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Wall Effects in Convective Heat Transfer from a Sphere to Power Law Fluids in Tubes

Daoyun Song[1], Rakesh K. Gupta[1] and Rajendra P. Chhabra[2]
[1]West Virginia University, Morgantown, WV
[2]Indian Institute of Technology, Kanpur, India

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Outline

- Introduction
- Governing equations & boundary conditions
- Simulation procedure
- Results and discussion
- Conclusions

Introduction

Flow and heat transfer around a sphere to liquids have applications in chemical industry such as

- Fixed or fluidized beds
- Falling ball viscometry
- Emulsion or suspension processing, such as foodstuff
- Emulsions & suspensions exhibiting a shear-thinning behavior with power law index n ~0.2-~0.8

Governing equations and boundary conditions

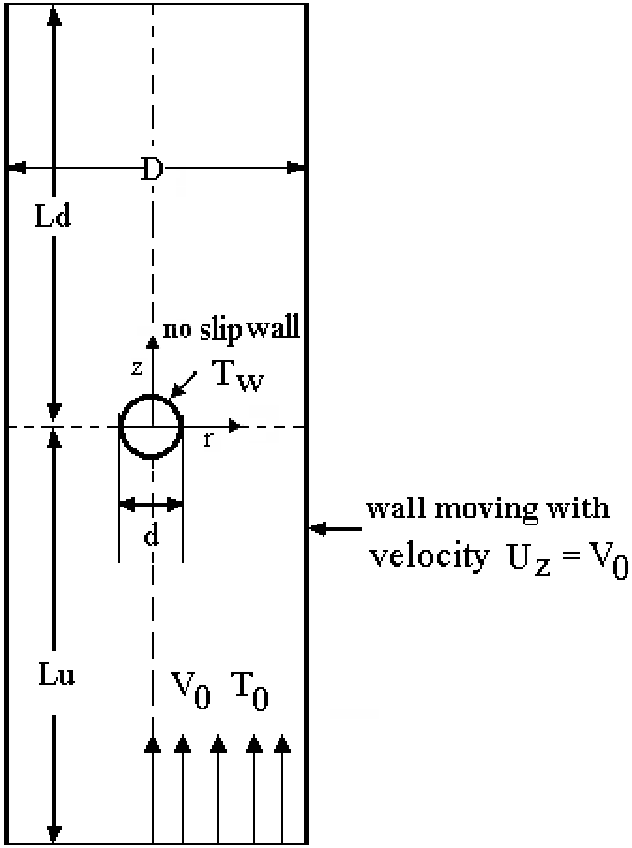

Figure 1. Schematic diagram of flow around a sphere in a tube

Governing equations

Continuity equation

$$\nabla \bullet U = 0$$

Momentum equation

$$\rho U \bullet \nabla U = \nabla \bullet \sigma$$

Thermal energy equation

$$\rho C_p U \bullet \nabla T = k \nabla^2 T$$

U, velocity vector; σ, total stress tensor; T, temperature

ρ, density; k, thermal conductivity; Cp, specific heat capacity

Momentum equation $$\rho U \bullet \nabla U = \nabla \bullet \sigma$$

$$\sigma = -pI + \tau$$

Need a constitutive equation relate the stress to the velocity gradient

Power-law model (n<1)

(shear-thinning behavior)

$$\eta = m\left[\sqrt{\frac{1}{2} I_2}\right]^{n-1}$$

$$\dot{\gamma} = \nabla U + \left(\nabla U\right)^T$$

$$I_2 = \dot{\gamma} : \dot{\gamma}$$

Boundary conditions

Inlet	$U_r = 0, U_z = V_0, T = T_0$
On the tube wall	$U_r = 0, U_z = V_0, \frac{\partial T}{\partial r} = 0$
On the sphere surface	$U_r = U_z = 0, T = T_w$
Axis of the tube	Position of r=0 set to be axial symmetry
Exit	Pressure =0, no viscous stress, $\frac{\partial T}{\partial z} = 0$

Dimensionless numbers: λ (geometrical parameter), Re (flow parameter), Pr (heat parameter), Nu=f(λ, Re, Pr, n)

$$\lambda = \frac{d}{D} \qquad \mathrm{Re} = \frac{d^n V_0^{2-n} \rho}{m} \qquad \mathrm{Pr} = \frac{C_p m}{k}\left(\frac{V_0}{d}\right)^{n-1}$$

Method to obtain local and sphere surface average Nusselt numbers

Directly accessible from COMSOL postprocessing

$$Nu_\theta = \frac{h_\theta d}{k} = \frac{d}{k(T_w - T_0)}\left[-k\frac{\partial T}{\partial n_s}\right]\Bigg|_\theta$$

$$Nu = \frac{hd}{k} = \frac{1}{k(T_w - T_0)\cdot 2\pi R}\int_0^\pi 2\pi R^2\left(-k\frac{\partial T}{\partial n_s}\right)\Bigg|_\theta \sin\theta d\theta = \frac{1}{2}\int_0^\pi Nu_\theta \sin\theta d\theta$$

