

Control of Systems Modeled by COMSOL Multiphysics as Distributed Parameter Systems

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Abstract: In systems and control theory dynamical systems described by partial differential equations are known as distributed parameter systems. So for modeling and dynamical analysis of distributed parameter systems wide possibilities are offered by COMSOL Multiphysics based on numerical solution of sets of partial differential equations by finite element method. Paper presents actual possibilities of control of systems modeled by COMSOL Multiphysics as distributed parameter systems in MATLAB & Simulink software environment by Distributed Parameter Systems Blockset for MATLAB & Simulink – Third-Party Product of The MathWorks www.mathworks.com/products/connections/

As model control problems control of radiation heating of complex-shape metal body and glass-melting furnace temperature field are demonstrated. Finally a service is presented for interactive formulation and solution of model control problems via the Internet www.dpscontrol.sk.

Keywords: modeling, control, distributed parameter systems, partial differential equations, numerical methods.

1. Introduction

In systems and control theory dynamical systems described by partial differential equations are known as distributed parameter systems (DPS) [1-3]. Thus for modeling and dynamical analysis of distributed parameter systems wide possibilities are offered by COMSOL Multiphysics [4] based on numerical solutions of sets of partial differential equations by finite element method. For control purposes of these systems usually some appropriate lumped parameter character manipulatable – actuating quantities are chosen as actuating quantities for control of output quantity fields - distributed parameter controlled quantities. These controlled DPS in input/output relations in

practice are lumped input and distributed parameter output systems (LDS). For control of dynamical systems given by numerical structures on complex-shape definition domains over 3D effective methods and software support are offered by control technology of DPS [5-8,10-12]. For demonstration of these possibilities as controlled systems simplified numerical models of complex-shape metal body radiation heating and glass-melting process temperature field in COMSOL Multiphysics software environment will be considered. Control synthesis will be solved by software product Distributed Parameter Systems Blockset for MATLAB & Simulink - Third-Party Product of The MathWorks (DPS Blockset) www.mathworks.com/products/connections/ [9]. For solutions of demonstrative control problems of processes modeled by COMSOL Multiphysics via the Internet an interactive service – Interactive Control will be presented by web portal www.dpscontrol.sk [7].

2. Controlled Distributed Parameter Systems

In control practice the distributed parameter controlled systems are mostly considered as LDS with lumped parameter manipulatable - input quantities $\{U_i(t)\}_i$ and quantity fields - distributed parameter output quantities on the output $Y(\mathbf{x}, t)$, where $\mathbf{x} = \{x, y, z\}$, Fig. 1.

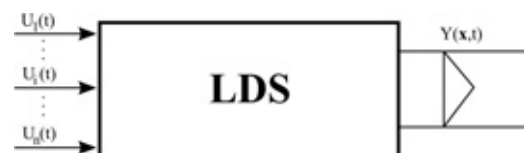


Figure 1. Lumped input and distributed parameter output system – LDS.

At discrete control block of zero-order hold units H is used along with block LDS at modeling dynamical relations among discrete

input quantities $\bar{U}(k)$ and distributed parameter output quantity $Y(\mathbf{x}, t)$.

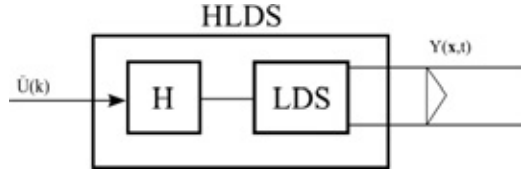


Figure 2. HLDS - LDS with block H of zero-order hold units.

