

A computationally efficient and physiologically consistent Left Atrium benchmark model

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Abstract

The left atrium (LA) is critical to normal left-heart function. A computationally efficient and physiologically consistent LA benchmark model is developed in COMSOL Multiphysics® for evaluating LA function. The findings indicate that simplifying LA geometry does not compromise the model's ability to reproduce in-vivo LA physiological performance under normal conditions. **Keywords:** Left Atrium, Mitral Valve, Fluid-Structure Interaction, in-vivo Pulmonary Vein flow.

Introduction

The left atrium (LA) is central to left-heart function and has three roles: reservoir during ventricular systole, conduit during diastole, and pump during late diastole. LA pressure reflects these roles through the recurring waveform in *Figure 1*.

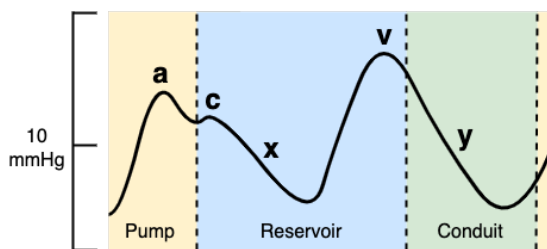


Figure 1. The LA pressure wave-form.

a denotes LA contraction, *c* mitral valve (MV) closure, and *v* LA filling; *x* and *y* are the descents. Disruption of LA function can impair left-ventricular performance and systemic circulation, making LA mechanics and hemodynamics relevant for predicting cardiovascular outcomes [1], [2]. Because direct assessment is invasive, computational models offer a noninvasive alternative [3]. Fluid-structure interaction (FSI) models are widely used to study blood interaction with cardiovascular structures [4], [5], [6], [7]. For the LA, prior studies used simplified geometries with the immersed boundary method [8], [9], [10], [11]. Feng et al. [12] used a more realistic CT-based LA model, but without realistic pulmonary vein (PV) inlet flow. This study presents an LA model that captures LA hemodynamics. Here, both fluid dynamics, and wall mechanics with active contraction represented by a tension force as in [12] were modeled. It is hypothesized that a simplified LA geometry can still reproduce clinically relevant flow parameters when the boundary conditions are obtained in-vivo. Such a model could move flow simulation closer to clinical workflow because patient-specific LA geometry would not be required.

Only PV flow data and LA end-diastolic volume are needed, and both can be acquired with a standard clinical magnetic resonance imaging (MRI) scanner.

Materials and Methods

A simplified LA model was constructed using computer-aided design (CAD), in-vivo LA volume data, and physiological properties. The geometry was combined with boundary conditions and material models to create the final FSI model. The following sections summarize model development and numerical setup.

Design of model geometry

The LA geometry is simplified to a sphere, see *Figure 2*. Four PVs were added as inlets and an idealized MV as the LA outlet. The sphere diameter was set from the end-diastolic LA volume in [13], giving 47.4 mm for 56 mL. Although in vivo wall thickness ranges from 1 to 4 mm [14], a uniform 2 mm thickness was used. The PV inlets were modeled as circular because inlet conditions were based on measured volume flow rates (Inlet), making exact inlet shape less critical. Using mean cross-sectional areas from Chnafa et al. [15], all four inlets were assigned a radius of 6.62 mm. This geometry was created in COMSOL Multiphysics®. The native MV has a complex asymmetric saddle shape [16]. Here, it was simplified as in *Figure 2C*, omitting anterior/posterior coaptation differences, chordae tendineae, and papillary muscles, and replacing the bicuspid valve with four identical leaflets. MV geometry was defined by coaptation depth, coaptation distance, and leaflet thickness, see *Figure 2B*. Coaptation distance and depth were taken from Gogoladze et al. [17]. Because coaptation depth is nearly identical between native leaflets, one average value was used for all four. Leaflet thickness was defined with a b-spline curve to match the native

range of 0.8-1.3 mm [18]. A 400 μm gap preserved topology, and mitral annulus diameter was set to 29 mm [19]. The geometry was designed in Autodesk Inventor (Autodesk, San Rafael, CA, USA).