Simulation procedure

1. COMSOL Multiphyscis 3.5a
2. Quadrilateral elements
3. Element choices:

   velocity-pressure coupling, Lagrange-$P_2P_{1;}$ T, Lagrange-Quadratic

4. Separately solve momentum equation and energy equation
5. Using corresponding Newtonian flow and temperature fields as initials to facilitate convergence
6. sphere-in-sphere configuration to mimic λ=0 (unconfined)

Results and discussion

Validation

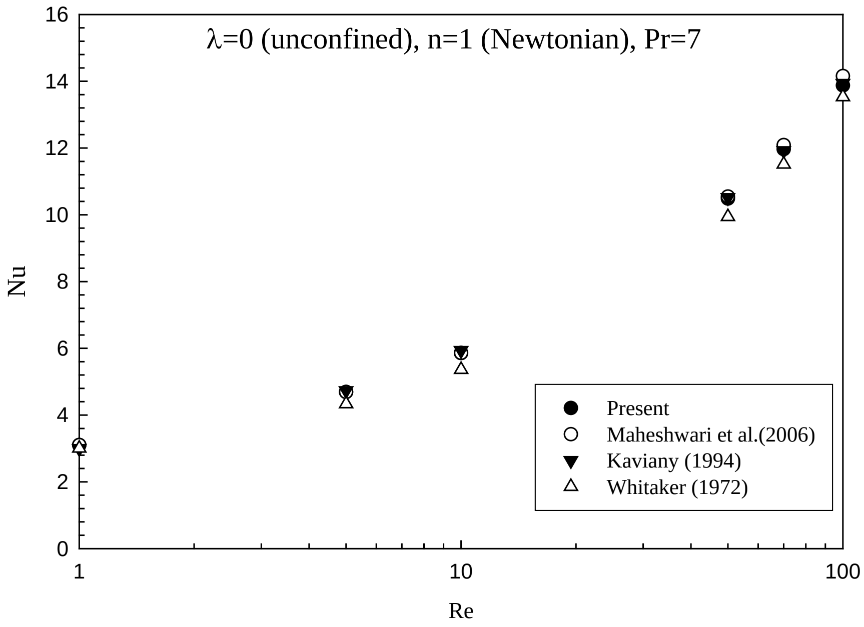

Figure 2(a). Comparison of mean Nu for the unconfined and Newtonian fluid case

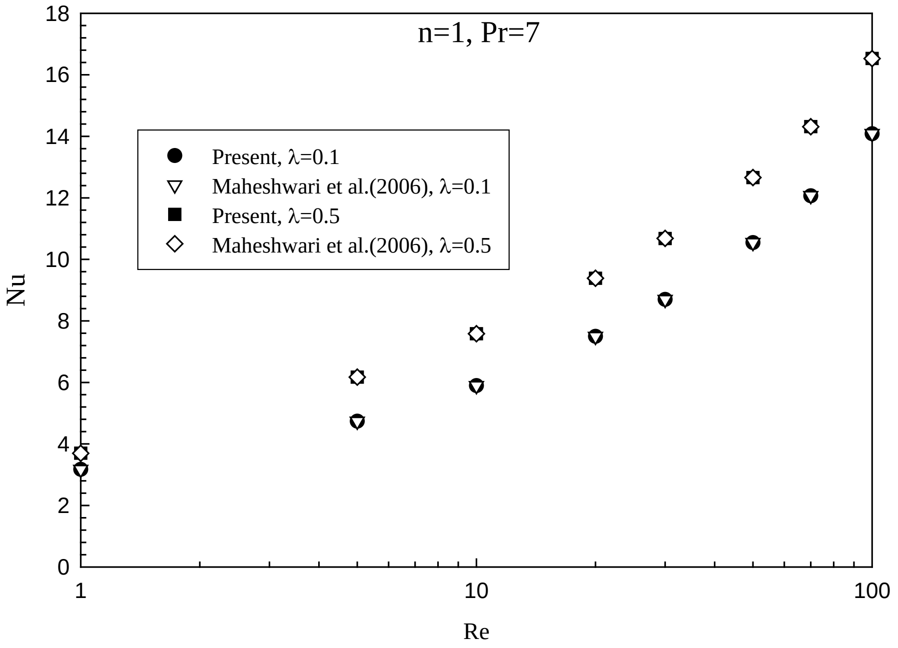

Figure 2(b). Comparison of mean Nu at different diameter ratios for the Newtonian fluid

