COMSOL CONFERENCE
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Acoustic Wave Propagation in Water Filled Buried Plastic Pipes

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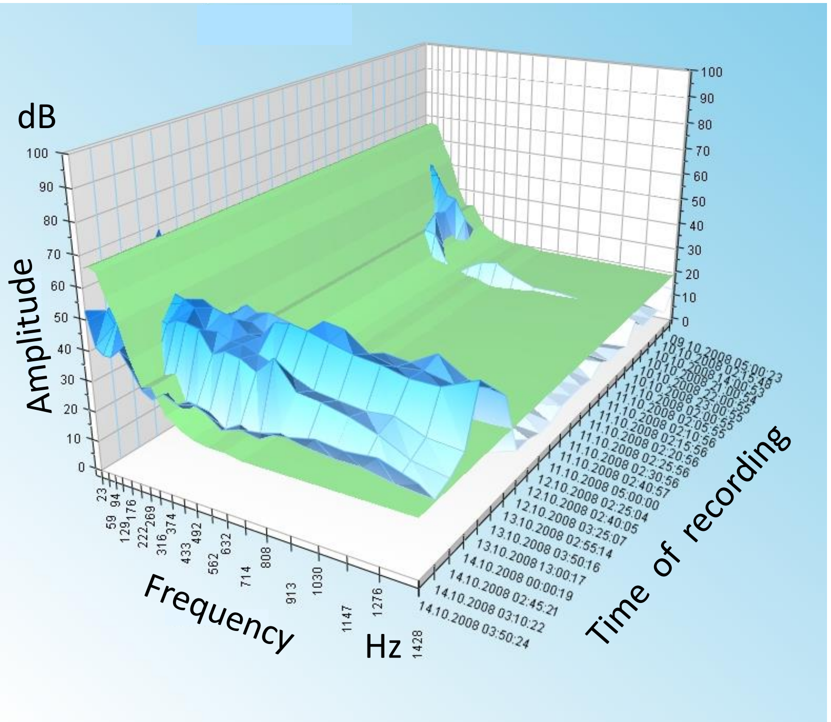
Motivation

