Navier-Stokes equations:

$$\rho\left(\frac{\partial \vec{u}}{\partial t} + (\vec{u}\cdot\vec{\nabla})\vec{u}\right) = \vec{\nabla}\cdot\overline{\overline{\sigma}} + \vec{F}$$

here $\vec{u} = (u, 0, w)$ in cylindrical coordinates with axi-symmetry.

If η is the viscosity then

$$\overline{\overline{\sigma}} = \begin{bmatrix} 2\eta\frac{\partial u}{\partial r} - p & 0 & \eta\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r}\right) \\ 0 & 2\eta\frac{u}{r} & 0 \\ \eta\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r}\right) & 0 & 2\eta\frac{\partial w}{\partial z} - p \end{bmatrix}$$

I am adding extra volume forces

$$\overline{F} = \vec{\nabla}\cdot\overline{\overline{\sigma_e}}, \text{ where}$$

$$\overline{\overline{\sigma_e}} = \begin{bmatrix} 2\rho_m\eta\frac{\partial u}{\partial r} & 0 & \rho_m\eta\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r}\right) \\ 0 & 2\eta\rho_m\frac{u}{r} & 0 \\ \rho_m\eta\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r}\right) & 0 & 2\rho_m\eta\frac{\partial w}{\partial z} \end{bmatrix}$$

(Remember, there is no dependence on ϕ as this is axi-symmetric case.)

Thus the N-S eq. is actually

$$\rho\left(\frac{\partial \vec{u}}{\partial t} + (\vec{u}\cdot\vec{\nabla})\vec{u}\right) = \vec{\nabla}\cdot\bar{\bar{\sigma}}_t$$

here $\bar{\bar{\sigma}}_t$ - total stress

and

$$\bar{\bar{\sigma}}_t = \begin{bmatrix} 2(1+k_m)\eta\frac{\partial u}{\partial r} & 0 & (1+k_m)\eta\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial r}\right) \\ 0 & 2(1+k_m)\eta\frac{u}{r} & 0 \\ (1+k_m)\eta\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial r}\right) & 0 & 2(1+k_m)\eta\frac{\partial w}{\partial z} \end{bmatrix}$$

thus the effective viscosity is $(1+k_m)\eta$, where k_m some constant.

Therefore total force on sphere $= 6\pi(1+k_m)\eta r v$

$$\Rightarrow \frac{\text{total force on sphere}}{6\pi(1+k_m)\eta r v} = 1$$

here r - radius of the sphere.
v - inlet velocity.

But in my comsol when calculate above quatity I get its value different from I.

though for $\rho_m = 0$, I get correct value.

in comsol file

$\rho_m \rightarrow pp$

$\eta \rightarrow$ eta_f

$r \rightarrow$ rad

$v \rightarrow$ vel

$\pi \rightarrow$ pi

PS: In comsol, I am plotting above quantity in ~~plot~~ 1D plot called Force.