Thickness Optimization of a Piezoelectric Converter for Energy Harvesting

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Abstract: The conversion of mechanical energy from environmental vibrations into electrical energy is a key point for powering sensor nodes, toward the development of autonomous sensor systems. Piezoelectric energy converters realized in a cantilever configuration are the most studied for this purpose. In order to improve the performances of the converter, the geometry has to be properly designed. In this context FEM simulations have been used in order to optimize the piezoelectric layer thickness. A parametrized geometry was created by means of moving mesh application mode. The electrical energy generated by the converter under an applied acceleration was computed, finding the optimal thickness for the piezoelectric layer. Different geometries were considered verifying that they do not affect the optimal thickness.

Keywords: energy harvesting, optimization, unimorph cantilever, moving mesh, piezoelectric energy converter.

1. Introduction

In sensors and automation field, wireless sensor nodes are usually powered with batteries. This implies several problems and costs for replacing, recharging and disposing the batteries.

The retrieving of unused environmental energy from mechanical vibrations and the conversion into electrical energy is a promising solution for powering sensor nodes. In this way, an autonomous sensor system that does not need batteries and maintenance is realized. For mechano-electrical power conversion different techniques can be used, among which the piezoelectric, electromagnetic, electrostatic techniques are most common. The most studied energy converters are based on the direct piezoelectric effect and they are made with thick-film or MEMS technologies [1-2]. In order to improve the power conversion, geometry and dimensions of the piezoelectric converters should be optimized. In this context the use of FEM simulations [3-5] is a powerful tool in predicting

and optimizing the expected behavior of different structures.

In this study a piezoelectric energy converter having a cantilever shape has been considered and simulated with COMSOL for the conversion of mechanical vibrations into electrical energy. The geometry, and in particular the thickness, of the piezoelectric layer was varied, using the moving mesh application mode, in order to verify if an optimal value that maximize the output electrical energy exists.

2. Methods

The model uses a piezoelectric application mode for the simulation of the mechanical and the electrical behaviour of the converter when a sinusoidal vertical acceleration is applied. The moving mesh application mode was used for changing the thickness of the piezoelectric layer, computing the mesh deformation with the arbitrary Lagrangian Eulerian (ALE) technique.

2.1 Geometry

A 3D geometry was considered for the simulations. The piezoelectric converter has a unimorph cantilever shape, as shown in Figure 1.

The device is made by a stainless steel substrate with a piezoelectric layer on the top, poled along the thickness direction. The length of the structure is 27 mm and the width is 3 mm, while the thickness of the substrate $t_{substrate}$ is 200 µm. The thickness of the piezoelectric layer t_{PZT} was set initially to 60 µm and then varied from 10 µm to 400 µm with the moving mesh application.



Figure 1. Piezoelectric converter structure.

2.2 Mesh

The mesh was composed of 320 quad elements for a total number of degrees of freedom of 11808. The mesh was created using the mapped mesh tool dividing the length of the converter into 20 elements exponentially spaced with an element ratio equal to 10, with a finer mesh near the clamped end. Each material layer was divided in 2 linearly spaced elements along the thickness, and in 4 linearly spaced elements along width. A swept mesh was done using opposite vertical surfaces of each layer as source face and target face. Figure 2 shows the obtained mesh.



Figure 2. Piezoelectric converter mesh.

3. Governing Equations

In a piezoelectric energy converter based on a flexure cantilever, with the structure as shown in Figure 1, the piezoelectric layer works in a transversal mode and therefore it is governed by the following equations written in the strain-charge format:

$$S = s^{E}T + dE$$

$$D = \varepsilon^{T}E + dT$$
(1)

In eq.(1) *S* is the mechanical strain vector, s^E the elastic compliance tensor (Pa⁻¹), *T* the mechanical stress vector (Nm⁻²), *D* the electric displacement vector (Cm⁻²), ε^T the dielectric permittivity tensor (Fm⁻¹), *E* the electric field vector (Vm⁻¹) and *d* the transverse piezoelectric coefficient tensor (CN⁻¹). For the substrate layer only mechanical behaviour was considered using the following stress-strain relationship:

$$S = sT \tag{2}$$

3.1 Subdomain conditions for the converter

The geometry is divided into two subdomains: one for the substrate layer and one for the piezoelectric layer. The substrate was made of stainless steel using the decoupled isotropic material implemented in the COMSOL material library, having the following compliance value: $s = 5x10^{-12} Pa^{-1}$.

