuring with Chirp or Maximum Length Sequences is therefore commonly done.

Similarly, a model to be simulated with FEA in the time domain will need a stimulus that can't be an ideal impulse for obvious convergence problem due to its discontinuity.

Let us explore a possible alternate approach.

Theory Background

Theoretically the impulse (Dirac delta) has multiple mathematical properties that makes its uses in multiple realms of physics and mathematics extensive.

In acoustics, we use it for the responses' transformation from time domain to frequency domain of linear time-invariant systems (to which, with some constrains we can assume a loudspeaker transducer is).

In the discrete world (digital or numerical simulation), if given an input sequence $x(n)$ to a system whose impulse response sequence is $h(n)$, the output sequence $y(n)$ is attained through a convolution sum (a form of integral transformation indicated with "*" for the discrete values [2])

$$y(n) = h(n) * x(n)$$

which, expressed term of z-transform (Laplace transform equivalent acting on the discrete values),

$$Y(z) = H(z)X(z)$$

will give us the transfer function H(z) which when evaluated on the unit circle allow us to derive the frequency response.

For implementation in the model, considering that the impulse response can be seen as a Gaussian where the sigma limit to zero, a gaussian pulse seems a good candidate. As it is also used in multiple COMSOL tutorials models, its use leaves no doubt in terms of model's solution convergence.

Governing Equations / Numerical Model / Simulation / Methods

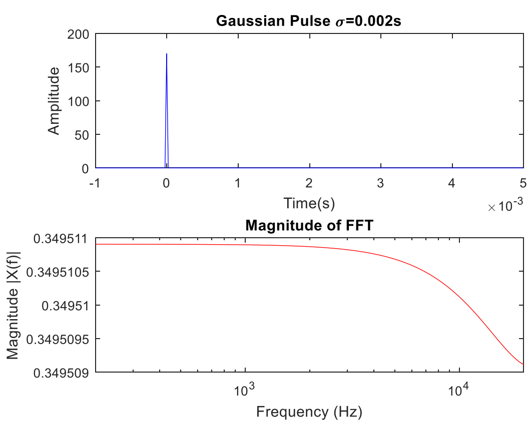
One characteristics of a Gaussian curve is that the Fourier transform in the frequency domain is also a Gaussian.

$$g(t) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{t^2}{2\sigma^2}}$$

Gaussian Pulse.

$$\begin{aligned} G(f) &= F[g(t)] \\ &= \int_{-\infty}^{\infty} g(t) e^{-j2\pi f t}\, dt \\ &= e^{-\frac{1}{2}(2\pi\sigma f)^2} \end{aligned}$$

Fourier Transform of a Gaussian Pulse retains its shape.



Pulse example and its spectral content on log frequency.

This helps the setup as, based on the target range of frequency spectrum that needs to be resolved, the pulse can be defined so that the maximum frequency in its spectral content matches it. The model in subject will be of a Tweeter (this kind of transducer is designed to reproduce the higher portion of the audible frequency spectrum, to give an example 2,000 to 16,000 kHz), the gaussian pulse thus will be defined so that if we call f_0 the highest frequency in exam

$$1[m/s] * \exp(-pi^{\wedge}2 * \mathrm{f0}^{\wedge}2 * (t - T0\,)^{\wedge}2)$$

can be placed as an analytic formula for the diaphragm velocity, Vin, in the definition section with "t" being time [s] and f_0 [Hz] easily passed as a parameter together with T0 defined as 1/f0.

The geometry can then be drawn applying Vin to the approximate portion of the displacing diaphragm (figure 1).

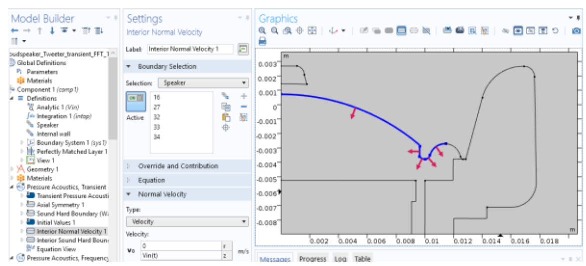


Figure 1. Diaphragm velocity setup.

