Acoustic Field Simulation of Ultrasonic Droplet Levitation—A Coupled Acoustic–Fluid Multiphysics Simulation

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Abstract
To address the built-in acoustics–flow Multiphysics interface in COMSOL's inability to update the acoustic field in real time during transient flow simulations, this paper proposes a one-way coupling strategy based on the frequency-domain acoustic field. The model employs a two-dimensional axisymmetric geometry, in which the acoustic standing-wave field at 28.75 kHz is solved using the "Pressure Acoustics, Frequency Domain" interface, yielding the periodically steady-state distributions of acoustic pressure and particle velocity. The acoustic radiation force is derived from the Gor'kov potential and imposed on the "Laminar Flow" interface as a user-defined body force. To accurately capture the surface effect of the radiation force, the built-in phase-field variable ϕ_{pf} is introduced to confine the body force strictly within the thin interfacial layer surrounding the droplet. Simulation results demonstrate that, while maintaining computational efficiency, the proposed strategy achieves stable levitation of the droplet in the ultrasonic field and reasonably reproduces its interfacial dynamics. It should be noted that the model is based on two fundamental assumptions: (1) the acoustic field is established rapidly on the timescale of droplet dynamics, and its spatial distribution remains invariant with changes in the droplet surface morphology; and (2) the acoustic radiation force is derived from the Gor'kov potential. This strategy provides an efficient and viable modeling framework for the numerical investigation of acoustically levitated droplets, including the evaporation of laser-heated fuel droplets.
Keywords: acoustic levitation; acoustics–fluid coupling; acoustic radiation force; Gor'kov potential; phase-field method; COMSOL Multiphysics.

Introduction

Acoustic standing-wave levitators realize contactless suspension of droplets by means of the acoustic radiation force, thereby eliminating interference from container walls and providing an ideal platform for investigating droplet dynamics under various temperature and humidity conditions. When performing transient Multiphysics simulations of acoustically levitated droplets in COMSOL Multiphysics, a direct fully coupled two-way acoustics–flow solution requires the acoustic field to be recalculated at every time step as the fluid interface moves, resulting in extremely high computational cost and, in many cases, failure to converge [1][2].

To address this issue, this paper proposes a one-way coupling strategy based on the frequency-domain acoustic field. The strategy assumes that the acoustic field is rapidly established and reaches a steady state on a timescale much shorter than the characteristic time of droplet motion, and that its spatial distribution remains unchanged with the deformation and movement of the droplet surface (i.e., the "frozen acoustic field" assumption). On this basis, the acoustic radiation force is calculated from the Gor'kov potential and imposed as a body force in the flow equations. This paper presents the model construction, key assumptions, and simulation results of the proposed method, and discusses its applicability and limitations.

Mathematical Model

Geometry and Mesh

The computational domain adopts a two-dimensional axisymmetric geometry, comprising the ultrasonic standing-wave acoustic field, the surrounding air, and the droplet suspended therein. The geometric model and mesh discretization are shown in Fig. 1.

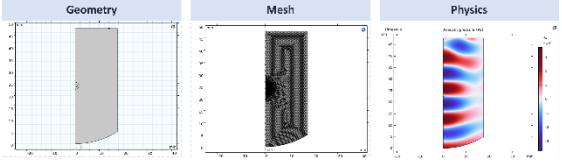


Fig.1. Geometry, mesh, and physics-interface setup of the coupled model.

Governing Equation of the Acoustic Field: Helmholtz Equation

The acoustic field is described by the "Pressure Acoustics, Frequency Domain" interface. Assuming the acoustic pressure varies harmonically in time as $\hat{p}(x,t) = Re\{p(x)e^{\{i\omega t\}}\}$, the time-derivative terms in the wave equation are replaced, and the complex acoustic pressure amplitude $p(x)$ satisfies the Helmholtz equation:

$$\nabla^2 p + k^2 p = 0 \quad (1)$$

where $k = \omega/c_0$ is the wavenumber, $\omega = 2\pi f$ is the angular frequency (f = 28.75 kHz), and c_0 is the speed of sound in air. Once the acoustic pressure is obtained, the particle velocity follows from the momentum equation:

$$u = \left(\frac{1}{i\omega\rho_0}\right)\nabla p \quad (2)$$

Where ρ_0 is the air density. Regarding the boundary conditions, a prescribed normal vibration velocity amplitude u_0 is imposed at the emitter, while the reflector and the side walls are treated as hard boundaries (zero normal gradient of the acoustic pressure). According to Assumption 1 of this study, the acoustic field is rapidly established on the timescale of droplet dynamics and does not vary with the droplet surface morphology; consequently, the Helmholtz equation is solved only once at the initial instant, and its steady-state solution remains unchanged throughout the transient flow computation, thereby avoiding the repeated solution of the acoustic equations at every time step.