Validation

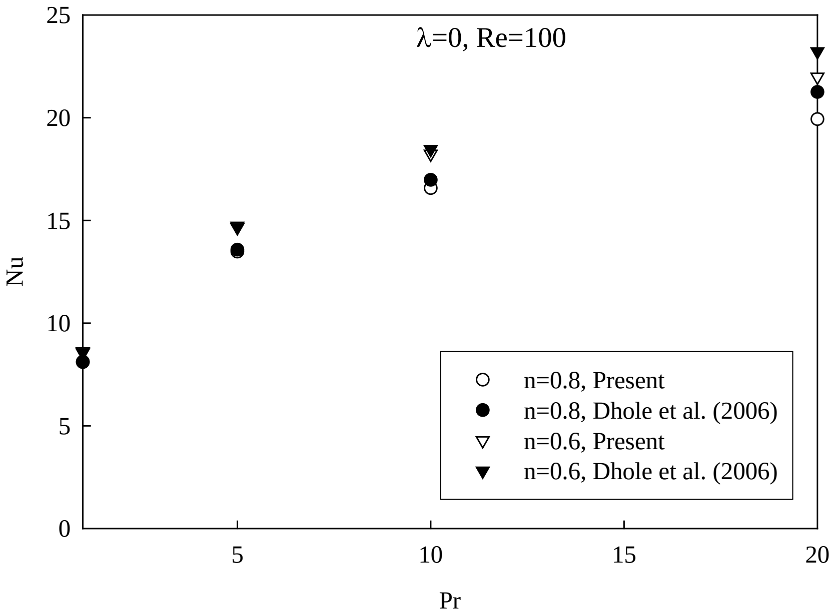

Figure 3(a). Comparison of mean Nu at different power law indices for the unconfined case

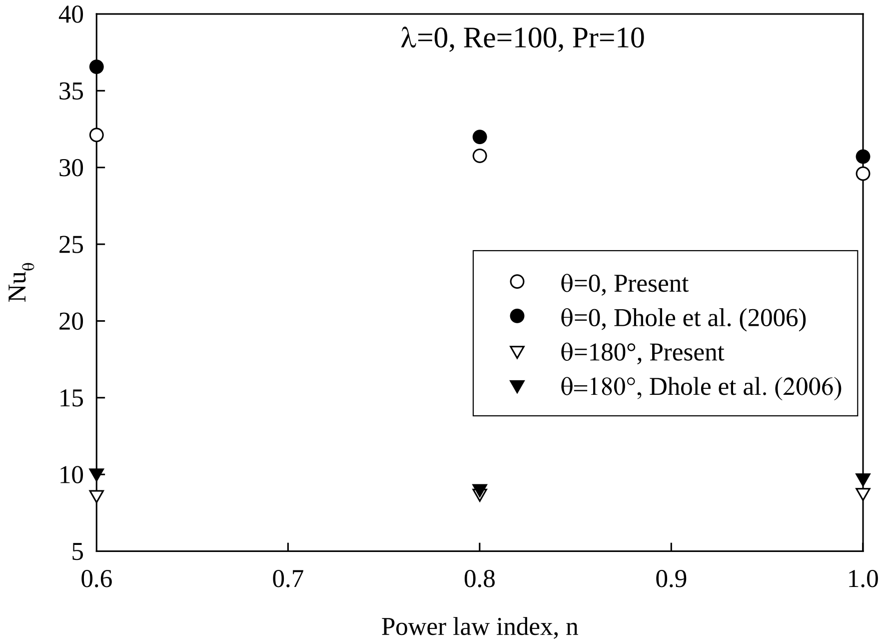

Figure 3(b). Comparison of local Nu at front and rear stagnation points for the unconfined case

