Bayesian Estimation of Tumours in Breasts Using Microwave Imaging

A. Jeremić and E. Khosrowshahli

Department of Electrical and Computer Engineering, McMaster University, Hamilton, Canada

Introduction

Microwave imaging has been recently proposed as an additional medical imaging technique which can potentially overcome some of the shortcomings of the mammography. Essentially the technique is based on illuminating breast with electromagnetic-wave(s) in microwave range.

From the physical point of view this can be represented as a wave propagation in medium (breast) that contains scatterers (both healthy and malignant tissue). Due to the fact that malignant tissue has larger conductivity the measurements obtained by receiving array of antennas will be different if the scatterers are present.

In this paper we propose a simplified parametric inverse 3D model which enables us detection of tumour presence and estimation of its size and/or position. Most of the existing solutions employ non-parametric image reconstruction techniques.

Finite-element Model

In order to solve the above equations we utilize finite-element method by developing three-dimensional model using RF module in COMSOL Multiphysics software. In this paper we model the breast as a sphere with radius of 100mm as shown in Figure 1.

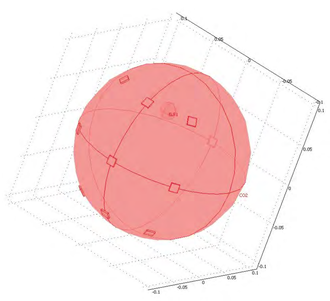

FIGURE 1: Geometry of the breast model

One antenna which acts as a transmitter is located on one side and nine receiving antennas are distributed on the other side of the sphere. These antennas are modelled as slim cubes which are centred on the surface of the sphere. Three boundary conditions are used to send waves in the medium. Perfect electric conductor boundary condition is applied to sides of antennas in order to guide wave through them, and scattering boundary condition is applied to the rest of surfaces to let waves propagate freely.

In this study tumour is considered as a sphere inside the breast with an arbitrary size and in arbitrary position. Three different studies are done using this model: effect of tumour location, tumour size and tumour permittivity.

Statistical Model

In the presence of tumour the measured signal becomes

$$\vec{y} = \vec{f}(R_t, \vec{R}, \sigma, \epsilon_r) + \vec{e}$$

where R_t is the radius of tumour, $\vec{R}$ is the position of tumour, σ and ϵ_r are conductivity and electric permittivities respectively. Note that in the above model we assumed a single tumour (scatterer) in order to simplify the computational cost but can be extended to account for multiple scatterers at the expense of computational time.

In presence of scattering the probability density function is given by

$$p_\sigma(\sigma) \cdot p_{\epsilon_r}(\epsilon_r) \cdot p(\vec{y}|\sigma, \epsilon_r, \vec{R})$$

Note that for a particular patient conductivity and permittivity can be treated as deterministic variables. In this paper we assume that sufficient set of previously measured properties is available and hence certain a priori knowledge on physiological values is available. The parameters can then be obtained by minimizing the cost function

$$c(\vec{R_t}, R, \sigma_r, \epsilon_r) = \log\left[p_{\sigma_r}(\sigma_r) \cdot p_{\epsilon_r}(\epsilon_r) \cdot p(\vec{y}|\mu_r, \epsilon_r, \vec{R})\right]$$

which is commonly know as maximum a posteriori estimate of unknown parameters. Note that because of nonlinear dependence of measured signals on physical parameters the above estimations hast to be performed using numerical optimization methods and Monte-Carlo simulations (in order to obtain posterior distribution.

Statistical Model

In order to evaluate the performance of our inverse model we simulate the measurement data using COMSOL and then add Gaussian noise in order to simulate measurement noise. In this preliminary approach we ignore several issues such as: skin reflection, calibration problems, antenna-to-antenna crosstalk, etc.

We simulate measurement data for 100 patients for which the conductivity and permittivity are generated using aforementioned Gaussian distributions. For each of the patients we then generate posterior distribution $p(\vec{y}|\sigma, \epsilon_r, \vec{R}, R_t)$ in order to calculate the parameter estimates. In Figures 5 and 6 we illustrate posterior distribution for arbitrarily chosen antenna for two different tumour sizes.

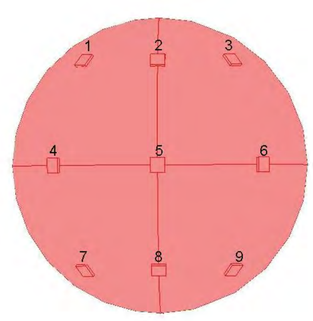

FIGURE 2: Posterior distribution for tumour in center size 1cm.

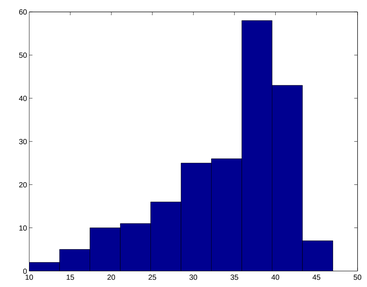

FIGURE 3: Posterior distribution for tumour in center size 1cm.

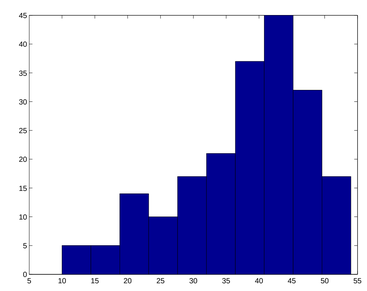

FIGURE 4: Posterior distribution for tumour in center size 2cm.

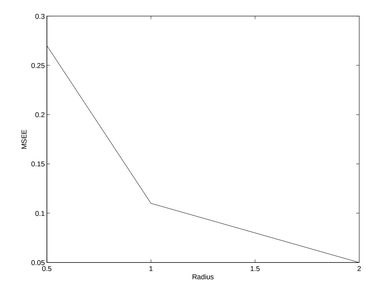

FIGURE 5: MSE of radius estimate as a function of tumour size.

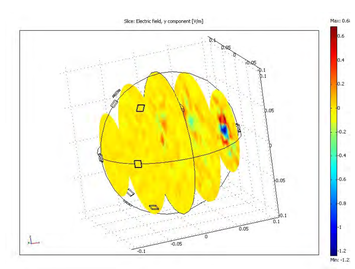

FIGURE 6: Electric field.