Acoustic Radiation Force: Gor'kov Potential

According to Assumption 2 of this study, the acoustic radiation force is derived from the Gor'kov potential. Treating the droplet as a rigid spherical particle of radius , the Gor'kov potential is defined as:

$$U = \left(\frac{4\pi a^3}{3}\right)\left[f_1\left(\frac{\kappa_0}{2}\right) < p^2 > -f_2\left(\frac{3\rho_0}{4}\right) < u^2 >\right] \quad (3)$$

where $\langle\cdot\rangle$ denotes the average over one acoustic period; κ_0 is the adiabatic compressibility of air; $f_1 = 1 - \kappa_d/k_0$ is the monopole coefficient; and $f_2 = 2(\rho_d - \rho_0)/(2\rho_d + \rho_0)$ is the dipole coefficient, with κ_d and ρ_d being the compressibility and density of the droplet, respectively. The acoustic radiation force is the negative gradient of the Gor'kov potential:

$$F_{ac} = -\nabla U \quad (4)$$

In a planar standing-wave field, the radiation force is distributed periodically along the standing-wave direction and points toward the pressure node, thereby confining the droplet at the nodal plane. It should be noted that the Gor'kov radiation force treats the droplet as a rigid particle and can therefore only predict its equilibrium position; it cannot capture internal circulation within the droplet or interfacial phenomena such as deformation, breakup, and coalescence. Nevertheless, this method is adopted in the present work primarily to ensure numerical stability.

Governing Equations of Two-Phase Flow

The droplet and the surrounding air are regarded as two immiscible, incompressible fluids, and the flow is governed by the incompressible Navier–Stokes equations:

$$\nabla \cdot u = 0 \quad (5)$$

$$\nabla \cdot [-pI + \mu(\phi)(\nabla u + \nabla u^T)] + f_{st} + f_{ac} = \rho(\phi)\left(\frac{\partial u}{\partial t} + u \cdot \nabla u\right) \quad (6)$$

where the density and viscosity are interpolated from the phase-field variable as ϕ and $\rho(\phi) = \rho_{air} + (\rho_d - \rho_{air})\phi, \mu(\phi) = \mu_{air} + (\mu_d - \mu_{air})\phi$; f_{st} is the surface tension body force; and f_{ac} is the body force form of the acoustic radiation force.

Phase-Field Equation and Interface Tracking

The phase-field method implicitly represents the interface by introducing a conserved order parameter ϕ (i.e., the built-in variable ϕ_{pf} in COMSOL): $\phi = 1$ denotes the droplet phase, $\phi = -1$ denotes the gas phase, and the isosurface $\phi = 0$ defines the interface. The evolution of ϕ obeys the convective Cahn–Hilliard equation:

$$\frac{\partial \phi}{\partial t} + u \cdot \nabla \phi = \nabla \cdot (M \nabla G) \quad (7)$$

$$G = \lambda\left[-\nabla^2 \phi + \frac{\phi(\phi^2 - 1)}{\epsilon^2}\right] \quad (8)$$

Where M is the mobility, λ is the mixing energy density, and ϵ is the interface thickness parameter. Equations (7) and (8) ensure that the interface advects with the flow while maintaining a smooth transition over a finite thickness ε, and that interface deformation, breakup, and coalescence are captured automatically. Surface tension is applied in body-force form:

$$f_{st} = \sigma\kappa\,\delta(\phi)\,\hat{n} \quad (9)$$

Where σ is the surface tension coefficient, κ is the interface curvature, and $n = \nabla\varphi/|\nabla\phi|$ is the interface unit normal.