This way the model would be very simple and solve quickly, just the use of the Acoustics Module will suffice. Basically, the idea of such study is to observe the impulse propagation in the time domain, retrieve locally within the domain, frequency response (figure 6), then optimize the geometry as much as possible with the option to investigate troubled frequency ranges with the Frequency Domain analysis for the details in polar response, far field and use a full Multiphysics simulation at the end to obtain detailed data like in fig. 7 and fig 8.

For the latest COMSOL update (5.3), PLM in the transient analysis (time domain) was a highlight feature addition in the Acoustics Module. The excitement of this feature brought up to mind the use of impulsive stimuli in time domain to then FFT the results.

The PML - Perfectly Matched Layer, is a domain defined with a construct that will seamlessly avoid incident pressure waves to return values back in computation, simulating total absorption of energy, like an anechoic environment would do. This feature has been implemented in the Acoustics Module Frequency Domain since few years.

In the Frequency Domain, its formulation is such that it does not require much attention in the setup. In the time domain, the PML brings the same advantages, truncating properly the computational domain, saving degrees of freedom by reducing domain size, reducing computational time. Alas, as it is, it does needs extra caution in the setup when in the time domain because its thickness is important for it to function well.

As a guideline, the thickness of the PML should be set at least an eight of the longest wavelength to be simulated. For the subject being a tweeter and not having interest in behaviors below 1000Hz makes it fine for the PML to be around 50mm wide. It is also recommended to keep it at least six layers in structured mesh.
Similar consideration goes for the main computational domain. For that, also the fact that the mesh size needs to be able to resolve the impulse in time needs to be considered. So, based on the speed of sound and the period T0, a reasonable mesh size would be a sixth the distance as maximum element size, and a tenth as minimum.
Boundary layer thickness towards the PML is also important and should be set to approx. 1/50 of the distance covered in T0.

Once those criterions are applied is possible to set a run to the solution (Eq. 1). That can be attempted considering, again, as length of analysis time the pulse duration plus the domain width time of flight, and with a factor that there will be residual perturbances coming from reflections within the domain geometries that we would also include (would be best if that distance gets related to T0 with an integer unit to have i.e. 6 times T0). The intervals that will give an optimal resolution of the impulse curve would be to the order of <1/2 or a fraction that then completes the range properly (1/2 works well with a time range lasting 6·T0).

$$\frac{1}{\rho c^2}\frac{\partial^2 p}{\partial t^2} + \nabla \cdot \left(-\frac{1}{\rho}\left(\nabla p_t - \mathbf{q}_\mathrm{d}\right)\right) = Q_m$$

Equation 1. Scalar Wave Equation Transient

To implement an FFT to the solution an additional step can be applied to the same study by right clicking on the study in the model builder's tree. In the popup menu in study steps under Frequency Domain insert a "Time to Frequency FFT." Selecting as input study the time dependent, as solution the "current," and using the solution store attained by the time dependent step, compute an FFT on it with the appropriate parameters as desired. COMSOL gives with this study the opportunity to even apply a windowing function, however, in order to attain more useful data from the results the mapping of that solution to a frequency domain is necessary.

Will be necessary setting up a PML for Frequency Domain and disabling the Transient's in the definition section. Making use of the same model, but this time adding a Frequency Domain physics will give the possibility to get the far field analysis and its features like polar plots and frequency response in the far field. This is realized by using the default parts of the physics definition, but changing its equation form to study controlled, so that the equations and pressure field values are coming from the time domain study.
Right clicking on the Frequency Domain physics we will have to add a far field calculation boundary, and a "Monopole Domain Source" (Fig. 2) where the wave numbers (acpr.k) will involve the pressure values from the time domain ((-acpr.k^2/rho0)*p) as from Eq. 2

$$\nabla \cdot \left(-\frac{1}{\rho_c}\left(\nabla p_t - \mathbf{q}_\mathrm{d}\right)\right) - \frac{k_{eq}^2 p_t}{\rho_c} = Q_m$$

Equation 2. Scalar Wave Equation Frequency Domain

The solution will in fact show another variable for pressure in the frequency domain as p2.
One last trick will be necessary to display the far field, as also mentioned in the COMSOL documentation "To evaluate the pressure in a point (x0,y0,z0), simply write pfar(x0,y0,z0). To evaluate the sound pressure level in the same point, it is advantageous to use the subst() operator and write, for example,subst(acpr.ffc1.Lp_pfar,x,x0,y,y0,z,z0)."
Using that but for r,z in the axisymmetric coordinates subst(acpr.ffc1.Lp_pfar,r,0,z,1[m]) in far field for polar plots will give the results 1m on axis.