Local Nu for Newtonian fluids

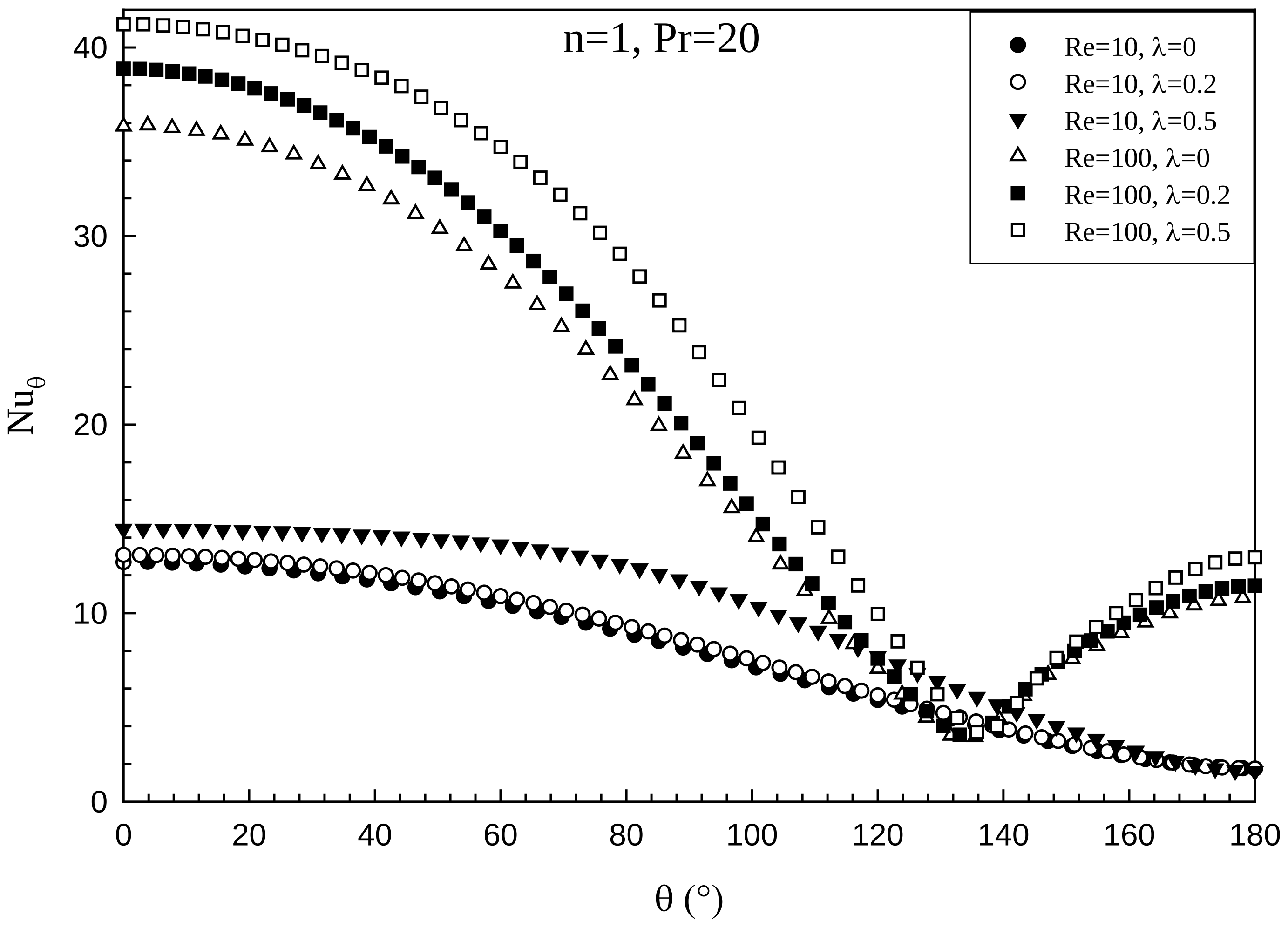

Figure 4. Local Nusselt number at the sphere surface at different Reynolds numbers and diameter ratios for Newtonian fluids

