Modeling Acoustic Interface Wave Dispersion using COMSOL

Benjamin M. Goldsberry[*1] and Marcia J. Isakson[1]
[1]Applied Research Laboratories, The University of Texas, Austin, TX USA
*Corresponding author: 10,000 Burnet Road, Austin, TX, 78758, goldsbe@arlut.utexas.edu

Abstract: Measuring geoacoustic parameters of underwater sediments is important for accurate numerical modeling of underwater acoustic propagation. While the density and compressional wave speeds can be directly measured in sediments, shear wave speeds are difficult to measure because they are highly attenuated. However, shear wave speeds can be indirectly determined through measurement of Scholte ocean-seafloor interface wave speeds. When the seafloor is composed of layers of sediments with different shear wave speeds, the Scholte wave speed becomes dispersive. Scholte wave speed dispersion was measured in the Narragansett Bay, Rhode Island [J. Greene et al., SYMPOL, Cochin, India, 2011]. By using an inversion process, each layer's shear wave speeds were calculated.
To validate the inversion method, COMSOL is employed to model the experiment at Narragansett Bay. The modeled and measured interface wave speeds are then compared.

Keywords: Acoustics, Interface Waves

1. Introduction

In shallow water waveguides, acoustic wave interactions with the ocean floor have a significant impact on the transmission loss of an acoustic signal. Therefore, knowledge of sediment types and their associated properties, such as compressional and shear wave speeds, is important for accurate numerical modeling of shallow water waveguides. While certain parameters, such as density and compressional sound speed, can be directly measured in sediments, shear waves exhibit high attenuation and are therefore difficult to measure directly [1]. A shear measurement system was recently developed that uses interface wave measurements to indirectly measure shear wave speeds.

On a fluid-solid interface, such as an ocean seafloor comprised of rock, there can exist an interface wave (Scholte wave). This wave only travels on the interface and is evanescent in both the sediment and fluid. It is well documented that Scholte waves propagate with a velocity of approximately 90% of the shear speed, and if the shear speed varies with depth, such as an ocean seafloor composed of multiple sediment layers, the Scholte wave is dispersive [1]. By exciting Scholte waves on a fluid-solid interface and measuring the frequency dependent phase velocities of the Scholte waves, shear wave speeds for each layer can be extracted through an inversion scheme.

Potty et al. developed an inversion model using a dynamic stiffness matrix and measured Scholte phase speeds in Narragansett Bay, Rhode Island [2]. To validate the inversion model, COMSOL Multiphysics is used to model the experiment. By using the predicted shear wave speeds from the inversion model, the finite element model calculates the Scholte wave phase speeds. The measured and modeled wave speeds are then compared for accuracy and verification of the inversion process.

2. Theory

Time-harmonic pressure waves in a two-dimensional fluid medium satisfies the Helmholtz equation:

$$\nabla^2 p + k^2 p = 0 \qquad (1)$$

where p is the acoustic pressure, and k is the wavenumber. In homogeneous, isotropic solids, the shear and compressional waves are governed by:

$$\nabla^2 \Phi + k_c^2 \Phi = 0 \qquad (2)$$

$$\nabla^2 \mathbf{\Psi} + k_s^2 \mathbf{\Psi} = 0 \qquad (3)$$

where Φ is the scalar displacement potential, k_c is the compressional wavenumber, $\mathbf{\Psi}$ is the vector displacement potential, and k_s is the shear wavenumber. The total particle displacement, d, is then:

$$d = \nabla \Phi + \nabla \times \mathbf{\Psi} \qquad (4)$$

On a fluid-solid interface, continuity of vertical displacement and normal stress, and no tangential stress is enforced:

$$\frac{1}{\rho_0 \omega^2}\frac{\partial p}{\partial n} = \nabla \Phi \cdot \hat{n} + (\nabla \times \mathbf{\Psi}) \cdot \hat{n} \qquad (5)$$

$$p = \sigma_n \qquad (6)$$

$$\sigma_s = 0 \qquad (7)$$

where $\omega = 2\pi f$, f is the frequency, ρ_0 is the density of the fluid medium, σ_n is the normal stress, and σ_s is the shear stress. When the shear phase speed is less than the fluid sound speed and the shear sound speed, there exists a frequency-independent interface wave, called a Scholte wave, which dies off exponentially in the direction normal to the boundary [3]. However, when the shear speeds in solids are not constant, such as in layered media where the different sediment types can have drastically different shear speeds, the Scholte wave phase speed becomes frequency dependent, or dispersive [1].