System *Lorno*: Detecting leak noise

leak noise spectrum vs. time



Application

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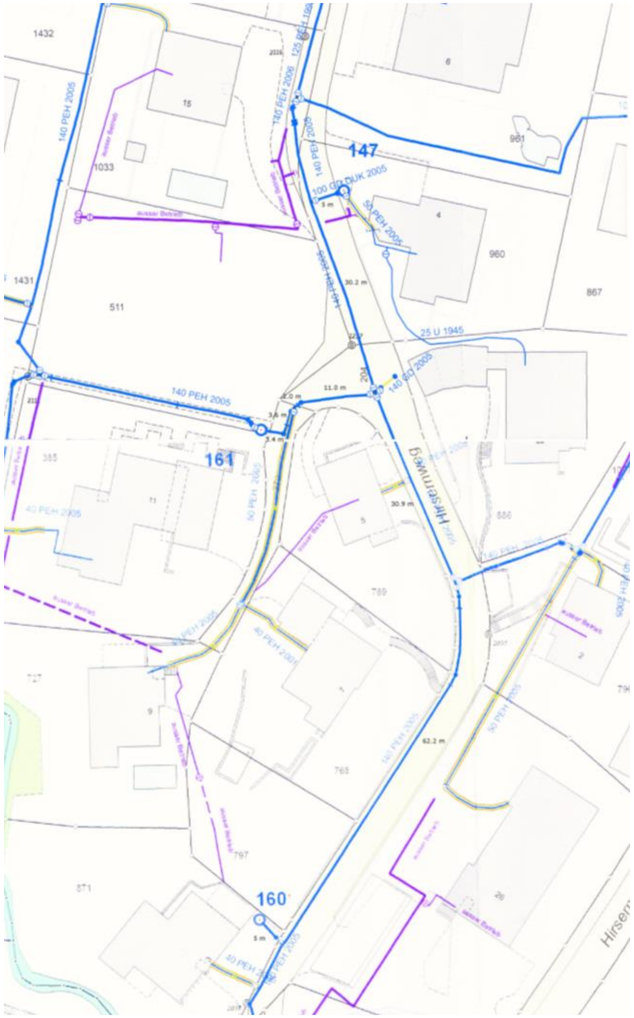
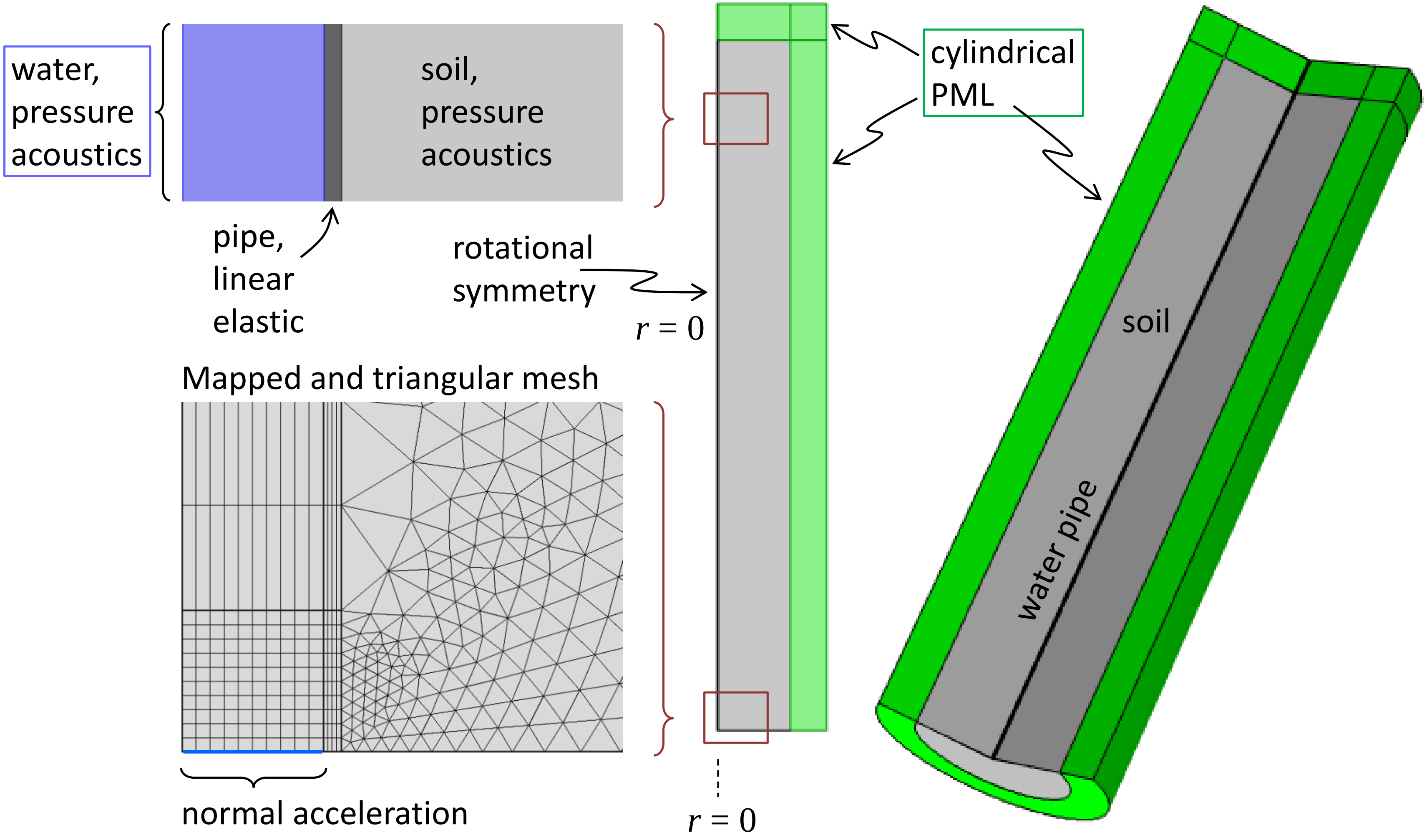
Example 1: Horw (Switzerland) GD-FZM-150 (red) GU-200 (blue), *c* = 1280 m/s. Leak was pinpointed and found at X on 22.5.2013

Example 2: Hergiswil (Switzerland) PE 140, *c* = 370 m/s



Model Geometry



water, pressure acoustics

soil, pressure acoustics

pipe, linear elastic

rotational symmetry

$r = 0$

cylindrical PML

Mapped and triangular mesh

normal acceleration

$r = 0$

Governing Equations

isotropic, elastic wave equation:

$$\frac{\partial^2 \vec{u}}{\partial t^2} - c_1^2 \nabla\left(\nabla \vec{u}\right) + c_2^2 \nabla \times \left(\nabla \times \vec{u}\right) = 0$$

$$c_1^2 = \frac{\lambda_L + 2\mu_L}{\rho} \qquad c_2^2 = \frac{\mu_L}{\rho}$$

Lamé constants:

$$\lambda_L = \frac{\nu E}{(1+\nu)(1-2\nu)}$$

$$\mu_L = G = \frac{E}{2(1+\nu)}$$

Helmholtz equation:

$$\nabla\left(\frac{1}{\rho}\nabla p\right) - \frac{1}{\rho c^2}\frac{\partial^2 p}{\partial t^2} = 0$$

$$\xrightarrow{\rho \neq \rho(\vec{x})} \nabla^2 p - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0$$

$$\xrightarrow{p = \hat{p}^{j\omega t}} \nabla^2 \hat{p} + \frac{\omega^2}{c^2}\hat{p} = 0$$

$$\nabla^2 \hat{p} + k^2 \hat{p} = 0$$

stress tensor

coupling, *BC* at interface: $-\vec{n}\,\frac{1}{\rho}\nabla p = \vec{n}\,\frac{\partial^2 \vec{u}}{\partial t^2}$ and $\underline{\underline{\sigma}} \cdot \vec{n} = p\,\vec{n}$

Material properties

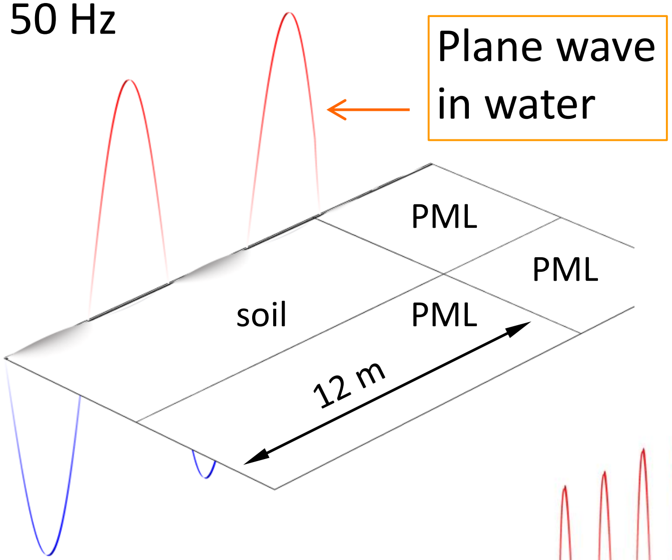
Water (20°C)	Density ρ	1000 kg/m^3
	Speed of sound c	1481 m/s
Soil	Density ρ	2500 kg/m^3
	Speed of sound c	1000 m/s
Air (20°C)	Density ρ	1.19 kg/m^3
	Speed of sound c	343 m/s
Polyethylene (PE100)	Density ρ	950 kg/m^3
	Elastic modulus E	1.1×10^9 Pa
	Poisson's ratio ν	0.33
Ductile Iron	Density ρ	7050 kg/m^3
	Elastic modulus E	170×10^9 Pa
	Poisson's ratio ν	0.25

Standard Dimension Ratio $SDR = \frac{D}{T} = 11$ or 17

Acoustic Wave in Water-Pipe-Soil

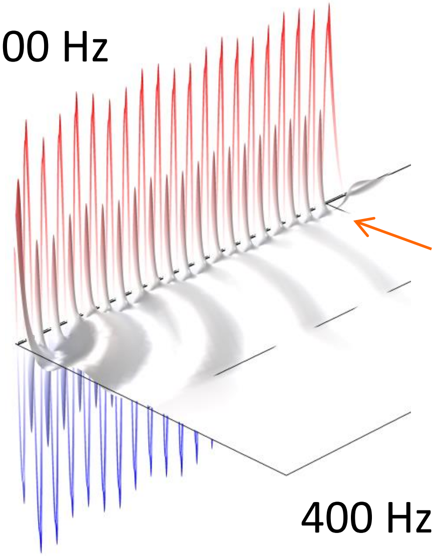
PE100 pipe, 160 SDR 11
E = 1.1 GPa, OD 160 mm
S_1 14.6 mm