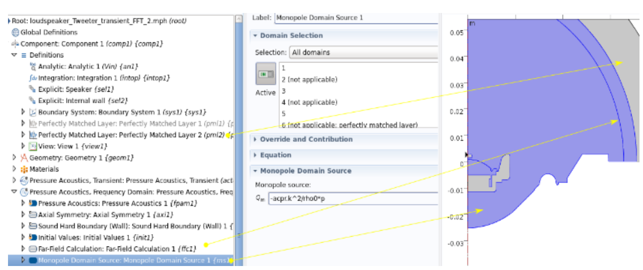


Figure 2. Frequency Domain setup.

Experimental Results / Simulation Results / Discussion

A prototype was put together to verify the first series of simulations, as a matter of fact, the “ugly” on axis frequency response resulting from the simulation showing cancellations happening at 3kHz and peaks around 8kHz was verified to be coherent with real life prototype.

Figure 3 shows an early attempt to improve the response by getting the location of the tweeter slightly off alignment to the woofer’s axis. Irregularities were also added at the edges to measure the effects.

The process of making a prototype by SLA and do polar measurements is costly and time consuming thus the first prototype was modified by hand to the one in figure 3. Once improvements were noted the approach switched back to computer simulations.

Figure 3. One early prototype.

Following are some of the results from the impulse response simulations. Figure 4 is a display of the impulse (**a**)as it becomes at 0.2m in front of the tweeter (**b**). Figure 5 **a** and **b** are plots coming from the far field calculations done in the Frequency Domain study by mapping the results obtained in Time Domain. Figure 6 is attained doing the FFT during the solution of the Time Domain study by then point probe at 0, 15, 30, 45 degrees off the tweeters’ symmetry axis. Time domain solution takes 3min 18 sec including the FFT calculation with a high number of bins, mapping to Frequency Domain gets solved in 20 sec., a similar detailed Multiphysics analysis in the frequency response takes similar times, but without the understanding that brings visualizing the pulse propagation in time.

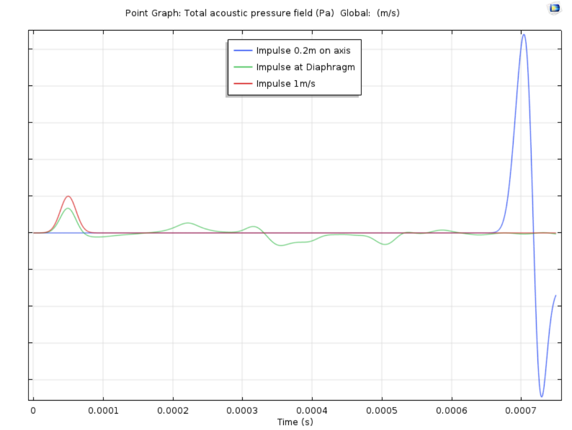


Figure 4a. Gaussian pulse, and propagation.

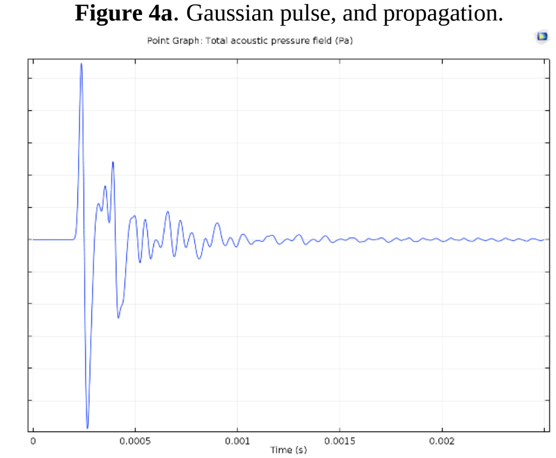


Figure 4b. Pulse in domain 0.2m on z axis.