Local Nu for non-Newtonian fluids

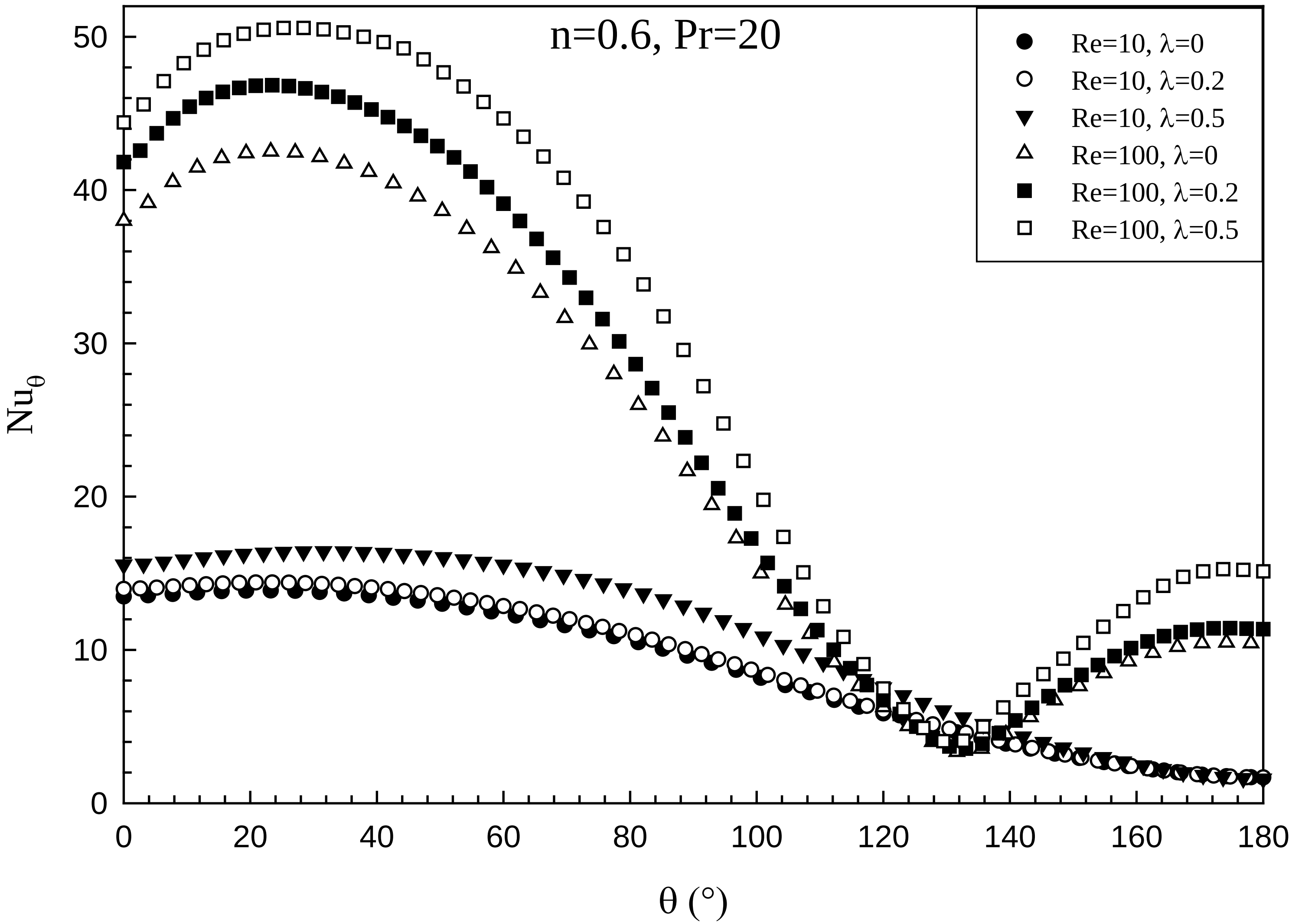

Figure 5(a). Local Nusselt number at the sphere surface at different Reynolds numbers and diameter ratios for n=0.6

Local Nu for non-Newtonian fluids

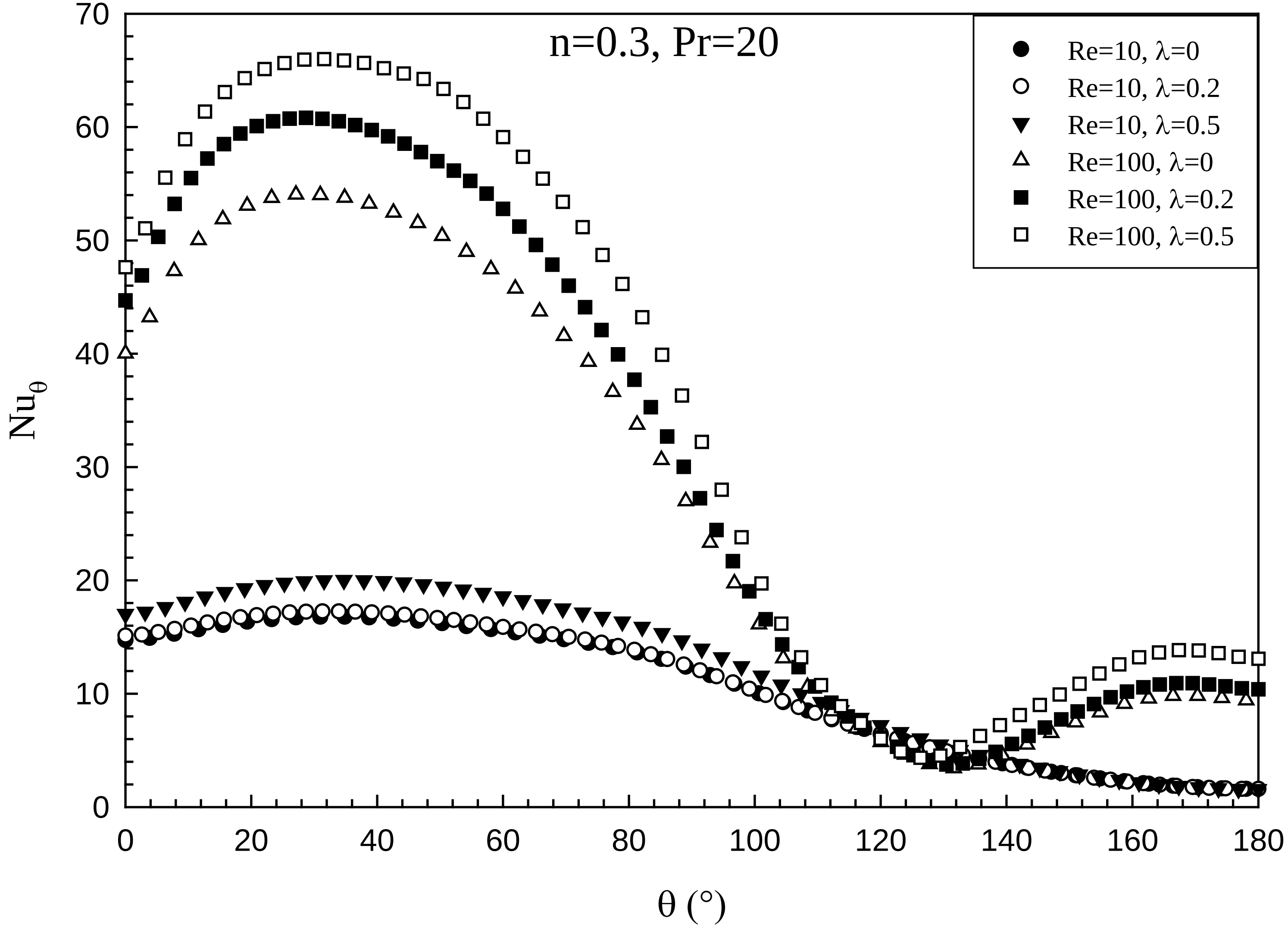

Figure 5(b). Local Nusselt number at the sphere surface at different Reynolds numbers and diameter ratios for n=0.3

