Determination of Constitutive Properties Using DIC-Displacement Data and U-FEM

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Abstract

The constitutive properties of a paperboard side-notched composite loaded vertically were determined in inverse method (IM) using a single component of measured displacement data. The cost function, which is the difference between the measured v-displacement and the reconstructed displacement from finite element method, was minimized using the Levenberg-Marquardt Algorithm. Displacements were recorded using digital image correlation (DIC). The primary advantage of this new formulation is the direct use of displacement data, eliminating the need for numerical differentiation when strain data is required. The inverse method algorithm determined the constitutive properties with errors smaller than 5%. Selection of initial conditions, test geometry configuration and comparison with other inverse methods will be addressed.
A set of 516 random DIC-measured v-displacement data points was used to determine the four material properties of the paperboard (E_1, E_2, G_{12}, and ν_{12}). The inverse method used is combining the reconstructed v-displacement from the finite element scheme using COMSOL Multiphysics® with the DIC-measured v-displacement. Through an iterative process that determines the new constitutive parameters, the displacement difference between the produced displacement from COMSOL and the DIC-measured displacements is minimized. The function to be minimized is
$f(\hat{u}_{FEM},P)=\|r\|$, where $r=\hat{u}_{DIC}-\hat{u}_{FEM}$
where $\hat{u}_{FEM}$ and $\hat{u}_{DIC}$ are vectors containing nodal v-displacements determined by COMSOL and DIC, respectively, P is a vector containing the constitutive parameters, E_1, E_2, G_{12}, and ν_{12}, and $\|r\|$ is the norm of r. Because the previous equation is nonlinear with respect to P, iterative procedures are necessary to minimize $f(\hat{u}_{FEM},P)$ and determine P. Levenberg-Marquardt Algorithm (LMA) is commonly used because it combines the benefits of Steepest Descent Method and Gauss-Newton Method. The LMA has the following form
$P_{i+1}=P_i-(J^T J+\lambda\cdot diag(J^T J))^{-1} J^T r$
where i is iteration number, J and J^T are Jacobian and Jacobian transpose, determined by backward difference, $J(m,n)=\partial r_m \partial P_n$; m is the number of nodal displacements and n is the number of constitutive parameters (4 in this case), and λ is a non-negative damping factor, adjusted each iteration, to alternate between Steepest Descent and Gauss-Newton Methods.
Different initial guesses were used to start the inverse method and determine the constitutive properties. As shown in Figure 1, the proposed method converges to the average values of the paperboard properties (determined experimentally) regardless the

initial guesses. Prior to using the actual measured displacement data, a simulated v-displacement from COMSOL was used to verify the reliability and robustness of the method. The primary advantage of the proposed inverse method is the fact that only one in-plane displacement was used even though DIC provides the two in-plane displacements. Furthermore, the constitutive properties were determined from a single experimental test. The use of a COMSOL LiveLink™ for MATLAB® facilitated the passing of information and to and from the optimization algorithm, allowing for streamlined, efficient and accurate of the LMA.

Figures used in the abstract

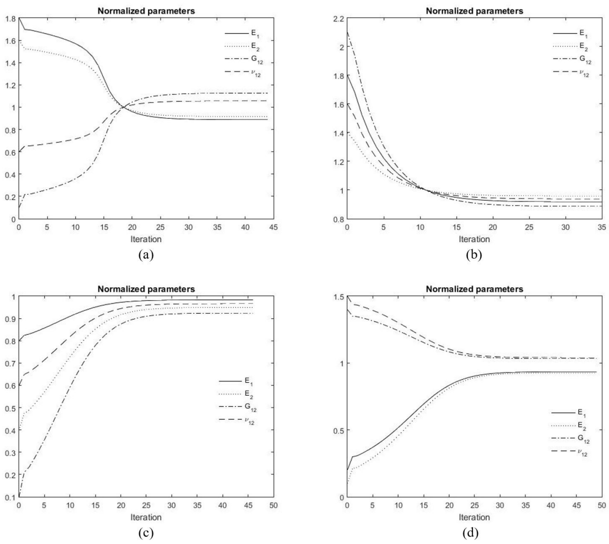

Figure 1: The convergence of the inverse method for different initial guesses.