It has been assumed that the active layer of the unimorph has been manufactured using ultra low-temperature piezoelectric material (0-3 composite). This kind of piezoelectric materials are currently under development and the parameters can only be estimated using preliminary tests. It must be emphasized that generally they exhibit relatively low activity therefore in this case the 0-3 composite has been modeled using the following set of properties:

- Elastic compliance tensor

	50	-20	-20	0	0	0	
$s^{E} =$	-20	50	-20	0	0	0	$\times 10^{-12} Pa^{-1}$
	- 20	- 20	50	0	0	0	
	0	0	0	70	0	0	
	0	0	0	0	70	0	
	0	0	0	0	0	70	

- Piezoelectric tensor

$$d = \begin{bmatrix} 0 & 0 & 0 & 0 & 11 & 0 \\ 0 & 0 & 0 & 11 & 0 & 0 \\ -2.5 & -2.5 & 5 & 0 & 0 & 0 \end{bmatrix} \times 10^{-12} CN^{-1}$$

- Relative permittivity matrix

$$\boldsymbol{\varepsilon}^{T} = \begin{bmatrix} 50 & 0 & 0 \\ 0 & 50 & 0 \\ 0 & 0 & 50 \end{bmatrix} \times \boldsymbol{\varepsilon}_{0}$$

- Density

$$\rho = 3000 \ kg \cdot m^{-3}$$

The vertical acceleration was applied using body load F_Z equal to $a\rho$ in each subdomain, where a = 0.1 g represents the acceleration magnitude and ρ is the density of the material. The acceleration value in the optimization analysis can be chosen arbitrarily provided it does not exceed the linear operation region of the piezoelectric converter.

3.2 Boundary conditions for the converter

The unimorph model imposes that one end of the converter is clamped in relation to mechanical boundary conditions. Therefore the fixed constraint condition was applied for vertical faces of both the layers, while all the other faces were unconstrained, allowing the bending of the device.

Piezoelectric layer has a bottom and a top electrode on its faces that were not considered in the geometry because their mechanical behaviour can be neglected due to their low thickness. On the contrary, the electrical behaviour of the electrodes must be considered and it was modelled with the electrostatic boundary conditions of the piezoelectric subdomain, by grounding the lower face of the PZT layer and imposing the "Floating condition" for the upper face. For the other faces of the piezoelectric layer the "Zero charge/Symmetry" constraint was used.

3.3 Boundary conditions for the moving mesh

A moving mesh was used in order to change the thickness of the piezoelectric layer, using the moving mesh application mode for the implementation. The bottom face of the piezoelectric subdomain was constrained as clamped while the vertical faces were clamped along normal direction and left unconstrained along tangential direction allowing them to stretch freely. The upper face of the piezoelectric layer was tangentially constrained and displaced in the normal direction to the surface by a given displacement, using a parameter *deltaThickness*. The parameter *deltaThickness* was changed from -50 µm to 340 µm obtaining a parameterized moving mesh.

4. Numerical model

A parametric segregated solver was used creating one group for the moving mesh variables and another group for the piezoelectric variables. Firstly the moving mesh was solved by a static analysis, in order to compute the new piezoelectric layer dimensions, and then the piezoelectric variables were solved by a frequency response analysis. The latter analysis was computed at a frequency of 10 Hz, which is adequately far from the converter resonance.

5. Experimental results

The displacement of the free tip of the piezoelectric converter, the open circuit voltage and the generated charge collected on the electrodes were computed for piezoelectric layer thickness varying from 10 µm to 400 µm. The obtained tip displacement is shown in Figure 3, in a log-log scale. Two different behaviours are visible depending on the piezoelectric layer thickness. The rigidity of the whole cantilever is determined by the substrate rigidity and the piezoelectric layer rigidity, and the rigidity of each layer depends on the Young's modulus of the material and the layer thickness. When the PZT layer thickness is increased the rigidity of the piezoelectric layer increases. If the piezoelectric layer rigidity is small compared to the substrate layer rigidity, the whole cantilever rigidity is dominated by the substrate, and the thickness of the piezoelectric layer is not influential. In this way the tip displacement remains almost constant and independent from the PZT layer thickness. On the contrary, when the piezoelectric layer rigidity is comparable or higher compared to the substrate rigidity, the rigidity of the piezoelectric layer influences the rigidity of the whole cantilever. In this condition the tip displacement decreases with increasing the PZT layer thickness.