Maximum Nu_θ away from the front stagnation point

Two competing mechanisms

1. Due to the shear-thinning behavior, lowering effective viscosity facilitates heat transfer

2. Temperature gradient decreases with the θ

Local Nu

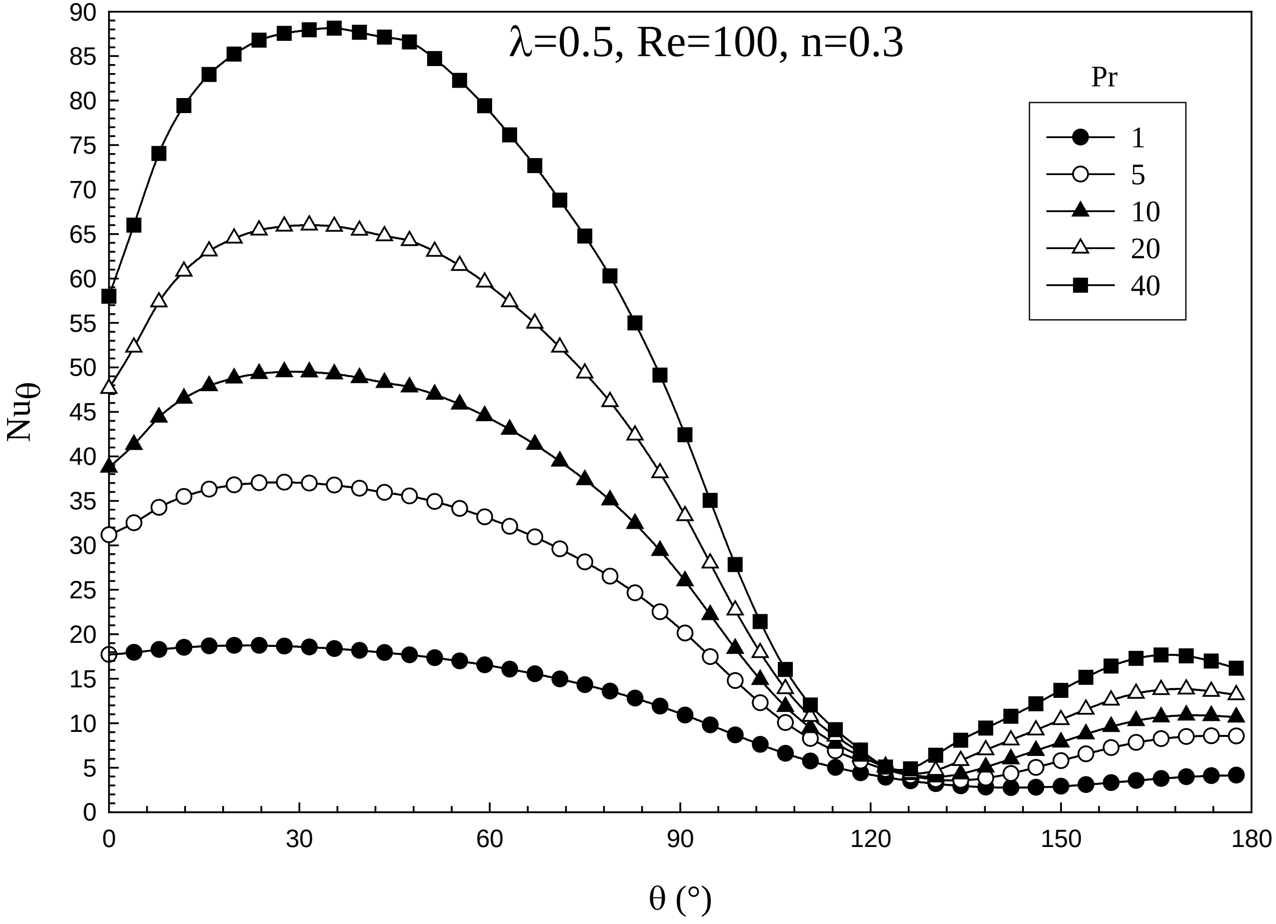

Figure 6. Effect of Prandtl number on local Nusselt number

Mean Nu for Newtonian fluids

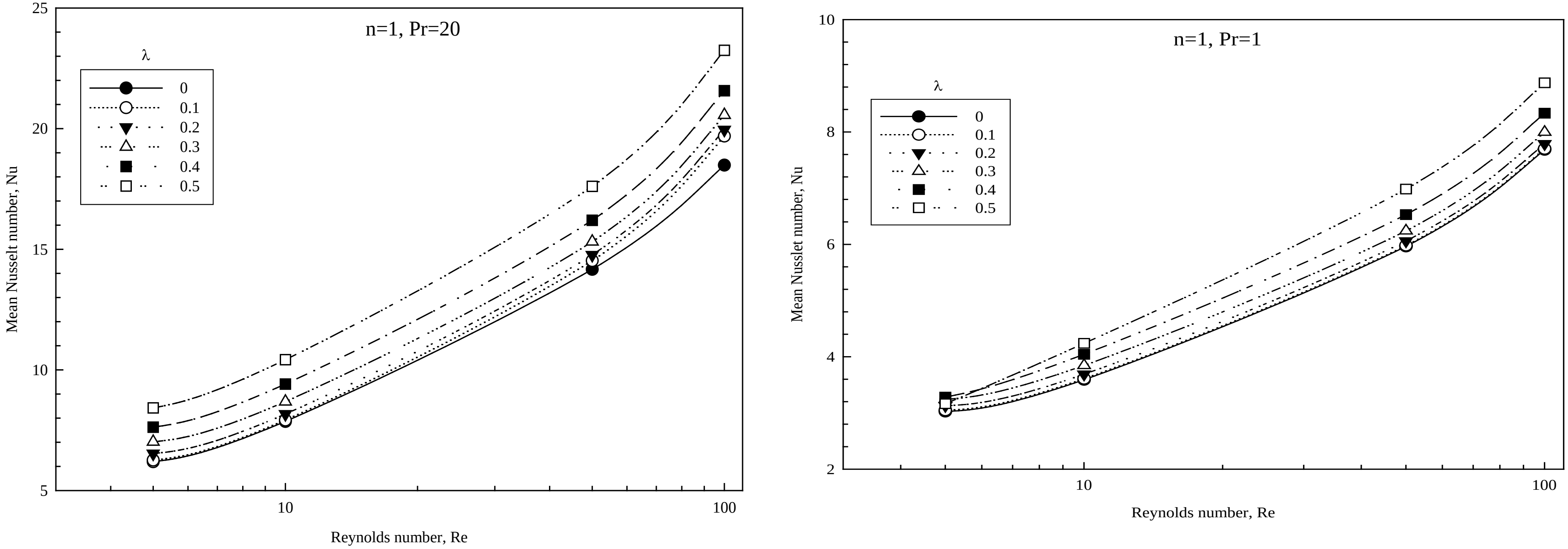