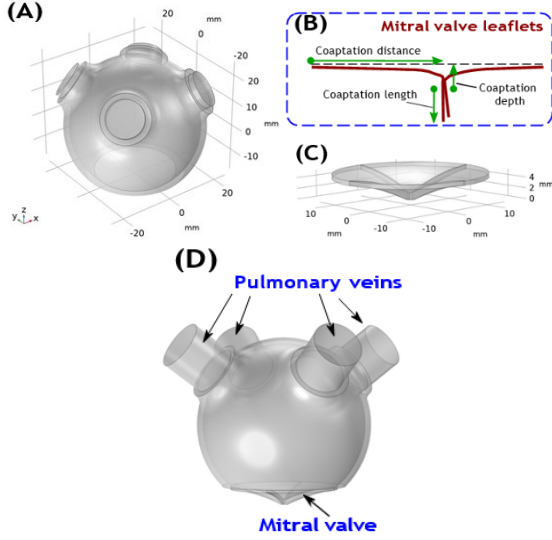


Figure 2. Simplified LA geometry. (A) Spherical LA model with uniform wall thickness. (B) Mitral valve geometry definition and landmarks. (C) Mitral valve with four symmetric leaflets. (D) Combined LA and mitral valve geometry.

Governing Equations

The simplified LA model contains one fluid domain and two solid domains: the LA wall and the mitral valve leaflets. The fluid domain was modeled with the Navier-Stokes equation:

$$\rho_f \left(\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} \right) = -\nabla p + \eta \nabla^2 \vec{v} + \rho \vec{g}, \quad (1)$$

where ρ_f (kg/m^3) is fluid density, $\vec{v}$ (m/s) is velocity, p (Pa) is pressure, η ($\text{Pa}\cdot\text{s}$) is viscosity, and $\vec{g}$ is gravitational acceleration. Blood properties were assumed: density 1060 kg/m^3 and viscosity $0.004 \text{ Pa}\cdot\text{s}$. Flow was assumed laminar, Newtonian, and incompressible. The mechanical response of the LA wall was modeled with the strain energy density function $\psi(\bar{C})$, proposed by Di Martino et al. [20]:

$$\psi(\bar{C}) = c_{LA} (I_{1c} (I_{3c})^{-1/3} - 3) + \frac{1}{2} K_{LA} (J - 1)^2, \quad (2)$$

Here, $\bar{C}$ is the right Cauchy-Green strain tensor, c_{LA} (Pa) is a stress-like material where I_{1c} and I_{3c} are invariants of $\bar{C}$, K_{LA} is the LA bulk modulus, and J is the determinant of the deformation gradient tensor. Although [20] reports regional values for c_{LA} , the LA wall was assumed homogeneous and assigned the mean value, see **Table 1**.

Since the primary focus of this study is on the flow dynamics within the LA rather than the behavior of

the MV, a simplified isotropic hyperelastic material model, without fiber reinforcement, has been utilized:

$$\psi(\bar{C}) = \frac{1}{2} \mu (I_{1c} - 3) + \frac{1}{2} K_{MV} (J - 1)^2, \quad (3)$$

where μ (Pa) is the first Lamé parameter and K_{MV} is the bulk modulus of the MV leaflets. The LA wall and leaflets were assumed to have the same density [20]. All material parameters are listed in **Table 1**.

Parameter	Value	Description
c_{LA}	8.38 kPa	Stress-like parameter
K_{LA}	10^6 kPa	Bulk modulus, LA wall
μ	150 kPa	First Lamé parameter, MV
K_{MV}	64 kPa	Bulk modulus, MV leaflet
ρ_s	1120 Kg/m^3	LA wall and MV leaflet density

Table 1. Material parameters for the LA wall and MV leaflets.

Boundary conditions

Inlet

Flow profiles for the four LA inlets (PVs) were obtained from a healthy volunteer (male, age 33 yrs.) using 2D phase-contrast MRI on a clinical scanner (Achieva dStream 1.5T, Philips Healthcare, the Netherlands). A multi-slice, multi-phase Balanced-Steady-State-Free-Precession sequence localized the PVs, after which imaging planes were positioned orthogonal to each vein and 3 mm upstream of the ostium. Flow was then acquired with a 2D phase-contrast sequence using 8 mm slice thickness, 1.2 mm by 1.2 mm pixel size, 0.60 m/s velocity encoding, and 40 recorded cardiac cycles. The resulting flow profiles are shown in **Figure 3A-D**. Each curve includes systolic forward flow (SFF), diastolic forward flow, and retrograde flow (RF).

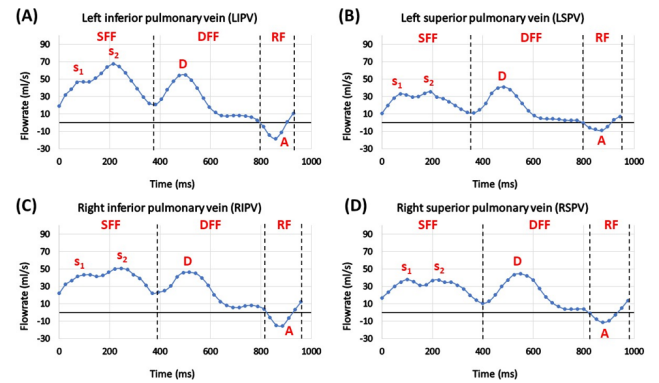


Figure 3. Flowrate measurements obtained in each pulmonary vein.