50 Hz



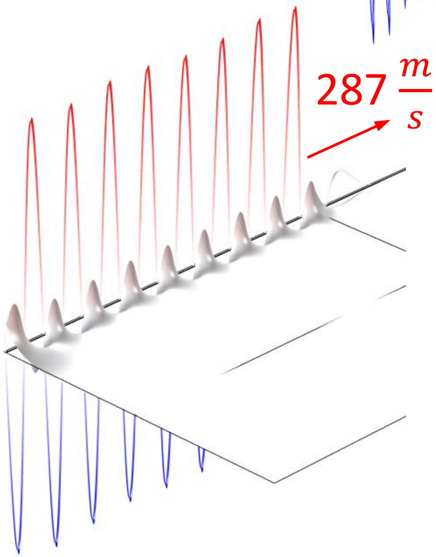
Plane wave in water

400 Hz



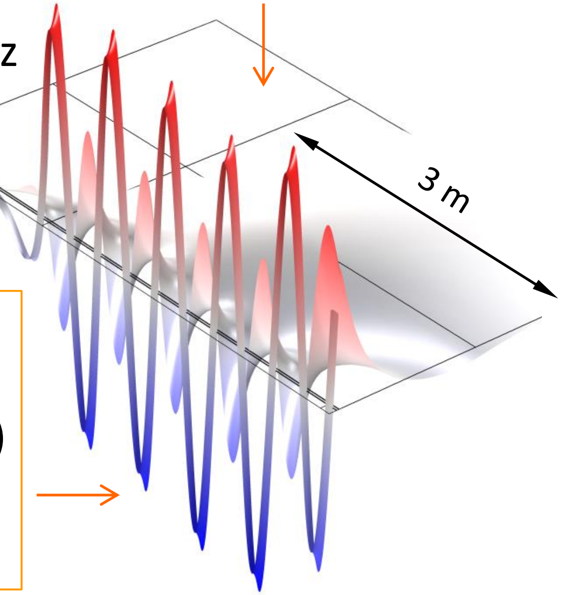
Acoustic wave in soil, coupled to water wave via pipe layer. No significant radiation loss