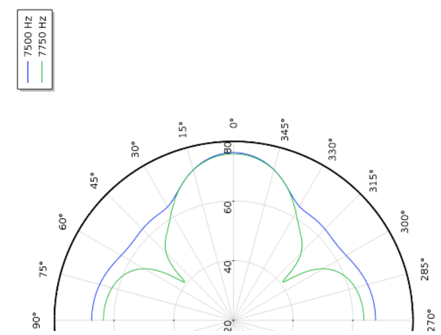


Figure 5a. Polar Responses Mapped Freq. Dom.

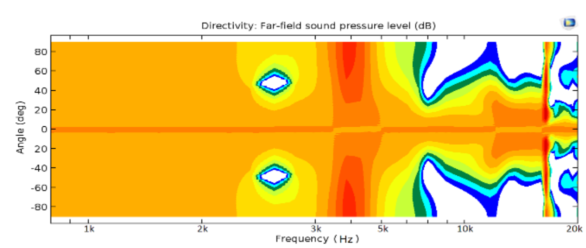


Figure 5b. Directivity Plot Mapped Freq. Dom.

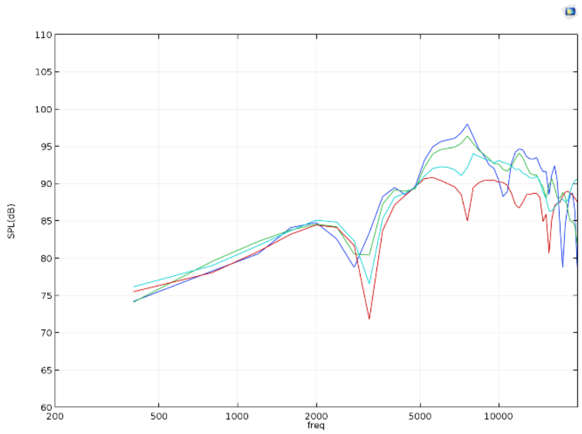


Figure 6. Frequency response by FFT of impulse

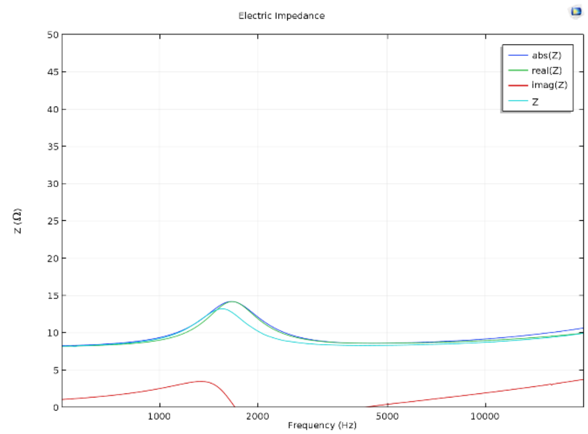


Figure 8. Impedance, simulated vs. measured (Z)

Now, detailed Multiphysics simulations results that include AC/DC, Structural Mechanics and Acoustics Modules. Figure 7a SPL and in figure 8 impedance curves with real prototype measurements imported in COMSOL for comparison. In figure 7b I have placed similar resolution points as in figure 6 for comparison to the FFT results.

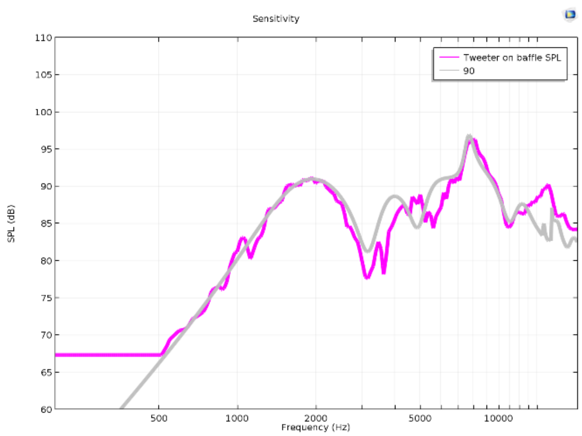


Figure 7a. Frequency response detailed simulation.