For decomposition of HLDS dynamics into time and space components particular distributed parameter step responses are used: $\{\mathcal{Z}H_i(\mathbf{x}, k)\}_i$. To partial particular step responses with maximal amplitudes: $\{\mathcal{Z}H_i(\mathbf{x}_i, k)\}_i$ in points $\{\mathbf{x}_i\}_i$ discrete transfer functions are assigned: $\{SH_i(\mathbf{x}_i, z)\}_i$ as time components of controlled HLDS dynamics. As space components of HLDS dynamics are considered reduced partial particular step responses in steady-state: $\{\mathcal{Z}HR_i(\mathbf{x}, \infty)\}_i$, where $\{\mathcal{Z}HR_i(\mathbf{x}, \infty) = \mathcal{Z}H_i(\mathbf{x}, \infty) / \mathcal{Z}H_i(\mathbf{x}_i, \infty)\}_i$.

Distributed parameter characteristics $\{\mathcal{Z}H_i(\mathbf{x}, k)\}_i$ and $\{\mathcal{Z}HR_i(\mathbf{x}, \infty)\}_i$ are obtained by COMSOL Multiphysics, transfer functions: $\{SH_i(\mathbf{x}_i, z)\}_i$ are identified in MATLAB & Simulink environment.

As controlled LDS simplified numerical models of radiation heating of the complex-shape metal body and the glass-melting process temperature field in the COMSOL Multiphysics software environment are assumed.

2.1 Dynamical characteristics of radiation heating of complex-shape metal body

At radiation heating of the complex-shape metal body input voltages of radiation heating elements as lumped parameter input variables are considered.

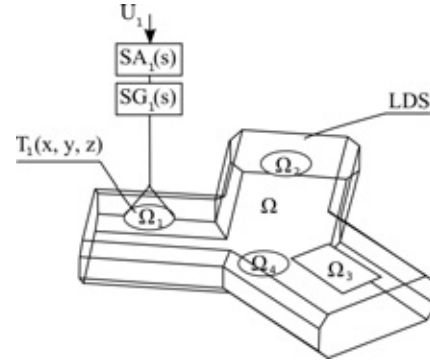


Figure 3. Controlled DPS as LDS – radiation heating of metal body of complex-shape.

$\{SA_i(s)\}_i, \{SG_i(s)\}_i$ - Laplace transfer functions of actuators

$\{T_i(x, y, z)\}_i$ - Shaping units in space relation

LDS - Lumped-input and distributed-output system

$\{U_i\}_i$ - Lumped parameter actuating variables

Ω - Complex-shape definition domain over 3D

$\{\Omega_i\}_i$ - Actuation subdomains of the metal body

The controlled distributed parameter variable is temperature field of the metal body $Y(\mathbf{x}, t) = Y(x, y, z, t)$. Computational net of the metal body is in the Fig. 4.

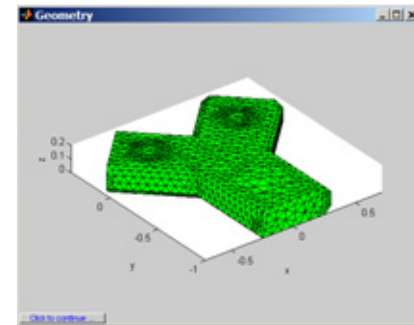


Figure 4. Computational net of the metal body.

At step changes of the input variables, particular distributed parameter step responses are obtained: $\{\mathcal{Z}H_i(\mathbf{x}, k)\}_{i=1,4}$, where vector \mathbf{x} represents an arbitrary point of the computational net.

From the set of partial particular courses the partial step responses with the maximal amplitudes in points $\{\mathbf{x}_i\}_{i=1,4}$ are chosen $\{\mathcal{Z}H_i(\mathbf{x}_i, k)\}_{i=1,4}$, Fig. 5. To these partial step responses discrete transfer functions are assigned by the identification methods in the MATLAB software environment: $\{SH_i(\mathbf{x}_i, z)\}_{i=1,4}$ which represent the time components of the dynamics of investigated controlled system.