3. Experiment

By measuring the dispersion of the Scholte waves, an inversion scheme can be applied to indirectly calculate shear wave speeds in layered media. Potty et al. developed an inversion scheme based off a dynamic stiffness matrix to model Scholte wave dispersion. To implement the inversion model, Scholte wave dispersion was experimentally measured in Narragansett Bay, Rhode Island. Several geophones were inserted into the sediment, each spaced 5 meters apart. A 135 kg weight was released from a ship. The impact of the weight on the ocean floor excited Scholte waves. By measuring the phase differences of the Scholte wave arrivals between pairs of geophones and knowing the distance between the geophones, the phase speed was calculated for each frequency.

The dynamic stiffness matrix was then used to calculate the shear speeds. First, core samples of Narragansett Bay were analyzed to find the number of sediment layers and the respective compressional wave speeds and densities. Then, the shear waves were indirectly calculated by iteratively adjusting the shear wave speed inputs into the model until a good match between the measured data and the model was found [2].

4. COMSOL Model

To verify the inversion scheme, the finite element method (FEM) was employed to model the experiment in Narragansett Bay. It is known that as the number of elements approaches infinity, the FEM solution converges to the exact solution of the given boundary value problem [4]. Therefore, the FEM is a good benchmark to the inversion scheme. Figure 1 shows the geometry of the model. There is a shallow water layer 6 meters deep, and three solid layers of various densities of sand. The bottom is a solid infinite half-space of dense sand. A point was placed on the fluid-solid boundary to implement the source condition. The material parameters are given in Table 1. In the water layer, the Pressure Acoustics, Frequency Domain physics interface was used. In the sediments, the Solid Mechanics physics interface was used. All of the sediment domains were included in one Solid Mechanics physics interface so that the coupling conditions between solid domains were automatically satisfied. Boundary conditions were then prescribed on the fluid-solid boundary. In order to implement Eq. (5), a Normal Acceleration node was placed in the Pressure Acoustics physics interface. The following inward acceleration was prescribed:

elas.u_ttX*elas.nX + elas.u_ttY*elas.nY

To implement Eq. (6), a Boundary Load node was placed in the Solid Mechanics physics interface. The load type was selected to "Load defined as force per unit area," and the following was prescribed to F_a:

-p*elas.nX	x N/m^2
-p*elas.nY	y N/m^2

The air-water interface was modeled as a pressure-release boundary by inserting a pressure node in the Pressure Acoustics physics interface and assigning the pressure to 0. To implement the force of the dropped weight on the ocean floor, a point load node was placed in the Solid Mechanics physics interface, and was prescribed on the point on the fluid-solid boundary. The following point load was prescribed:

| 0 | x N |
| -135*9.81 | y N |

A maximum element size of eight elements per smallest wavelength in each domain was used to effectively resolve all the waves. A frequency parametric study from 1-20 Hz was then computed. Figure 2 shows an example FEM solution at 20 Hz.
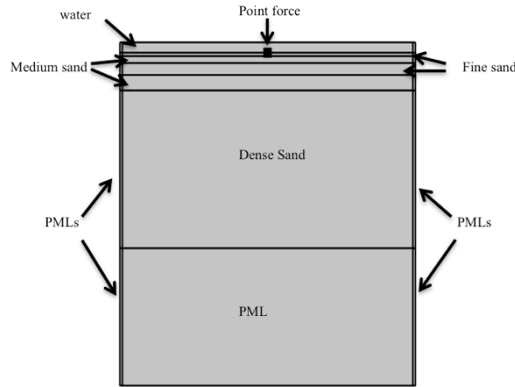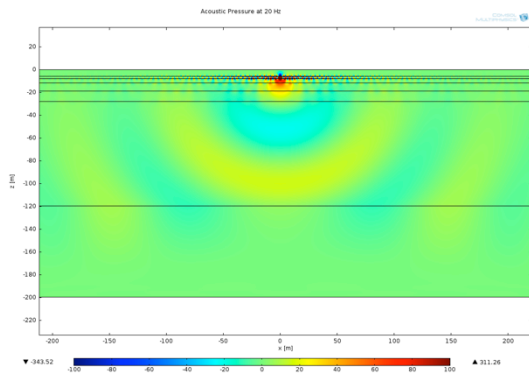

Figure 1. FEM geometry.

