COMSOL® Experience Of Biomechanical Modeling Of Shells And Plates In Ophthalmology

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Introduction

Modern problems of biomechanics, in particular, the problems of modeling vision correction in ophthalmology are associated with the study of the stress-strain state of shells and plates made of complex materials (fig.1 shows general structure of human eye). These problems are essentially nonlinear in geometry and in physical properties. It is necessary to model large and inelastic deformations on the one hand, and to set complex loading systems on the other. Solving such problems analytically is very hard, so modeling using FEM methods is the best way to study them.

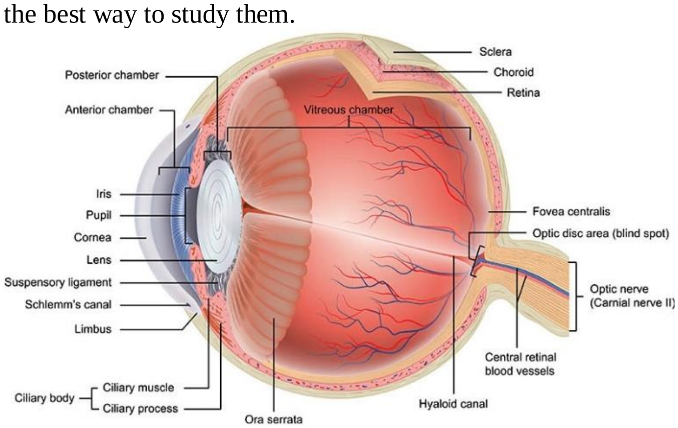

Figure 1. Anatomy of human eye.

This paper considers the deformation of a significantly plastic anisotropic plate under hydrostatic loading, which simulates the bending of the section of the central eye nerve when intraocular pressure increases (see fig.2 for eye nerve structure). The biological tissue of nerve fibers has different strength properties under tension and compression. Such materials are called plastically anisotropic or materials with SD-effect. The elastic characteristics of the nerve tissue are very low and its strength is negligible. Determining these characteristics experimentally is a rather difficult task. Medical data of biological SD-tissue is used together with the classical Hill's theory of plasticity and various mathematical models in which the transversal isotropy parameter and the plastic anisotropy parameter (SD-parameter). Asymptotic formulas for stresses are presented which are used for numerical modeling and graphical representation of elastoplastic properties of circular SD-plates. A numerical solution of the plate bend after calculating the system of fifth-order differential equations is obtained. Euler difference method and software package COMSOL 5.4 are used to solve the problem.

The classical theory of elastic and elastoplastic bending was developed in the scientific works of R. von Mises, R. Hill, L. H. Donnell. Elasticity theory for thin isotropic plates was introduced by S.P. Timoshenko. Plasticity was added to elasticity theory by V.V. Sokolovsky [1]. Bending problems of SD-plates are solved in works [2, 3, 4].

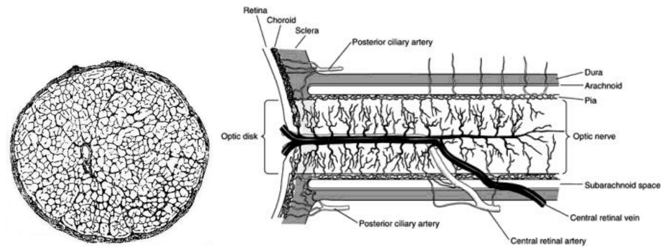

Figure 2. Schematic anatomy of optic nerve (right) and microscopic picture of cross section of optic nerve (left).

Mathematical model

This paper considers the problem of elastic-plastic bending of a round freely supported SD-plate possessing the properties of transverse anisotropy and uniformly loaded with pressure p on the upper surface.

Figure 3 shows the central cross section of a curved circular plate, which shows following parameters:

- h is the half thickness of the plate,
- x_1, x_2 are the radiuses of the plastic regions on the top and the bottom of the plate respectively,
- a_1, a_2 are the depths of the plastic zones from the bottom and from the top of the plate respectively.

The plastic regions are shaded. In the case presented on fig.3 the neutral surface does not coincide with the geometrically average surface. A solid line — the neutral surface, a dashed line — geometrically mid-plane.