200 Hz



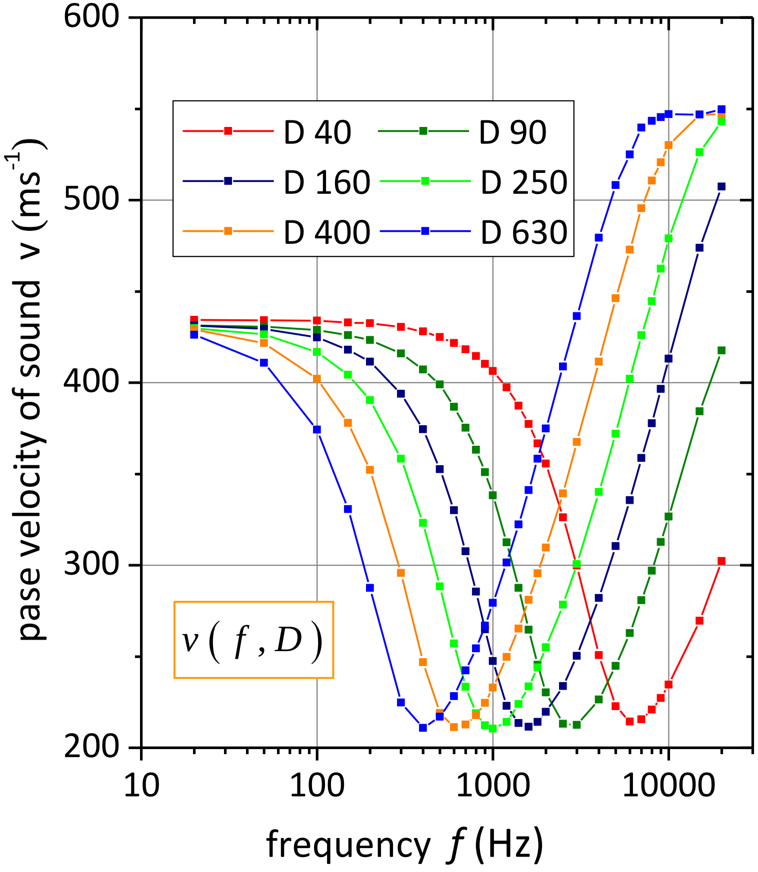
Acoustic wave in soil adjacent to pipe

400 Hz

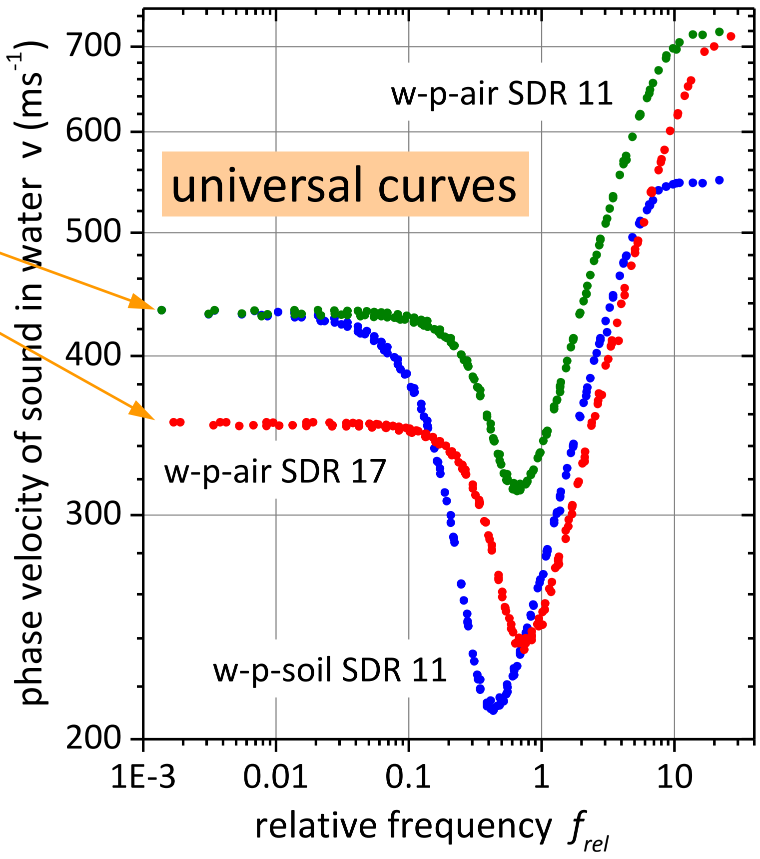


Maximum amplitude (water, soil) adheres to pipe layer

α1 mode: *v* vs. *f*