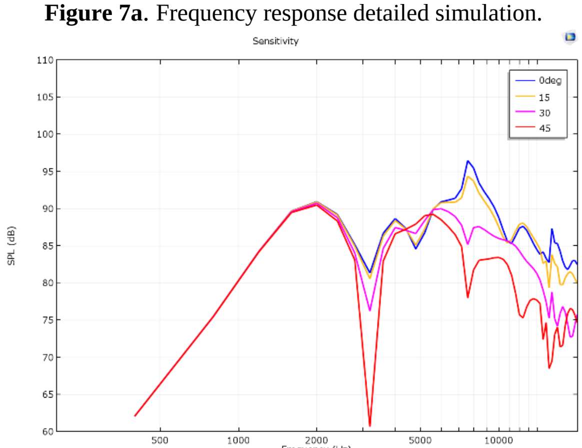


Figure 7b. Frequency response at few angles.

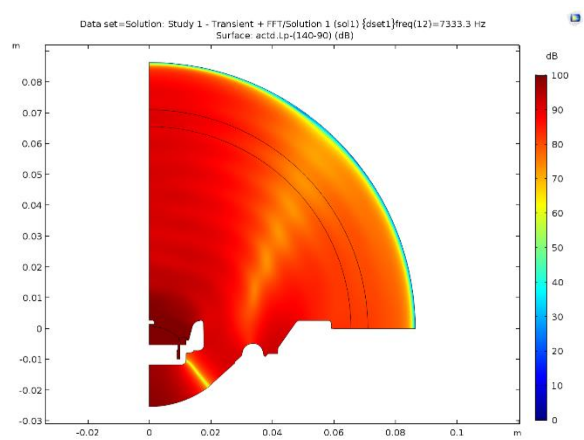


Figure 9 a. Sound Pressure Level FFT.

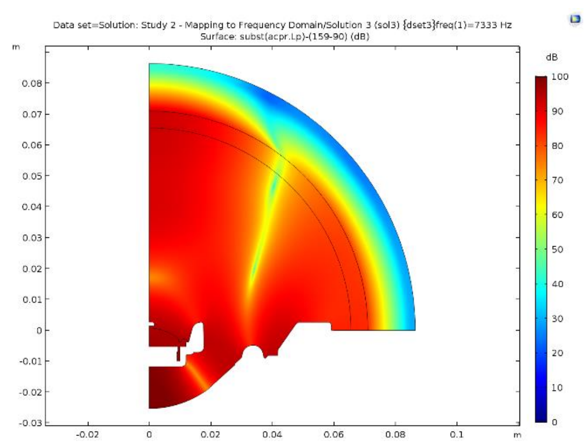


Figure 9 b. Sound Pressure Level mapped results.

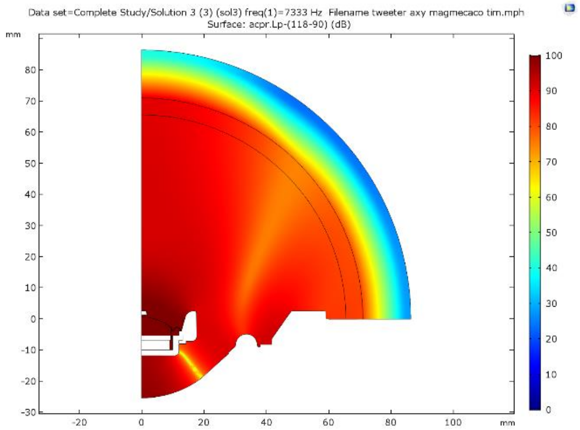


Figure 9 c. Sound Pressure Level Frequency Domain.

There was a good amount of correlation between the results from studies of the impulse response to fully fledged Multiphysics simulation, corresponding dips and peaks, similarities in polar response etc. that show the usefulness of this approach.

Figure 6 is an SPL response from performing an FFT from the time domain results on axis, at 15, 30 and 45 degrees. It is nice to see that beside the attenuation in frequency coming from the rise in frequency of the tweeter impedance, the behavior displayed matches. In detail, the peak at 8kHz on axis gradually becomes a dip at 45 degrees and the same behavior is captured in both approaches.