The beginning of the coordinate system is in the center of the plate on the neutral surface (point O on fig.3). The development of plastic zones is disturbed.

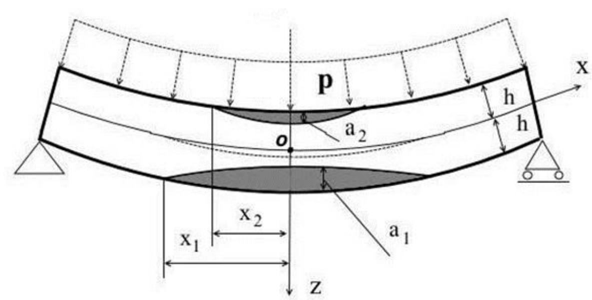

Figure 3. Elastic-plastic bending of a circular plate from SD material.

In the articles [3], [4], the mathematical model for the SD-plate is made more complicated and a new criterion of fluidity is proposed:

$$\bar{k} = \sqrt{\sigma_r^2 - A\sigma_r\sigma_\theta + \sigma_\theta^2} + \sigma\beta \qquad (1)$$

Here σ_r, σ_θ are the stresses in the plane of the plate. The average stress σ is equal to:

$$\sigma = \frac{\sigma_r + \sigma_\theta}{3} \qquad (2)$$

In (1) the transversal isotropy parameter A is used, which varies from 1 to 2 and can be found by following relation:

$$A = 2 - \frac{\left(\sigma_{pz}+\sigma_{cz}\right)^2}{\left(\sigma_p+\sigma_c\right)^2} \frac{\sigma_p^2\sigma_c^2}{\sigma_{pz}^2\sigma_{cz}^2}, \tag{3}$$

where σ_p is the yield point for uniaxial tension in the plane of the plate, σ_{pz} is the yield point for uniaxial tension in a direction perpendicular to the plane of the plate, σ_c is the yield point by uniaxial compression in the plane of the plate, σ_{cz} is the yield point for uniaxial compression in a direction perpendicular to the plane of the plate.
The parameter β characterizes the plastic anisotropy property - SD effect.
For uniaxial stretching (the formula on the left) and uniaxial compression (the formula on the right), criterion (1) is equal to:

$$\bar{k} = \sigma_p + \frac{1}{3}\sigma_p\beta, \qquad \bar{k} = \sigma_c + \frac{1}{3}\sigma_c\beta. \tag{4}$$

Thus, the relationship between β, σ_p and σ_c is:

$$\frac{\sigma_c}{\sigma_p} = \frac{3+\beta}{3-\beta} \tag{5}$$

In the case of a biaxial stress state, the criterions for stretching and compression can be written accordingly:

$$\bar{k} = \sigma_{pz}\sqrt{2-A} + \frac{2}{3}\sigma_{pz}\beta,$$
$$\bar{k} = \sigma_{cz}\sqrt{2-A} - \frac{2}{3}\sigma_{cz}\beta, \tag{6}$$

or

$$\sigma_{pz}\sqrt{2-A} + \frac{2}{3}\sigma_{pz}\beta = \sigma_{cz}\sqrt{2-A} - \frac{2}{3}\sigma_{cz}\beta, \tag{7}$$

hence

$$\beta = \frac{3\sqrt{2-A}}{2}\frac{(\sigma_{cz}-\sigma_{pz})}{(\sigma_{cz}+\sigma_{pz})} \tag{8}$$

The plate bending theory is based on the plane stress state model. The deformation of the transverse shear is ignored. The stress in the direction perpendicular to the plane of the plate is assumed to be zero. Proceeding from formulas (5), (6) and (8) it is possible to establish a connection between yield strengths

$$\frac{\sigma_c}{2\sigma_{cz}} = \left(\frac{\sigma_c}{\sigma_p} - 1\right)\left(\frac{\sigma_{cz}}{\sigma_{pz}} - 1\right)^{-1} \tag{9}$$