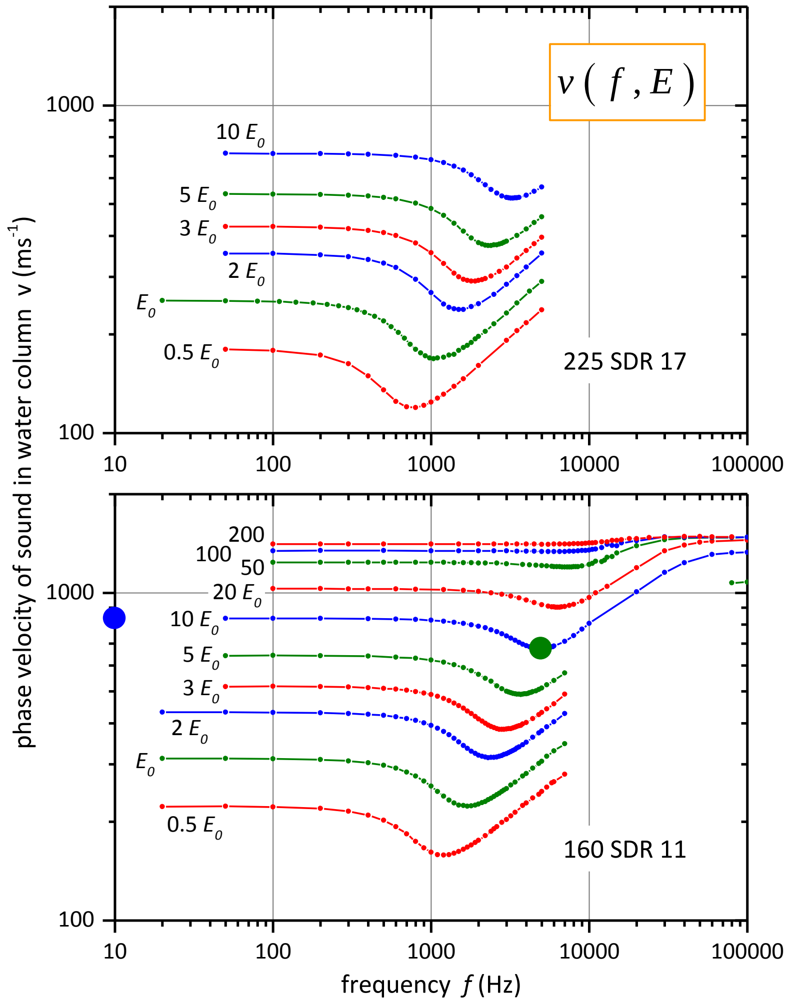


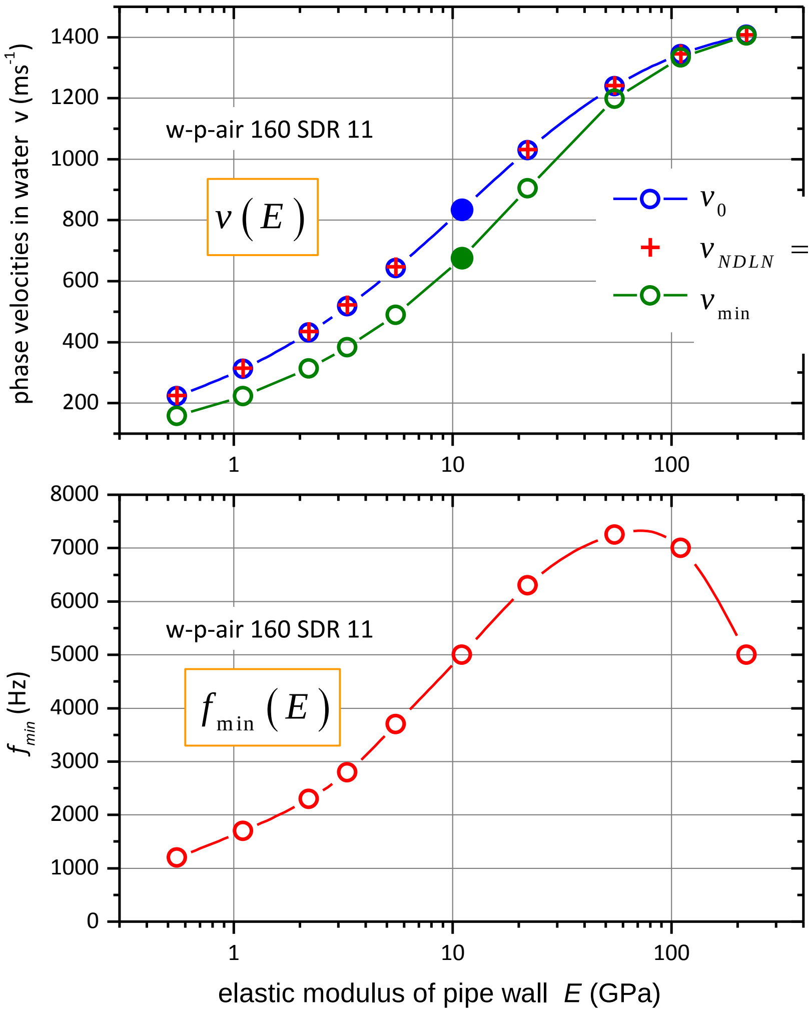
$$\frac{v_{11}}{v_{17}} \approx \sqrt{\frac{SDR17-2}{SDR11-2}} = 1.29$$



$$f_{norm} = c_w / d\pi \qquad f_{rel} = \frac{f}{f_{norm}} = \frac{f \cdot d\pi}{c_w}$$