Figure 2. Acoustic pressure field at 20 Hz.

Table 1. Acoustic parameters for each layer

Medium	Depth (m)	c_p (m/s)	c_s (m/s)	ρ (kg/m³)
Water	6	1540	-	1000
Layer 1	2	1600	45	1650
Layer 2	4	1650	100	1840
Layer 3	7	1600	170	1710
Layer 4	9	1650	250	1940
Half-space	∞	1836	380	2034

4.1 PML Considerations

Perfectly Matched Layers (PML) were used to model the domain as an infinitely long waveguide. A PML was placed on the left and right side of each layer and on the bottom of the last domain to simulate an infinite half-space. Several techniques were used to ensure minimal artificial reflections from the PMLs. First, attenuation was applied to both the shear and compressional wave speeds in the solid domains. This was implemented so that the shear wave is mostly attenuated at the PMLs, and the typical wave speed and size for each PML can be set to the respective compressional wave speed. In addition, PMLs are not effective for absorbing interface waves and can create artificial reflections if the interface wave is not mostly attenuated at the PMLs. Therefore, the domain was elongated so the Scholte wave would be mostly attenuated at the computational edge of the domain, thus minimizing artificial reflections from the PMLs.

4.2 Calculation of Phase Speed

For each frequency, the Scholte wave pressure was extracted on the fluid-solid interface. To calculate the phase speed of the Scholte wave, a spatial Fourier transform was performed on the extracted pressure along the propagation direction (x), transforming from physical space to wavenumber space:

$$\hat{p}(k) = \int_{-\infty}^{\infty} p(x)e^{-i2\pi kx}\, dx \qquad (8)$$

Since the frequency is known, the Scholte wave speed is:

$$c = \frac{2\pi f}{k} \qquad (9)$$

5. Results

Figure 3 shows the comparison between the inversion model and the measured data. The dynamic stiffness matrix shows good agreement with the measured data. The FEM model agrees closely with the inversion model, thus validating this approach to indirectly measure the shear wave speed.
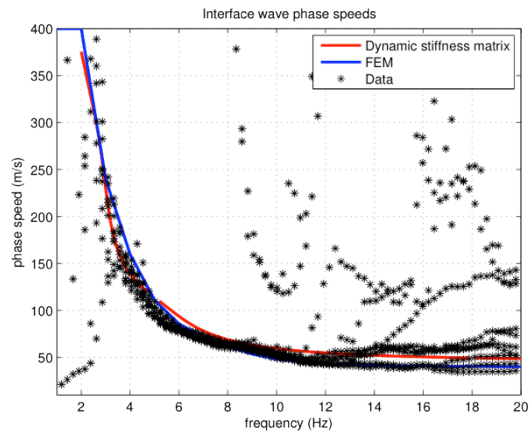
Figure 3. Measured data compared with the dynamic stiffness matrix inversion and FEM.

5. Conclusion

COMSOL Multiphysics was used to model the experiment done at Narragansett Bay and validate the inversion scheme to indirectly measure shear wave speeds. Special care was taken so the PMLs did not create artificial reflections. To calculate the phase speed, the spatial Fourier transform was taken over the pressure at the fluid-solid boundary. The FEM model shows good agreement with the inversion scheme and the data, thus validating the inversion scheme as a valid approach to indirectly measure shear wave speeds.

8. References

1. F.B. Jensen, W.A. Kuperman, M.B. Porter, and H. Schmidt, *Computational Ocean Acoustics*, American Institute of Physics, New York (1994).
2. G. Potty and J. Miller, "Measurement and modeling of Scholte wave dispersion in coastal waters," In Proc. of Third Int. Conf. on Ocean Acoustics, Beijing, China, 21-25 (May 2012).
3. Ivan Tolstoy and C.S. Clay, *Ocean Acoustics: Theory and Experiment in Underwater Sound,* American Institute of Physics, New York (1987).
4. Eric B. Becker, Graham F. Carey, and J. Tinsley Oden, *Finite Elements, An Introduction: Volume I,* Prentice-Hall, New Jersey (1981).

9. Acknowledgements