PV flow is pulsatile and includes a short back-flow period, see **Figure 3**. Because the velocity profile is

not fully developed, it was reconstructed with the Womersley-Evans model [21], [22]. An example of the profile extracted from the phase-contrast images and the reconstructed 3D profile at one time point is shown in **Figure 4**.

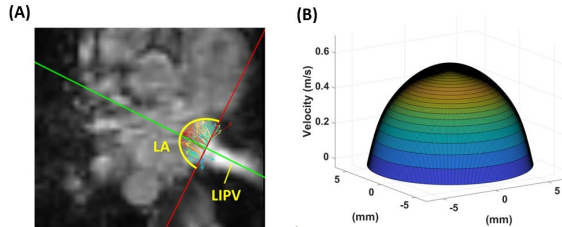


Figure 4. (A) shows the overlay of the extracted flow profile from the MRI. (B) shows an example of the reconstructed velocity profile in one pulmonary vein.

Outlet

The outlet condition uses a dynamic pressure because LA function is linked to the left-heart pressure gradient during the cardiac cycle [23], [24]. A baseline porcine left-ventricular pressure profile measured in-vivo [25] was applied at the outlet, see **Figure 5**. Because the LV domain is not included, only the diastolic part of the pressure profile was used; the systolic part was removed with a customized squared-sine step function in COMSOL Multiphysics®.

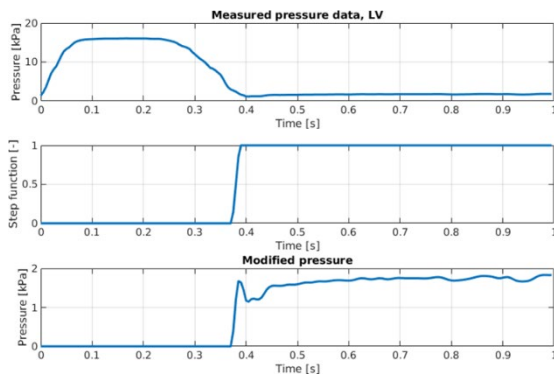


Figure 5. Outlet pressure profiles. Top: in vivo profile. Middle: step function used to remove systolic pressure. Bottom: applied outlet pressure profile.

Motion constraints

Several motion constraints were imposed on the LA model. Because the cross-sectional area of each PV changes only minimally, see **Figure 6**, the inlet areas in the simplified model were assumed to be constant throughout the cardiac cycle.

A tension force per unit area was applied to the outer LA wall to model active contraction during systole. Expressed as a spring constant, it was implemented as a triangular function with a maximum of 350 N/m and a width of 0.1 s, matching late diastole. This approach has also been used in similar studies [12], [9].

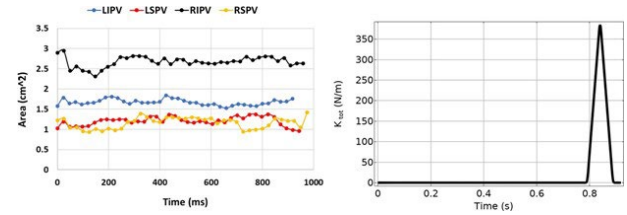


Figure 6. Left: cross-sectional area variation in the four pulmonary veins. Right: applied spring force used to model LA contraction.

Numerical model implementation

The FSI equations were solved in COMSOL Multiphysics®. Fluid-solid coupling was handled with the Arbitrary Lagrangian-Eulerian (ALE) method, and a direct linear solver was used. Time stepping used an implicit backward differentiation formula (backward Euler) with an initial step of 0.001 s and a maximum polynomial order of 2. The geometry was discretized with tetrahedral, quadrilateral (brick), and prism elements. Discretization is done by normal and fine settings. A mesh convergence study comparing LA pressure and volume gave differences of 1.4% and 0.7%, respectively. To balance accuracy and computation time, the Normal mesh was selected. The model was run on the Computer Cluster of Technical University of Denmark, with a computation time of approximately 12 hours.