If the σ_p, σ_c, σ_{pz}, σ_{cz}, are known, then the values of β and A can be calculated. Let's assume that $\sigma_p \leqslant$, σ_c, then from formula (9) it follows that $\beta \geqslant 0$, and from formula (10) that A$\leqslant$ 2.
From the point of view of the evaluation of the stressed state of the plate the most critical is in its center, therefore the stresses in the plastic regions near the centers of the top and the bottom surfaces of the plate should be considered.
Suppose that $\beta \ll 1$. In this case in the center of the plate $\sigma_\theta = \sigma_r$ and formula of the stresses take the form [5]:

$$\sigma_\theta = \sigma_r = \frac{\bar{k}}{a}\left(\frac{1}{sign(z)F} - \frac{2\beta}{3a}\left(1+\frac{1}{F^2}\right) + \frac{4\beta^2}{sign(z)9a^2F}\right), \tag{10}$$

where

$$a = \sqrt{2-A}, \qquad F = 1 - \frac{sign(z)2\beta}{3a}. \tag{11}$$

After expanding into a series and neglecting the terms of a higher order, asymptotic formulas are obtained:

$$\frac{\sigma_+}{\sigma_p} = \frac{\beta+3}{3\sqrt{(2-A)}}\left(1 - \frac{2\beta}{3\sqrt{(2-A)}}\right) \tag{12}$$

$$\frac{\sigma_-}{\sigma_p} = -\frac{\beta+3}{3\sqrt{(2-A)}}\left(1 + \frac{2\beta}{3\sqrt{(2-A)}}\right) \tag{13}$$

Numerical Modeling

It becomes possible to estimate the influence of the parameters A and β on the stresses in the plate without solving the large problem of elastoplastic equilibrium of the plate [3].
The results of calculations using formulas (12) and (13) are given in Table 1.

Table 1: Dependence of stresses on the parameters β and A.

	A = 1.1	A = 1.1	A = 1.2	A = 1.2	A = 1.3	A = 1.3
β	σ_-/σ_P	σ_+/σ_P	σ_-/σ_P	σ_+/σ_P	σ_-/σ_P	σ_+/σ_P
0	1.054	1.054	1.119	1.119	1.195	1.195
0.01	1.065	1.050	1.131	1.114	1.205	1.185
0.05	1.110	1.033	1.178	1.094	1.245	1.150
0.1	1.165	1.0128	1.242	1.067	1.333	1.136

As the transversal isotropy parameter increases, the stresses increase. An increase in the parameter A by 10% causes an increase in stresses at β = 0 by 7%, and at β = 0.1 by 10%, therefore, the rate of stress growth with increasing β rises. With an increase in β by 5% and a constant A, the compressive stress increases by 5.5%, and the tensile stress drops by 2.3%. Analysis of the results of numerical simulation shows that for weak plastic anisotropy, the influence of the transversal isotropy parameter is greater than the effect of the SD, but with a strong plastic anisotropy, the effect of the SD increases substantially. This conclusion becomes even more obvious on the plots of the stress functions which depend on the parameters A and β.

Simulation Results

Research shows that plasticity area in the compression zone of the plate is substantially smaller than those in the tension zone. It is assumed that the yield strength during compression is greater than that under tension. To calculate the bending, the COMSOL 5.4 software package is used. Depending on the pressure, sizes of plasticity zones are calculated (see fig.4).

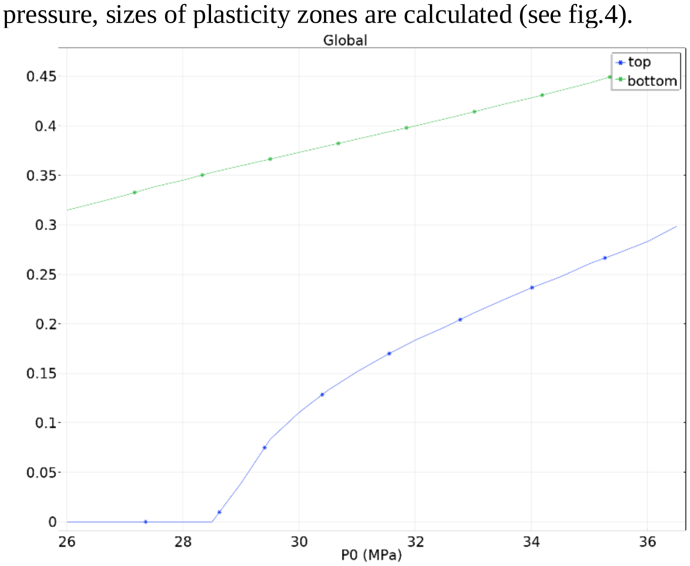

Figure 4. The radiuses of plasticity "spot" at the top (blue) and the bottom (green) of the plate for P *26-36.5* MPa.