For an interesting subject on the PML, figure 9 shows the SPL map of the three methodologies at 7333Hz, **a** is the FFT from the time domain impulse, the domain was kept on purpose the same among the studies and the PLM size is smaller than recommended above for the time domain, it is interesting to see the diffraction waves caused by the superposition to a not fully absorbing PML. This to show how its size needs to be kept on consideration, that data mapped in the Frequency Domain study is figure 9 **b** the PML of that size works properly but the data from the Transient study alters what the correct results are, nevertheless patterns and behaviors are still somewhat coherent with figure **c** which is a complete Multiphysics simulation.

On a personal note, I love to see that standing wave node hairline drop in SPL by the cavity behind the tweeter. Actually, that is what causes the off-axis irregularities (see appendix for measurements). Is also fascinating seeing the results from the time domain, how wave propagates bouncing off surfaces and creating interference patterns.

Conclusions

I strongly believe that with some tweaks and perhaps interfacing data to MATLAB and applying extra calculations like Helmholtz-Kirkoff integral [3], or FFT in a specific way could yield great gain in processing time to eyeball the design with multiple fast iterations using just the acoustic module.

Having experimented with time domain simulation in COMSOL and its ability to do FFT, parse data and results, allowing to be manipulated to further studies and analysis of a design, has opened the door to great possibilities. Study on distortion, visualization of wave reflection and refraction on boundary, eventually waterfall analysis, all in the virtual realm of computer simulation are possible ideas as increasing computing power becomes more affordable.

This methodology is perhaps at its infancy, but it has great potential to become an essential tool as the impulse response measurements approach is now in the loudspeaker industry.

References

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2. Mikio Tohyama, Tsunehiko Koike, *Acoustic Signal Processing*, 143-227. Academic Pross, San Diego, CA (1998)
3. David W. Gunness, Ryan J. Mihelich, LOUDSPEAKER ACOUSTIC FIELD CALCULATIONS WITH APPLICATION TO DIRECTIONAL RESPONSE MEASUREMENT, AES 109th Convention, Los Angeles (2000)

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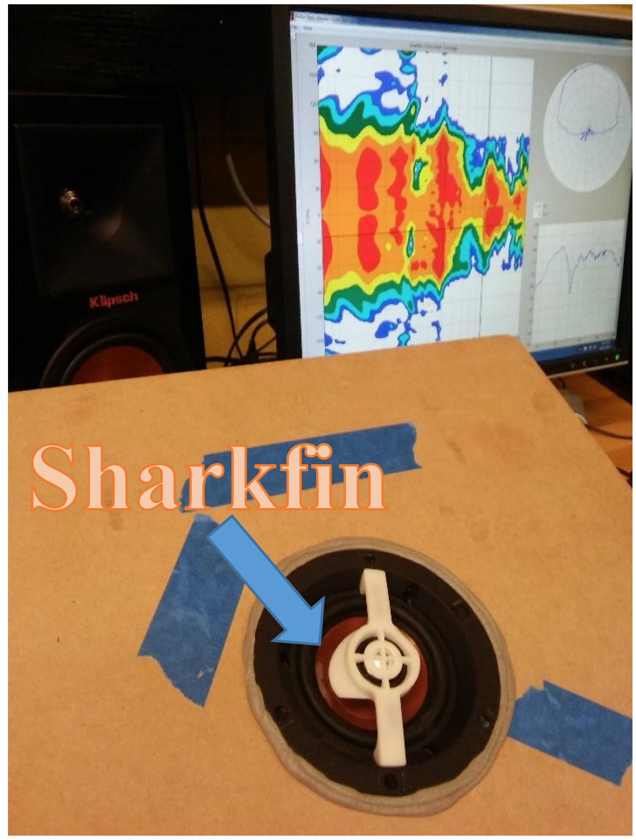
Appendix

The below shows the fruit of several simulation iterations in 3d (nicknamed sharkfin) and the comparison with a slightly improved version to the simulated results shown in this paper. The original geometry prototype was empirically "improved" to some extents with some cut out and an off centering to explore with direct SPL measurements if was worth to evolve the geometry into something "originally" better.



Above original prototype and sharkfin. On the right tweeter assembled on system with open baffle. Below 360 degrees measurements in anechoic chamber.