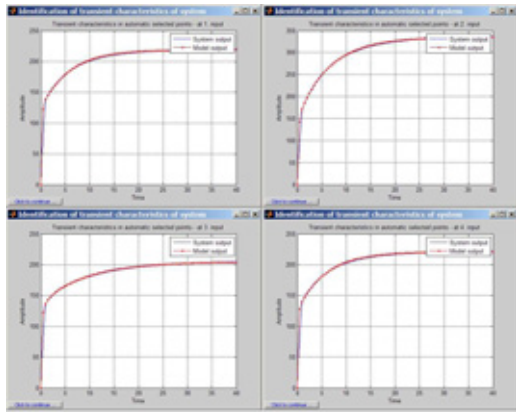


Figure 5. $\{\mathcal{Z}H_i(\mathbf{x}_i, k)\}_{i=1,4}$.

Discrete transfer function related to input U_1 :

$$\frac{140.5 z^2 + 158.8 z + 36.44}{1.314 z^3 + 7.833 z^2 + 6.965 z + 1}$$

Discrete transfer function related to input U_2 :

$$\frac{288.8 z^2 + 269.2 z + 55.88}{2.658 z^3 + 13.32 z^2 + 8.573 z + 1}$$

Discrete transfer function related to input U_3 :

$$\frac{214 z^2 + 251.6 z + 34.14}{1.818 z^3 + 12.21 z^2 + 10.65 z + 1}$$

Discrete transfer function related to input U_4 :

$$\frac{134.2 z^2 + 164.2 z + 36.8}{1.027 z^3 + 7.39 z^2 + 7.067 z + 1}$$

Further reduced distributed parameter step responses in steady-state are defined

$$\{\mathcal{Z}HR_i(\mathbf{x}, \infty) = \mathcal{Z}H_i(\mathbf{x}, \infty) / \mathcal{Z}H_i(\mathbf{x}_i, \infty)\}_{i=1,4}$$

as space components of investigated controlled system dynamics, Fig. 6.

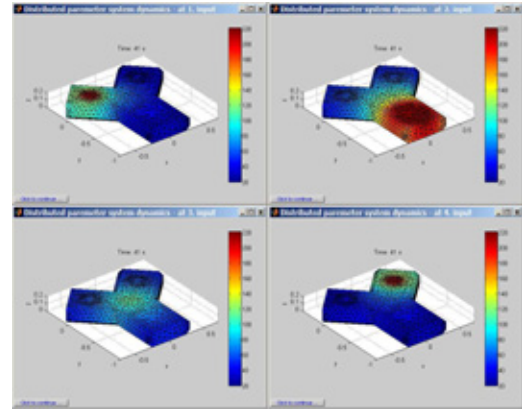


Figure 6. $\{\mathcal{Z}HR_i(\mathbf{x}, \infty)\}_{i=1,4}$.

2.2 Dynamical Characteristics of Glass-Melting Furnace Temperature Field

By similar procedure the dynamical characteristics for control of the glass-melting furnace can be defined, where manipulatable – input quantities are the gas flows to the individual burners and controlled variable is temperature field of arch of melting part of the furnace. The technological scheme of the furnace is illustrated in the Fig. 7.

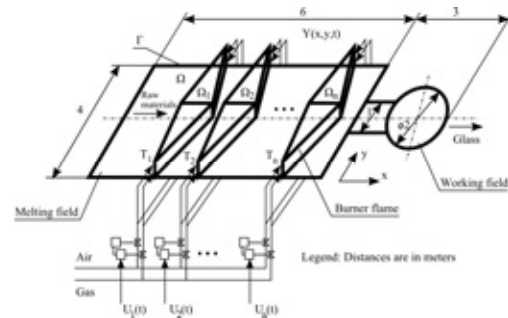


Figure 7. Technological scheme of glass-melting furnace.

Computational net of the glass-melting furnace model definition domain is in the Fig. 8.

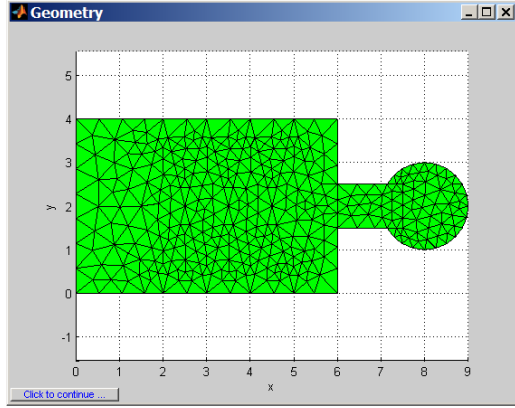


Figure 8. Computational net of the glass-melting furnace model definition domain.