Fig. 4 shows increasing radiuses of plasticity "spot" at the top and the bottom of the plate while raising the load from 26 MPa to 36.5 MPa.

According to the results of the calculation, the magnitude of the plasticity "spot" and the depth of plasticity areas significantly depends on the condition of compression or tension (see fig.5 and fig.6).

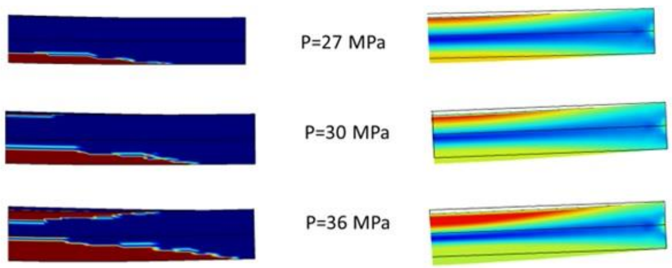

Figure 5. Results of FE modeling - plastic anisotropy (left) and orthotropy (right) of the plate.

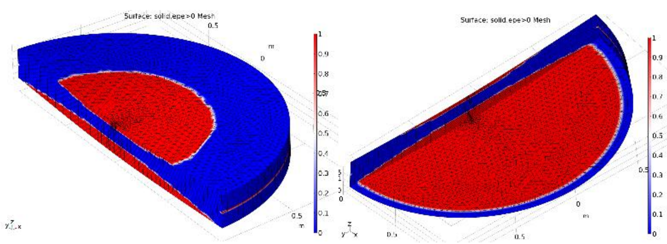

Figure 6. The plasticity "spot" at the top (left) and bottom (right) of the plate for $P=36$ MPa.

FE Modeling of Optic Nerve

Based on real microscopic picture (see fig.2 (left)) 2D geometrical model is built with 3 different isotropic materials (see fig.7) corresponding to:

- nerve bundles (orange color);
- soft shell, walls of blood vessels, and septa (blue color), which separates nerve bundles;
- central retinal artery and vein and small capillary (red color).

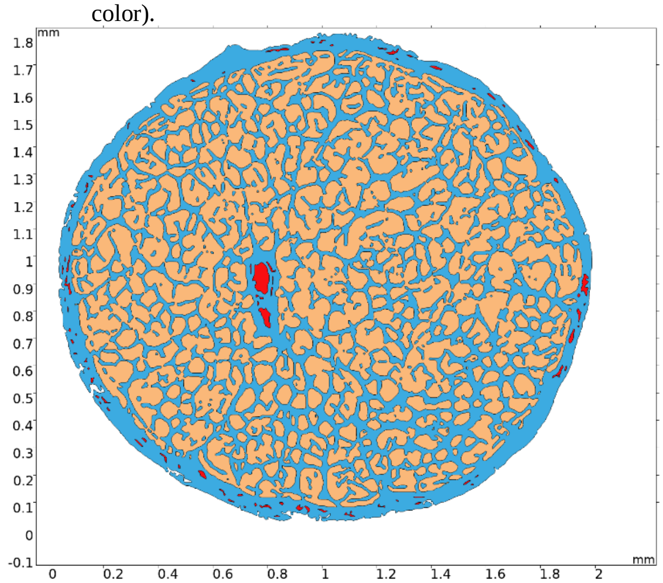

Figure 7. 2D geometry of optic nerve base on real microscopic picture; nerve bundles - orange, soft shell – blue, blood – red.

Model is built in plate interface (branch of Structural Mechanics). It allows to use thin flat structures of cross section of optic nerve, being loaded with intraocular pressure (IOP) P_0 in a direction out of the plane. In this calculation $P_0=80$ mmHg, which corresponds to IOP level during creation of corneal flap (step of vision correction), intraocular injection, or eyes rubbing [6]. The reason to model such high pressure is to investigate influence of such level of IOP on stress-strain state in the area of central retinal blood vessels. Fig.8 show the FE mesh in that area.