Figure 7. Effect of Re, λ and Pr on mean Nusselt number for Newtonian fluids

Mean Nu for non-Newtonian fluids

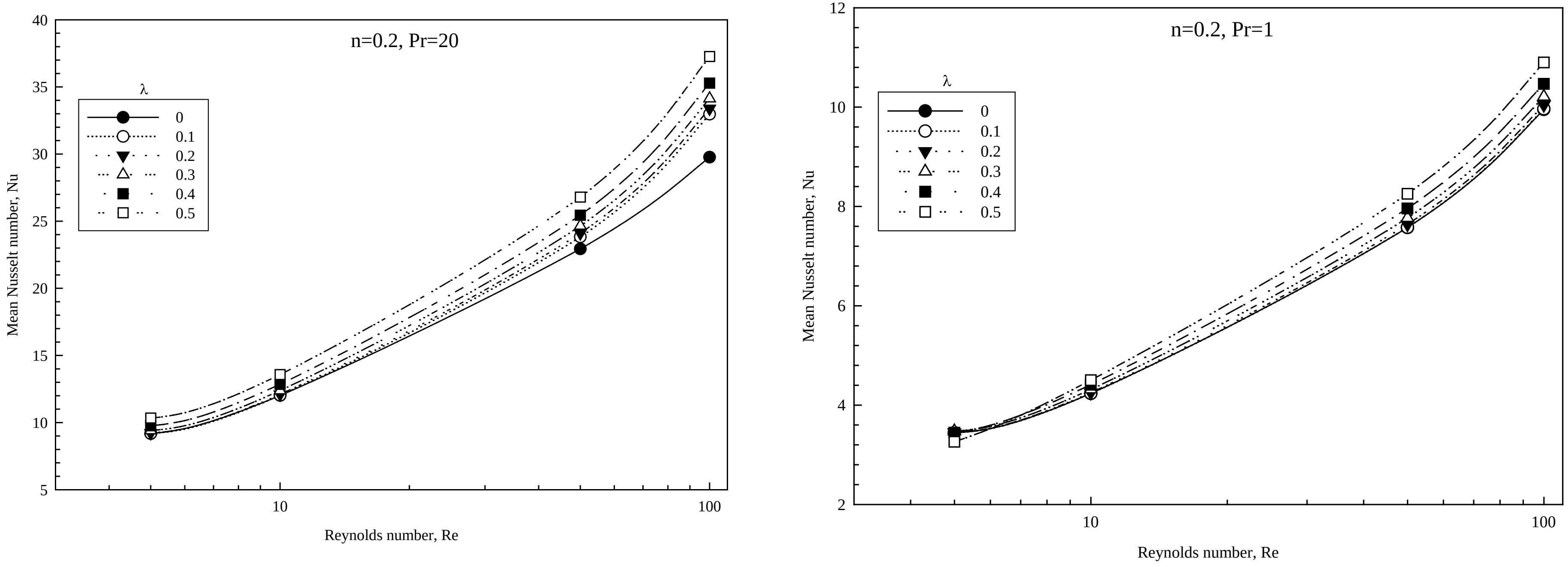

Figure 8. Effect of Re, λ and Pr on mean Nusselt number for non-Newtonian fluids

Effects of wall and n on mean average Nu become less at smaller Prandtl numbers