At step changes of the lumped parameter input variables, $\{\mathcal{Z}H_i(\mathbf{x}, k)\}_i$ - distributed parameter particular step responses are obtained. Partial particular step responses $\{\mathcal{Z}H_i(\mathbf{x}_i, k)\}_{i=1,4}$ with the maximal amplitudes in points $\{\mathbf{x}_i\}_{i=1,4}$ are shown in the Fig. 9.

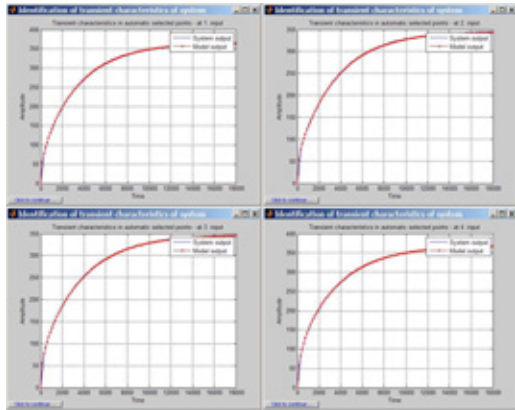


Figure 9. $\{\mathcal{Z}H_i(\mathbf{x}_i, k)\}_{i=1,4}$.

Discrete transfer function related to input U_1 :

$$\frac{1,329 \cdot 10^8 z^2 + 5,072 \cdot 10^5 z^2 + 365.6}{1,901 \cdot 10^8 z^3 + 2,731 \cdot 10^6 z^2 + 4284 z + 1}$$

Discrete transfer function related to input U_2 :

$$\frac{9,163 \cdot 10^7 z^2 + 4,341 \cdot 10^5 z + 347,9}{1,334 \cdot 10^8 z^3 + 2,026 \cdot 10^6 z^2 + 4273 z + 1}$$

Discrete transfer function related to input U_3 :

$$\frac{8,829 \cdot 10^7 z^2 + 3,786 \cdot 10^5 z + 345,6}{1,394 \cdot 10^8 z^3 + 1,806 \cdot 10^6 z^2 + 4100 z + 1}$$

Discrete transfer function related to input U_4 :

$$\frac{8,498 \cdot 10^7 z^2 + 4,984 \cdot 10^5 z + 368,7}{1,205 \cdot 10^8 z^3 + 2,02 \cdot 10^6 z^2 + 4225 z + 1}$$

Further reduced distributed parameter step responses in steady-state are defined

$$\{\mathcal{Z}HR_i(\mathbf{x}, \infty) = \mathcal{Z}H_i(\mathbf{x}, \infty) / \mathcal{Z}H_i(\mathbf{x}_i, \infty)\}_{i=1,4}$$

as space components of investigated controlled dynamics, Fig. 10.

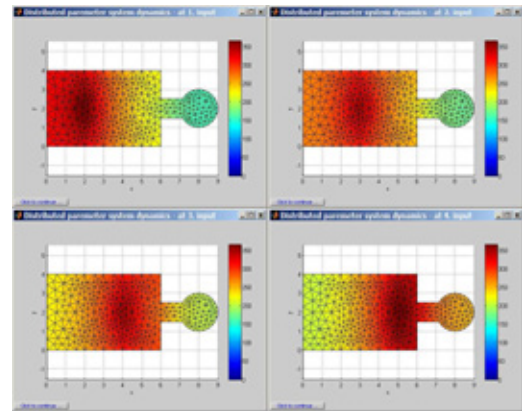


Figure 10. $\{\mathcal{Z}HR_i(\mathbf{x}, \infty)\}_{i=1,4}$.