Quantitative parameters

Clinically, LA function is assessed using several quantitative measures. Here, LA pressure, LA volume, and maximum transmitral velocity during the cardiac cycle were evaluated. Because the outlet was pressure-controlled rather than velocity-controlled, the transmitral velocity profile was used to evaluate model performance. Velocity was calculated at the center of the MV effective orifice area throughout the cycle. The effective orifice area was defined by a circle in the xy-plane enclosing the moving leaflet tips during valve opening and closing. Lastly, the kinetic energy, K_e (mJ), inside the LA was quantified by:

$$K_e = \frac{\rho_f}{2} U_{LA}^2 V_{LA}, \quad (4)$$

where U_{LA} (m/s) is the maximum velocity magnitude within the LA at a given time, and V_{LA} (m³) is the total LA volume at the same instant. Kinetic energy quantifies the work required to move blood due to inertia [26].

Simulation Results

LA function in clinical practice is assessed both qualitatively and quantitatively. Here, clinically relevant outputs from the simplified model are compared with literature values.

LA pressure

The FSI-derived LA pressure wave has two positive peaks, the v-peak and a-peak, see **Figure 7**.

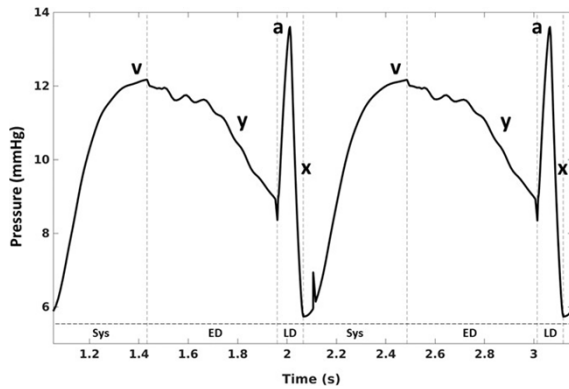


Figure 7. FSI-derived LA pressure waveform. *v* marks LA filling, *a* active LA contraction, and *x* and *y* the descents.

The a-peak and v-peak were compared with physiological values from [27]. The simplified model reproduces the overall pressure pattern in **Figure 1**. **Table 2** shows an a-peak of 13.6 mmHg, a v-peak of 12.2 mmHg, and a mean LA pressure of 6.90 mmHg. All three values fall within the physiological ranges reported in [27].

Parameter	FSI result	Physiological range
a-peak	13.6 mmHg	4-16 mmHg
v-peak	12.2 mmHg	6-21 mmHg
Mean	6.90 mmHg	2-12 mmHg

Table 2. LA a-wave and v-wave peak pressures from the FSI model versus in-vivo data.

LA volume

Figure 8 shows LA volume over two flow cycles.

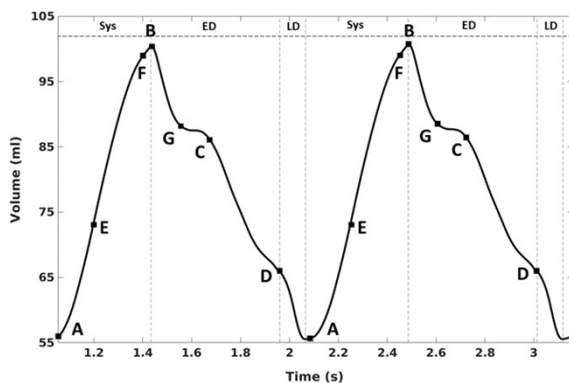


Figure 8. LA volume over the cardiac cycle from the FSI model. *A* minimum volume; *B* maximum volume; *C* mid-diastole; *D* end of the conduit phase; *E* and *F* 0.2 s intervals in the reservoir phase; *G* 0.1 s after ventricular systole.

LA volume peaks at point B with 100 ml and reaches its minimum at point A with 56 ml. The pattern matches normal in-vivo physiology: the LA fills

during ventricular systole and empties during ventricular diastole.

In **Figure 8**, A-B represents the reservoir phase, with the minimum volume set by the model geometry. B-D corresponds to the conduit phase, when blood passes from the PVs to the LV without external compression. C marks mid-diastole, and G marks LA volume 0.1 s after ventricular systole. During early diastole the MV opens and stored elastic energy is released, producing the volume decrease. E and F mark 0.2 s intervals within the reservoir phase, which coincides with ventricular systole. The simulated LA volume curve agrees closely with published data [13]. As shown in **Table 3**, both maximum and minimum volumes lie within physiological ranges. Active stroke work (W_{AS}), representing LA contraction during late diastole, is 43.5 ml/mmHg and falls within the range reported in [28].

Parameter	FSI result	Physiological range
Minimum volume (V_{min})	56 ml	32-74 ml
Maximum volume (V_{max})	100 ml	65-139 ml
Active stroke work (W_{AS})	43.5 ml/mmHg	44.8 ± 5.5 ml/mmHg

Table 3. LA volumes - FSI results versus physiological values.