Figure 3. Tip displacement of the free edge as a function of piezoelectric layer thickness when a sinusoidal vertical acceleration, with a magnitude of 0.1 g and a frequency of 10 Hz is applied to the cantilever.

Figure 4 shows the open circuit voltage and the charge generated on the converter electrodes when the piezoelectric layer thickness is varied.

The charge Q reaches a maximum for low piezoelectric layer thickness, decreasing then when the rigidity of the piezoelectric layer becomes significant and it influences the whole converter rigidity. On the other hand, the converter capacitance C monotonically decreases with thickness with an inverse proportionality. For this reason, being the output voltage V the ratio between charge Q and capacitance C, the voltage increases for a constant charge level Q, and it remains instead constant when both Q and C decrease. In this way charge and voltage have opposite trends with piezoelectric layer thickness, as shown in Figure 4.



Figure 4. Open circuit voltage and generated charge as a function of piezoelectric layer thickness, when a sinusoidal vertical acceleration, with a magnitude of 0.1 g and a frequency of 10 Hz, is applied to the cantilever.

The electrical energy converted from mechanical vibrations and stored in the piezoelectric material was computed with the following equation and it is plotted in Figure 5 as a function of the piezoelectric layer thickness:

$$E = \frac{1}{2}QV \tag{3}$$

The energy shows a maximum for a thickness value of 210 μ m that can be considered, in this particular case, the optimal piezoelectric layer thickness. In the optimal condition the thickness ratio $t_{\rm PZT}/t_{\rm substrate}$ has a value of 1.05.

In order to verify if the ratio between the optimal thickness of the piezoelectric layer and

the substrate thickness is independent from the converter dimensions, different geometries were chosen. The length and the width of the converter and the thickness of the substrate were changed, once at a time, maintaining the rectangular shape. The considered dimensions for the different geometries are reported in Table 1, including the previously simulated cantilever geometry used as reference.



Figure 5. Electrical energy converted by the device as a function of piezoelectric layer thickness, when a sinusoidal vertical acceleration, with a magnitude of 0.1 g and a frequency of 10 Hz, is applied to the cantilever.

In order to verify if the ratio between the optimal thickness of the piezoelectric layer and the substrate thickness is independent from the converter dimensions, different geometries were chosen. The length and the width of the converter and the thickness of the substrate were changed, once at a time, maintaining the rectangular shape. In Table 1 the considered dimensions for the different geometries are reported, including the previously simulated cantilever geometry used as reference.

	Substrate	Width	Length
	thickness	[mm]	[mm]
	[µm]		
reference	200	3	27
geometry 1	300	3	27
geometry 2	200	6	27
geometry 3	200	3	10

 Table 1. Dimensions values for the implemented converter geometries.

One converter with a length of 10 mm, one with a width of 6 mm and one with a substrate

thickness of 300 μ m were considered, without changing the other dimensions. The same magnitude and frequency values for the mechanical acceleration were used. The electrical energy was computed comparing the optimal thickness ratio $t_{PZT}/t_{substrate}$ with that obtained previously. Figure 6 shows the obtained results for the reference geometry and for the other three considered geometries.



Figure 6. Electrical energy converted by the device as a function of piezoelectric layer thickness, for different geometries, when a sinusoidal vertical acceleration, with a magnitude of 0.1 g and a frequency of 10 Hz, is applied to the cantilever.

It can be observed that changing the dimensions of the converter does not change the trend of the electrical energy as a function of the thickness ratio, and the value where the energy is maximum does not vary. It results that the geometrical dimensions of the piezoelectric energy converter do not influence the optimal thickness ratio $t_{PZT}/t_{substrate}$, which only depends on the stiffness of the PZT and the substrate.

6. Conclusions

This paper shows how FEM simulations can be used for optimizing the geometrical dimensions of a piezoelectric energy converter.

A moving mesh application mode was coupled with a piezoelectric application mode in order to give geometry with parametrized thickness. The electrical energy converted by the piezoelectric device was computed to find an optimal thickness of the piezoelectric layer.

Geometries with different dimensions were considered, verifying that the optimal thickness ratio $t_{PZT}/t_{substrate}$ is independent from the converter dimensions, and it is influenced only by the mechanical properties of the piezoelectric layer and the substrate. This model is a specific example of using moving mesh for device geometry optimization, and in general it is a useful tool for the design of power harvesting converters.

7. References

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