3. Synthesis of Control

Synthesis of control is decomposed into space and time tasks based on decomposition of controlled system dynamics into space and time components. HLDS is lumped input and distributed parameter output system with zero-order hold units. CS – control system, SS – space part of control synthesis solved as approximation problem of control deviation $E(\mathbf{x}, k)$ on the set of space components of controlled system dynamics - $\{SHR_i(\mathbf{x}, \infty)\}_i$. Vector of optimal approximation parameters $\bar{E}(k)$ as vector of lumped parameter control deviation enters into block of time synthesis TS, where controllers $\{R_i(z)\}_i$ are designed in relation to time components of controlled system dynamics - $\{SH_i(\mathbf{x}, z)\}_i$, Fig. 11.

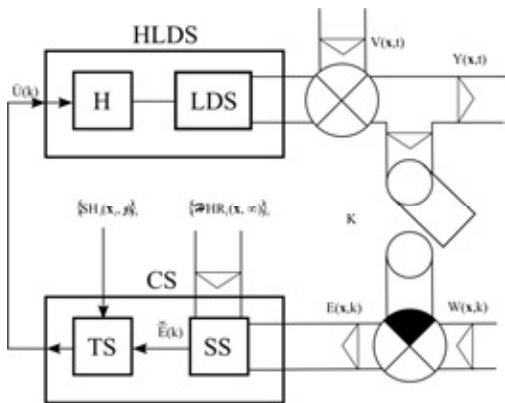


Figure 11. Distributed parameter system of control.

- HLDS – Lumped input / distributed parameter output system with zero-order hold units
- CS – Control synthesis
- TS – Time control synthesis
- SS – Space control synthesis
- K – Time/space sampling
- $Y(\mathbf{x}, t)$ – Distributed controlled variable
- $W(\mathbf{x}, k)$ – Reference variable
- $V(\mathbf{x}, t)$ – Disturbance variable
- $\bar{E}(k)$ – Vector of control errors
- $\bar{U}(k)$ – Vector of control variables
- $E(\mathbf{x}, k)$ – Distributed parameter control error

By this procedure control problems of heated metal body and glass-melting process temperature field are solved in the software environment Distributed Parameter Systems Blockset for Simulink – Third-Party MathWorks Product with results in the next figures.



Figure 12. DPS Blockset on the web portal of The MathWorks.

3.1 Control of metal body radiation heating

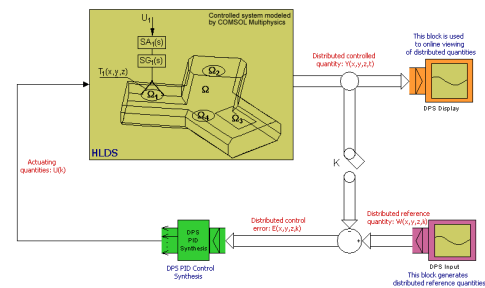


Figure 13. Control loop of the metal body heating.

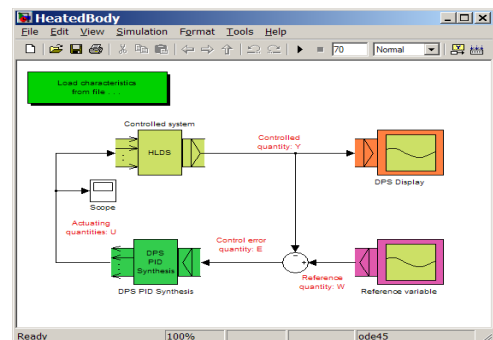


Figure 14. Control loop of the metal body heating in DPS Blockset environment.

Where the solution of control task is obtained as reference variable, control quantities, quadratic norm of distributed control error and controlled quantity (in counterclockwise direction), Fig. 15.