At smaller Pr's, advection is weak, conduction is dominant. Thus the flow characteristics are largely irrelevant. This phenomenon is stronger at n<1.

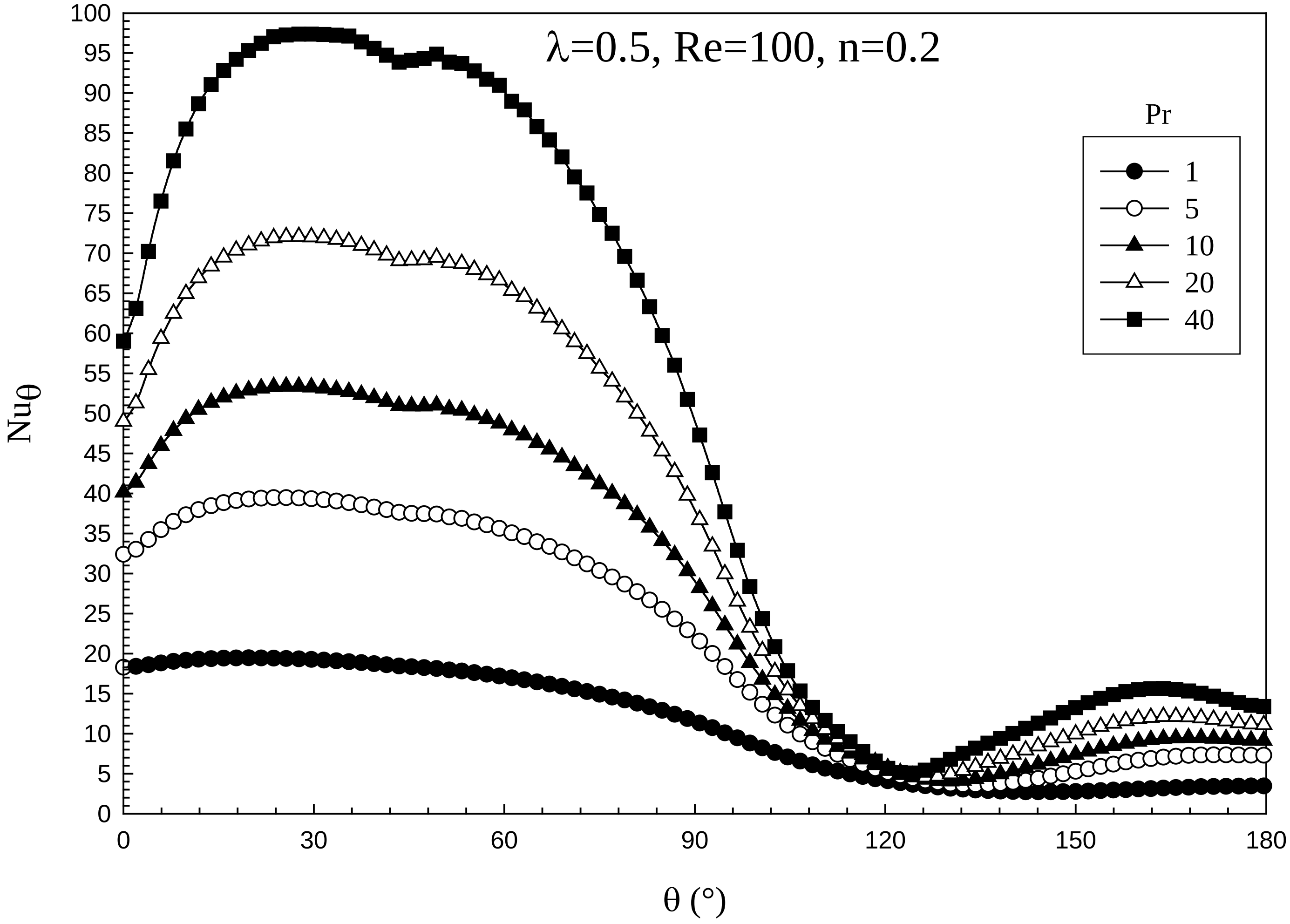

Figure 9. Instability increases with Pr

Conclusions

- Nu increases with the increase of Re, Pr, and λ, decrease of n
- Wall effects become less severe at lower Pr, Re and n
- Effects of wall with Re, Pr and n on heat transfer are coupled

Thanks