α1 mode: $v(f, E)$

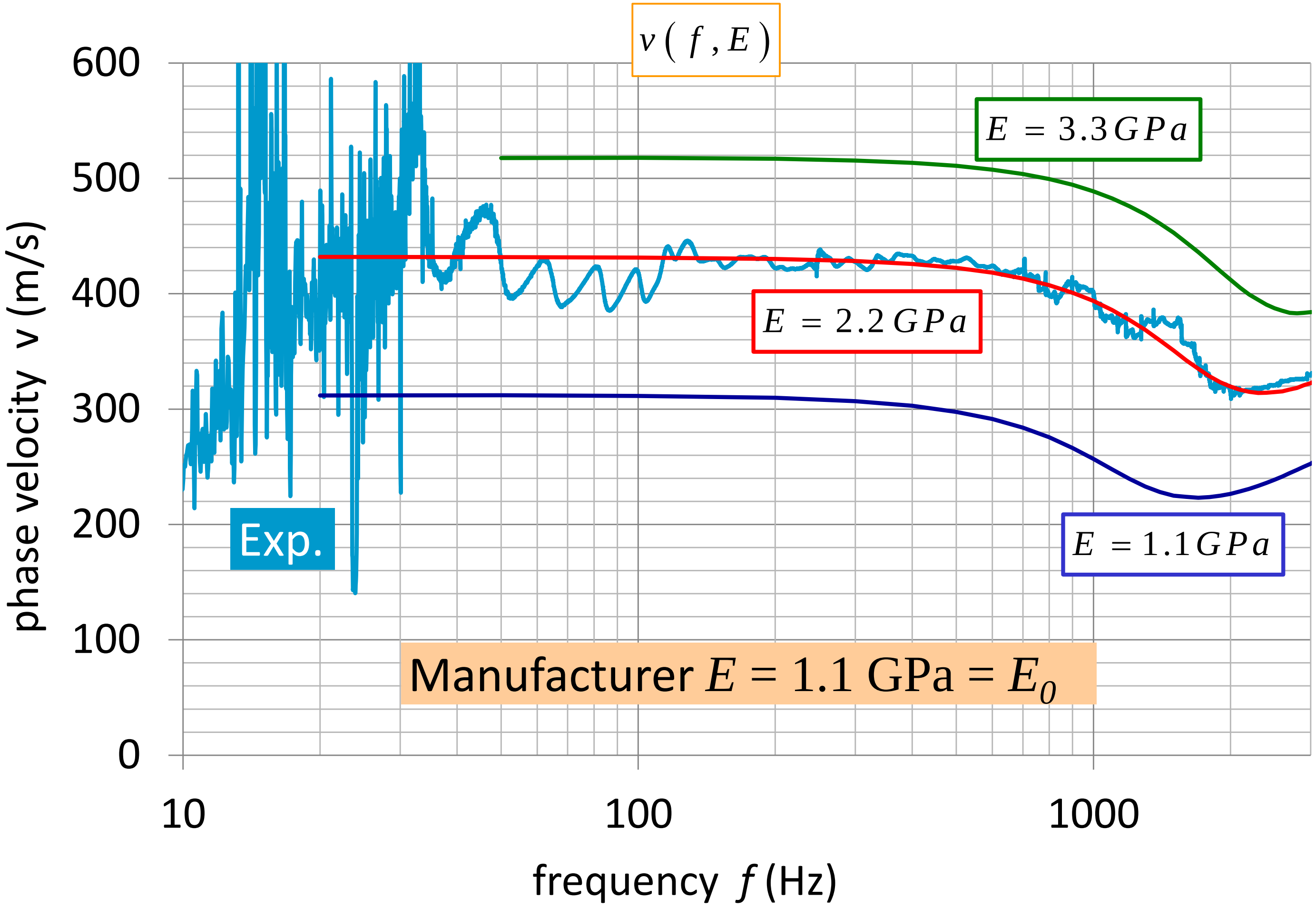




$$= \frac{c_w}{\sqrt{1+\frac{B_w \cdot d}{E \cdot T}}}$$

$B_w \approx$ 2.5 GPa
> 2.2 GPa

FEM vs. Experiment