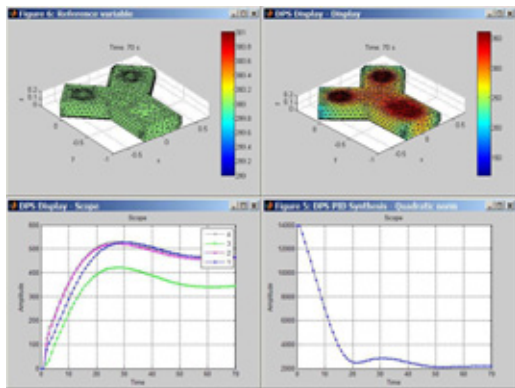


Figure 15. Simulation of metal body heating control.

3.2. Control of temperature field of glass-melting furnace

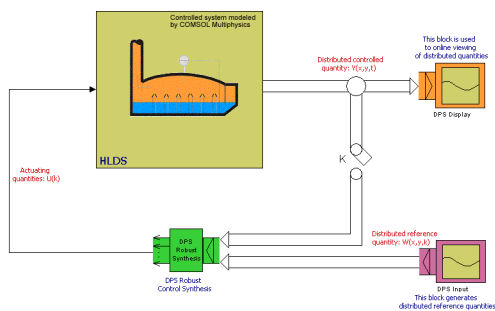


Figure 16. Control loop of temperature field of glass-melting furnace.

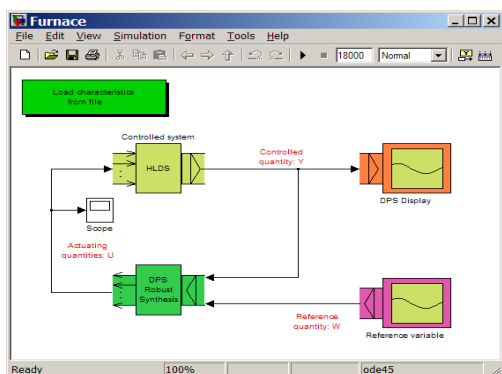


Figure 17. Control loop of the glass-melting furnace in DPS Blockset environment.

Finally the solution of control task is obtained as reference variable, control quantities, quadratic norm of distributed control error and controlled quantity (in counterclockwise direction), Fig. 18.

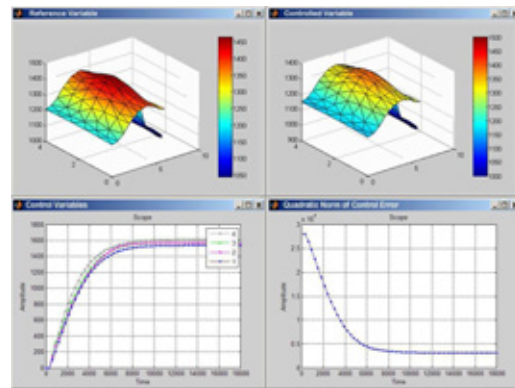


Figure 18. Simulation of glass-melting furnace temperature field control.

4. Interactive control via the Internet

For solution of similar model control problems of distributed parameter systems modeled in COMSOL Multiphysics software environment flexible possibilities are offered by service Interactive Control of web portal www.dpscontrol.sk, Fig. 19.



Figure 19. Web portal Distributed Parameter Systems with Interactive Control – www.dpscontrol.sk

By means of FTP server interested person enters geometry of controlled system along with

computational net, distributed step responses related to particular lumped parameter manipulatable – input quantities and distributed parameter reference quantity in points of computational net as text files. Then service Interactive Control provides via the Internet animated results of control process simulation in form of DPS Blockset outputs, Fig. 15, 18.

5. Conclusions

COMSOL Multiphysics is a powerful interactive software environment for modeling and dynamical analysis all kinds of scientific and engineering problems based on partial differential equations. In this way COMSOL Multiphysics opens new possibilities for control of distributed parameter systems described by numerical structures along with Distributed Parameter Systems Blockset for MATLAB & Simulink – Third-Party MathWorks Product in all kinds of scientific and engineering disciplines. Presented paper demonstrates these actual possibilities along with Interactive Service for formulation and solution of model control problems via the Internet.

6. References

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