Fig.7 and fig.8 shows that geometry of the model is very difficult with no pattern or simple models.

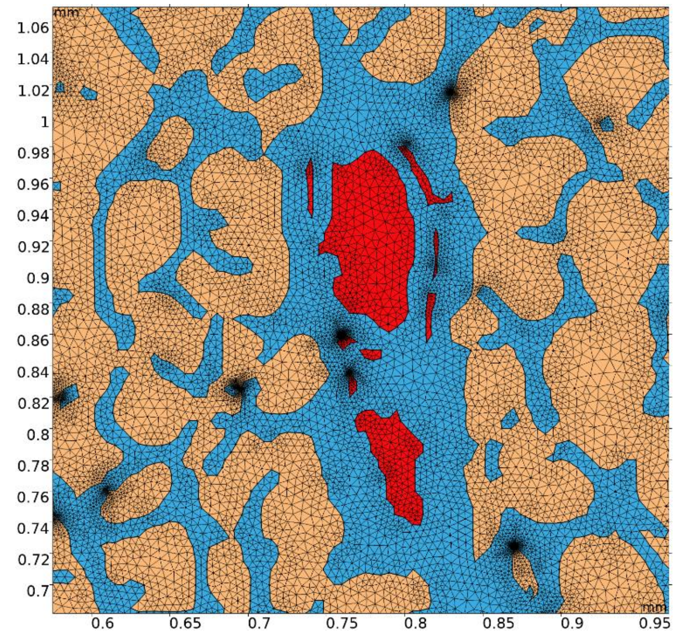

Figure 8. Mesh of central zone with central retinal blood vessels; nerve bundles - orange, walls of blood vessels and septa – blue, blood – red.

For boundary conditions the outer edge of the plate is pinned. So, zero displacement of that edge in any direction are prescribed, but rotations are free.

FE Modeling Results

Fig.9 shows displacement of the cross section of optic nerve in a perpendicular direction.

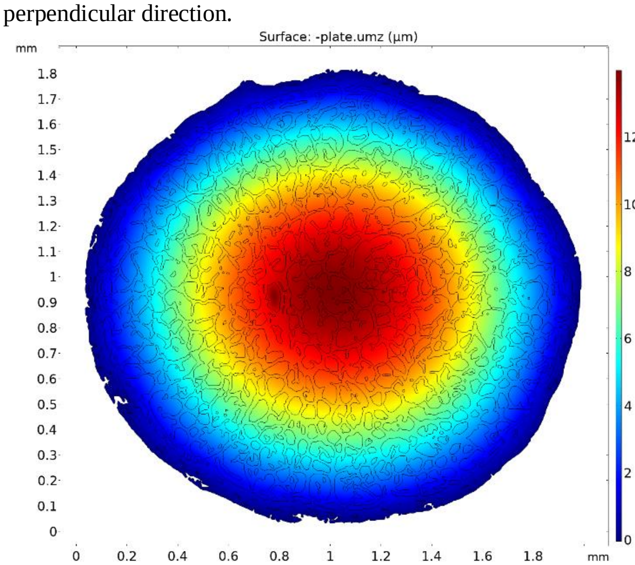

Figure 9. Displacements of the plate in the direction out of surface.

Fig.10 and fig.11 shows stress on the surface of the cross section of optic nerve.