Conversion of the Surface Radiation Force into a Body Force

The Gor'kov radiation force is $F_s = F_{ac}/A_d$, in essence, a force acting per unit area on the droplet surface. To apply it within the finite-thickness phase-field interface, a Dirac delta function constructed from the phase-field variable $\delta(\phi)$ is

introduced to "smear" the surface force into a body force:

$$\delta(\phi) = |\nabla\phi|\left(or\ \delta(\phi) = \frac{4\varphi(1-\phi)}{h}\right) \quad (10)$$

Where h is the characteristic mesh size of the interface. The function $\delta(\phi)$ satisfies $\int \delta(\phi)dn = 1$ (with n the interface-normal coordinate), such that integrating along the normal direction recovers the surface force per unit area. The acoustic radiation force thus enters the momentum equation (6) as the following body force:

$$f_{ac} = F_{ac} \cdot \delta(\phi) \quad (11)$$

In other words, once the acoustic field is obtained from the "Pressure Acoustics, Frequency Domain" interface, the spatial distribution of the radiation force F_{ac} is computed from Eqs. (3)–(4) and then, via Eqs. (10)–(11), confined strictly to the thin interfacial layer defined by ϕ, thereby realizing a one-way coupling from the acoustic field to the flow.

Solution Procedure

The Helmholtz equation (1) is first solved to obtain the steady-state acoustic field. Subsequently, the radiation force body force defined by Eqs. (3)–(4) and (10)–(11) is substituted into the two-phase flow equations (5)–(9), and the evolutions of the flow field and the phase field (interface) are obtained simultaneously through a fully coupled solution procedure combining stationary and time-dependent studies.

Solver Settings

The model contains two types of equations with fundamentally different mathematical properties, and the solver must be configured separately according to the equation type [3]:

(1) *Frequency-domain problem (Helmholtz equation).* The frequency-domain study computes the steady-state response to a single-frequency harmonic excitation, with all field quantities being complex amplitudes. The resulting large sparse linear system requires no time marching and is solved once using the direct linear solver MUMPS. Since this solution serves as a known field in the subsequent transient study (under the "frozen acoustic field" assumption), only a single frequency point (28.75 kHz) needs to be computed, and the acoustic pressure and particle velocity results are passed to the transient study step via a "stored solution" or parametric sweep.

(2) *Transient problem (Navier–Stokes + convective Cahn–Hilliard equations).* COMSOL offers three time-stepping methods for transient studies: the implicit backward differentiation formula (BDF), the generalized-α method, and the explicit Runge–Kutta method. BDF employs a variable order (1–5) with automatic step-size selection, using high-order schemes when solution quality permits and automatically reducing the order when robustness is required; it is the default for problems involving diffusion, convection, and reaction. The generalized- α method is suited to second-order wave-propagation problems, whereas explicit Runge–Kutta is primarily intended for ODEs and is inefficient for FEM problems. The transient equations of the present model (the convective–diffusive Cahn–Hilliard equation and the viscous incompressible flow) constitute a stiff, predominantly parabolic problem; therefore, the implicit BDF time stepping is adopted, combined with free time stepping subject to a maximum step-size constraint, so as to balance the resolution of interface evolution against computational efficiency.

(3) *Nonlinear and linear solvers.* Within each time step, the nonlinear algebraic system arising from the density and viscosity interpolation and from surface tension is solved by the fully coupled Newton method. The sparse linear systems generated by the Newton iterations are solved using the direct solver MUMPS. The reason for choosing a direct solver over an iterative one is that the two-phase flow–phase-field coupled system is ill-conditioned and convection-dominated; for such small-to-medium-scale 2D axisymmetric problems, direct solvers (MUMPS being the most robust, followed by PARDISO) are memory-affordable and substantially more robust. A relative tolerance is set to control the time-stepping error, and a small initial time step is used to ensure that the droplet excitation phase, when the forces are first applied, is properly resolved.