Transmitral velocity

Figure 9 shows transmitral velocity variation. The profile has three peaks and closely resembles in-vivo behavior [29].

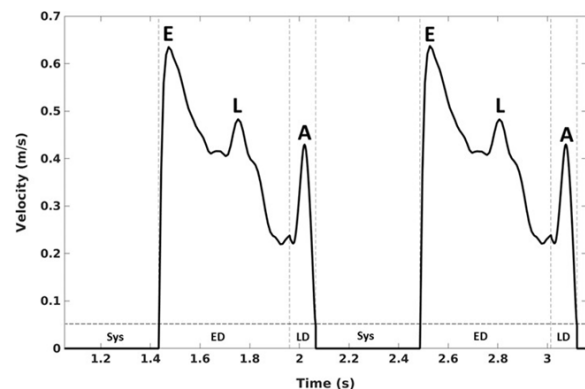


Figure 9. Transmitral velocity at the center of the mitral valve effective orifice area. *E*-, *L*-, and *A*-peaks are marked. *Sys* is systole; *ED* and *LD* are early and late diastole.

The highest peak is the E-peak at 0.63 m/s, followed by the L-peak at 0.48 m/s and the A-peak at 0.43 m/s. The E-peak represents passive LA emptying during early diastole, the L-peak corresponds to diastolic forward flow from the PVs (**Figure 3**), and the A-peak reflects active LA contraction during late diastole.

Parameter	FSI result	Physiological range
E-peak	0.63 m/s	0.54 ± 0.13 m/s
A-peak	0.43 m/s	0.44 ± 0.21 m/s
E/A	1.47	0.8-1.7

Table 4. Transmitral velocity- FSI results versus physiological values.

Table 4 lists the center-point velocity at the mitral valve effective orifice area and the E/A ratio. All the values fall within physiological ranges.

Kinetic energy

Along with stroke work, LA blood-pool kinetic energy contributes to the external work needed for efficient cardiac pumping [30]. **Table 5** reports three kinetic-energy peaks during the cardiac cycle. The highest occurs in early diastole at 5.30 mJ, which is 34.3% higher than the physiological value reported in [30], based on 15 healthy volunteers. Peak systolic and late-diastolic kinetic energies are both double the physiological range.

Parameter	FSI result	Physiological range
Peak systole	3.00 mJ	1.50 ± 0.10 mJ
Early diastolic peak	5.30 mJ	3.50 ± 0.25 mJ
Late diastolic peak	1.90 mJ	0.80 ± 0.10 mJ

Table 5. Kinetic energy - FSI results versus physiological values.

Discussion

This study shows that a simplified LA model can evaluate physiological LA function. The LA was represented as a sphere with four PV inlets and an MV outlet. Boundary conditions were based on representative in vivo data, and results were compared with the literature.

Simplifying LA geometry helps manage the many variables in FSI simulation and reduces computational cost.

The LA pressure profile on **Figure 7** captures the main LA pressure events. The a-peak, v-peak, and mean pressure (**Table 2**) agree with physiological values in [27]. The missing c-wave is expected because it arises from MV bulging after LV contraction [27], whereas this model excludes the LV chamber and constrains MV motion.

Physiologically, the mitral annulus translates during ventricular systole and influences LA pressure and volume. Because annular motion was constrained here, the PV inlets were left free to move perpendicular to their cross-sectional planes to mimic its effect on LA pressure and volume.

LA volume is a prognostic biomarker for conditions such as atrial fibrillation, pulmonary hypertension, and diastolic dysfunction [31], [32]. The simulated LA volume curve (**Figure 8**) matches physiological behavior [13], and both minimum and maximum

volumes fall within reported ranges. The results suggest that LA geometry has limited influence on volume change. By scaling the simplified model and applying patient-specific boundary conditions, the approach could support patient-specific modeling while retaining low computation time and clinical feasibility.

Transmitral flow is routinely used to assess LA mechanics and hemodynamics. The simulated profile (**Figure 9**) reproduces the main in-vivo features reported in [29], [33]. The E-peak is 20.4% higher than the reported mean (**Table 4**) but still lies within the standard deviation of the in vivo value [33].

Kinetic energy can be evaluated as a potential diagnostic or prognostic parameter. Since kinetic energy and pressure both contribute to external cardiac work, altered kinetic energy may reveal disease even when pressure magnitude and distribution change little [30].

A major strength of this study is the use of in vivo boundary conditions and material properties. The results show that simplifying LA geometry does not prevent the model from reproducing normal physiological behavior. This finite element approach therefore shows promise for patient-specific studies of cardiac structures under both physiological and pathological conditions.

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