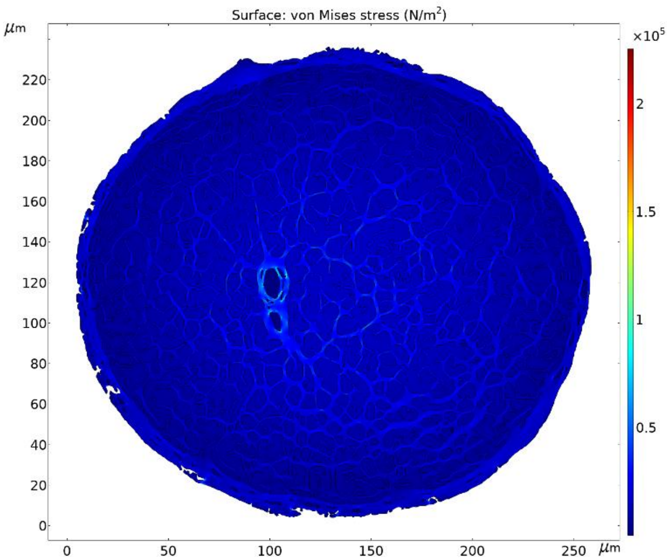

Figure 10. Midsurface von Mises stress of the cross-section of optic nerve

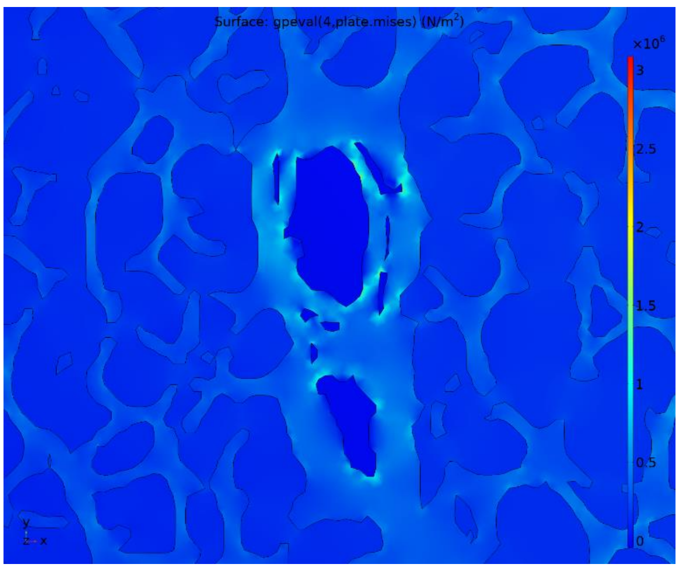

Figure 11. von Mises stress in central zone around central retinal blood vessels.

Conclusions

Numerical modeling and graphical representation of the elastoplastic properties of circular transversely isotropic and plastic anisotropic plates showed that for surface stress functions a solution to the problem of optimizing the selection of the parameters of transversal isotropy and plastic anisotropy under the condition of minimum stresses is possible.
The application of the yield criterion made it possible to construct asymptotic formulas for their calculation, taking into account the transversal isotropy and the SD effect for the elastoplastic bending of a circular plate. The formulas obtained are universal and estimate the influence of the parameters of transversal isotropy and SD effect on the stress-strain state of any material satisfying the described conditions. Asymptotic formulas allows to make a rapid evaluation of the stress state of a plate without heavy calculations, which is important in engineering practice.
FE modeling shows that areas next to an aorta and a vein are highly stressed and leads to a reduction in the size of the cross section of the vein and aorta. Which means that high intraocular pressure has negative impact on blood circulation and can lead to different eye diseases.
As a result, we can conclude that the capabilities of the COMSOL software package allow us to investigate many problems of nonlinear deformation of SD-materials and that difficult geometrical FE problems in the ophthalmology field can be solved.

References

1. Sokolovskiy V. V., *Theory of plasticity*, 608 p. Moscow: Vysshaya shkola, 1969
2. Pavilaynen G.V., Elastic-plastic bending of a circular transversally isotropic plate, *Vestn. Leningr. un-ta*, **№ 13**, Pp. 70-75 (1983)
3. Yushin R.YU., On the possibility of the account of plasticity anisotropy when bending round plates, *Vestn. S.-Peterb. un-ta*, **Ser.1. V.1,** Pp.134-140 (2010)
4. Pavilaynen G.V., Yushin R.YU., Analysis of the account of elastic transversal isotropy and plastic anisotropy in the bending of circular plates, *Vestn. S.-Peterb. un-ta*, **Ser.1. V.4,** Pp.128-137 (2010)
5. Pavilaynen G.V., Kropacheva N.YU., Numerical modeling of the elastic-plastic bending of SD-plates, *In the Book of Abstracts "Scientific Research of the SCO Countries: Synergy and Integration"*, **V.1**, Pp. 230-239 (2019)
6. Iomdina E.N., Bauer S.M., Kotliar K.E., *Eye Biomechanics: Theoretical Aspects and Clinical Applications*, 208 p. Real Time, 2015

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