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Applicability and Limitations

（1） *Applicability of the Gor'kov radiation force.* Equations (3)–(4) are based on the following premises [4]: (i) the particle size is much smaller than the wavelength ($ka \ll 1$; the wavelength a in air at 28.75 kHz is approximately 11.8 mm, a condition satisfied by droplets of millimeter scale and below); (ii) the particle can be approximated as rigid and incompressible (valid when the density and compressibility contrasts between the droplet and air are sufficiently large); (iii) the acoustic field is a linear, lossless (or weakly dissipative) standing wave; and (iv) only the overall force and equilibrium position of the particle are of interest, rather than the flow inside it. Consequently, the Gor'kov force can only predict the equilibrium position of the droplet and cannot capture internal circulation or interfacial phenomena such as deformation, breakup, and coalescence; it is retained in this work primarily for numerical stability. In addition, this force neglects the viscous drag

on the droplet exerted by acoustic streaming, which cannot be ignored under strong acoustic fields or for large droplets; in such cases the model underestimates the actual force.

（2） *Applicability of the one-way coupling (frozen acoustic field) assumption.* This assumption requires that the acoustic-field establishment time be much shorter than the characteristic time of droplet motion, and that the disturbance of the acoustic field by the presence and motion of the droplet be negligible — i.e., that the scattering of the acoustic field by the droplet is weak (the case of a single particle in dilute suspension). When the droplet size approaches the wavelength, undergoes significant deformation, or when strong scattering among multiple droplets occurs, the acoustic field varies with the interface evolution, and the one-way coupling is no longer valid.

（3） *Applicability of the two-phase flow model.* Equations (5)–(6) assume that both phases are incompressible Newtonian fluids and that the flow is laminar (low Reynolds number), which applies to problems such as the suspension, deformation, and coalescence of millimeter-scale and smaller droplets in a gas. The phase-field method requires the interface thickness to be much smaller than the droplet radius and the mesh to resolve the interface adequately, and is suitable for moderate Weber numbers ($We \lesssim O(1-10)$) in which the interface remains continuous [5]; although great topological changes such as large deformation and breakup can be captured by the phase-field method, they require finer meshes and smaller time steps, with a substantial increase in computational cost.

（4） *Geometric assumption.* The two-dimensional axisymmetric model cannot describe azimuthal instabilities of the droplet (e.g., non-axisymmetric oscillations or rotation), and is inherently limited in predicting strong deformation or breakup; such problems require extension to three-dimensional models.

Results and Discussion

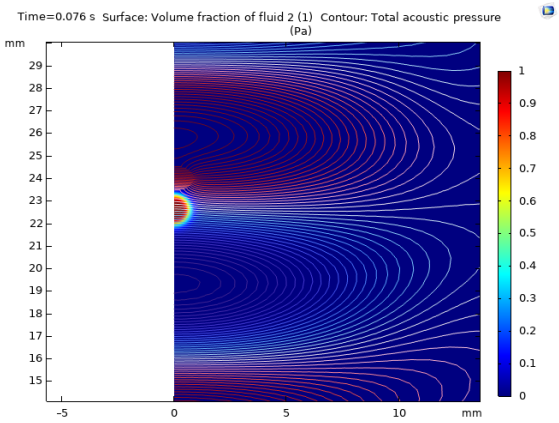


Fig. 2. Volume fraction and acoustic pressure distributions.

The simulation results show that the proposed one-way coupling strategy enables the droplet to levitate stably in the ultrasonic field. Fig. 2 presents the volume fraction and acoustic pressure distributions during droplet levitation.

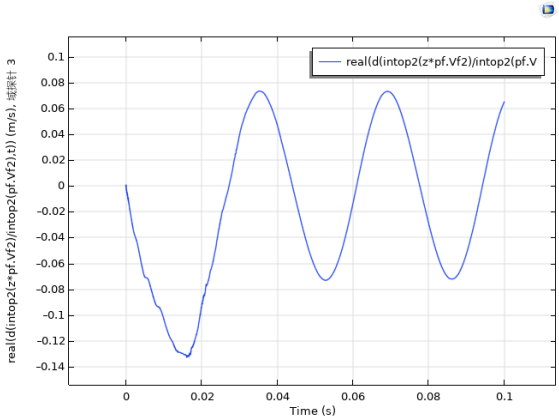


Fig. 3. Velocity evolution of the droplet during levitation.

Fig. 3 shows the evolution of the velocity field during levitation. Fig. 4 shows that the droplet position changes during levitation and eventually stabilizes at the pressure node. Fig. 5 shows the evolution of the interfacial parameter (phase-field variable) during levitation.

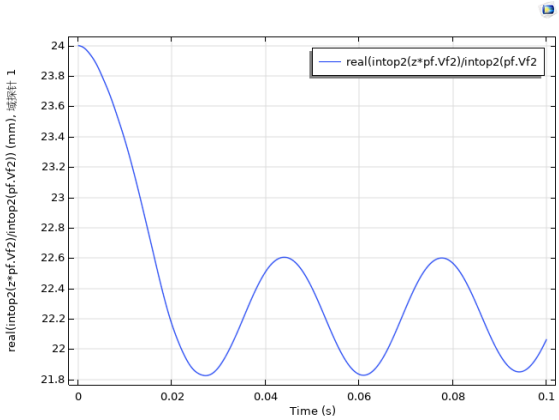


Fig. 4. During levitation, the droplet migrates and eventually settles at the pressure node.

Discussion: The above results indicate that the proposed "frequency-domain acoustic field with one-way coupling + user-defined acoustic radiation force" strategy achieves stable droplet levitation while preserving computational efficiency, and reasonably reproduces the interfacial dynamics of the levitated droplet. However, it must be emphasized that, owing to the adoption of the "frozen acoustic field" assumption and the Gor'kov radiation force, the model can only predict the equilibrium position of the droplet and cannot resolve details such as internal circulation and interfacial deformation. Future work will introduce the acoustic-impedance-based body force $f_{ac} = -2kE_{ac}\, sin(2ky)\hat{e}_y$[6] and will quantitatively validate the model against the established experimental platform (28.8 kHz ultrasonic levitator + 808 nm contactless laser heating + synchronized high-speed/infrared imaging), further extending this framework to the evaporation of laser-heated fuel droplets of various materials under ultrasonic levitation conditions.

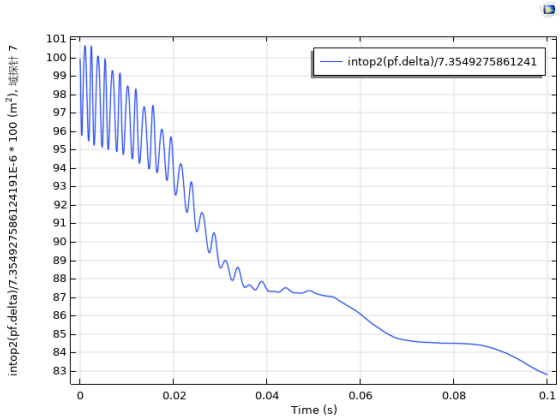


Fig. 5. Evolution of the interfacial parameter during droplet levitation.

Regarding experimental validation, a 28.75 kHz ultrasonic levitation platform was built in this study, combined with 808 nm contactless laser heating and synchronized high-speed/infrared imaging, enabling real-time capture of droplet deformation and temperature response, thereby providing quantitative validation data for the COMSOL acoustics–flow–phase-field coupled simulations.

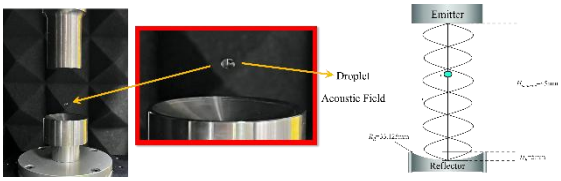


Fig. 6. During levitation, the droplet migrates and eventually settles at the pressure node.

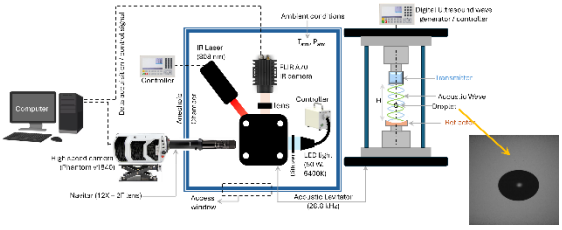


Fig. 7. Experimental setup and high-speed images of the droplet.

Conclusion

(1) The simulation results show that the modeling strategy proposed in this work enables stable levitation of the droplet in the ultrasonic field;

(2) The model reasonably reproduces the interfacial dynamics of the levitated droplet;

(3) Under the two assumptions “the acoustic field is rapidly established and does not vary with the droplet surface" and "the acoustic radiation force is computed from the Gor'kov potential" — the feasibility of the "frequency-domain one-way coupling + user-defined acoustic radiation force" method for droplet levitation simulations has been validated.

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