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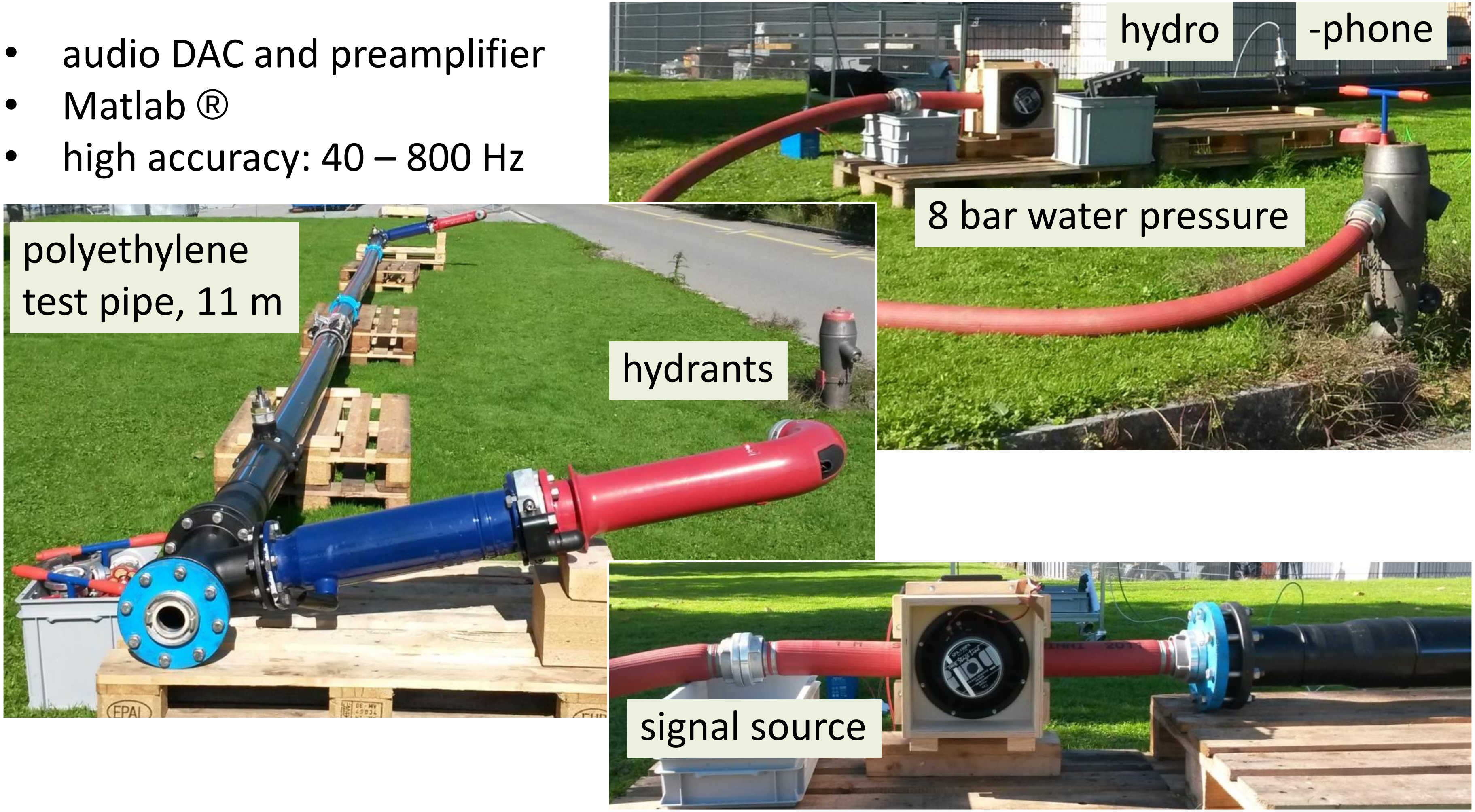
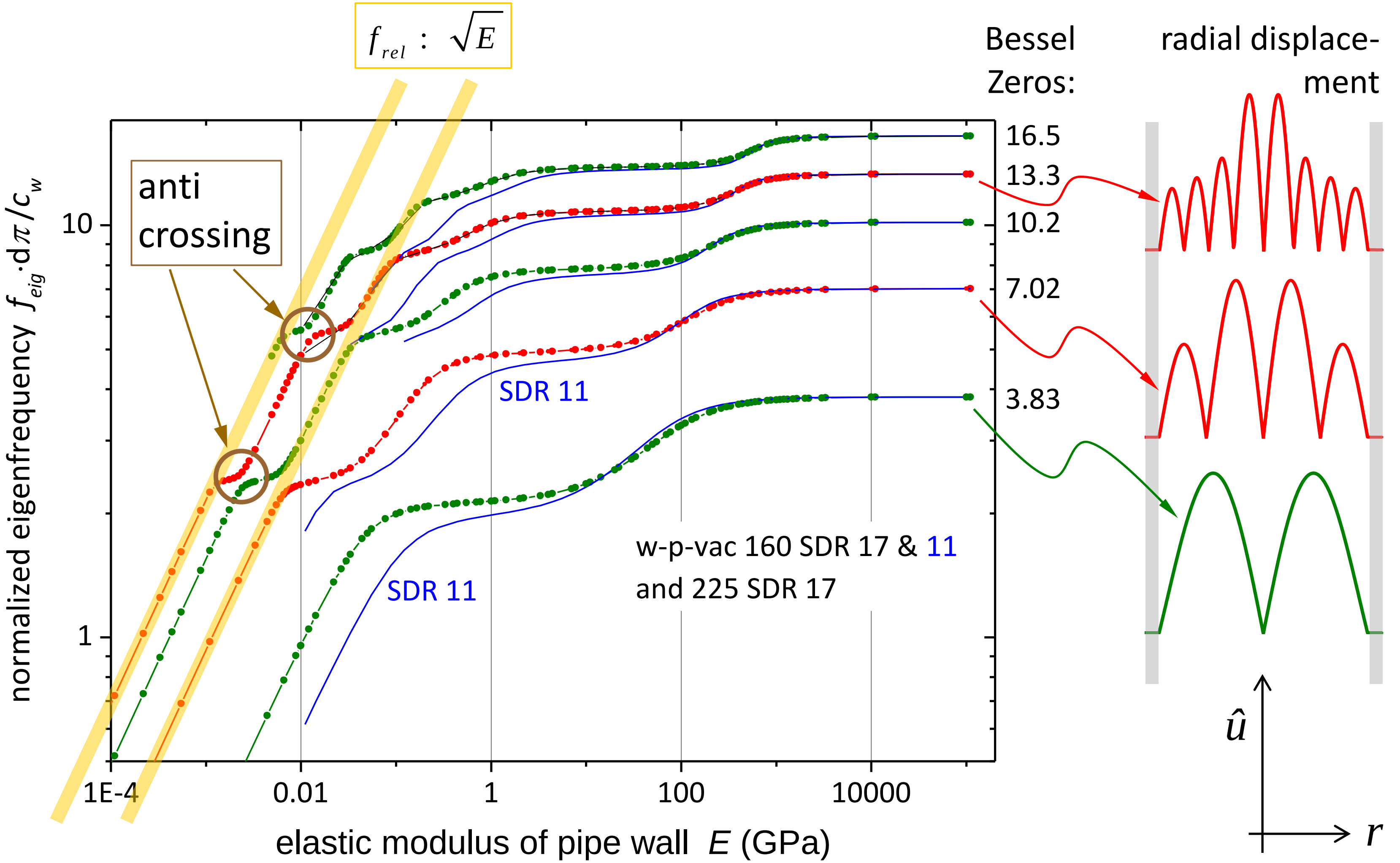
Experimental Setup

- audio DAC and preamplifier
- Matlab ®
- high accuracy: 40 – 800 Hz



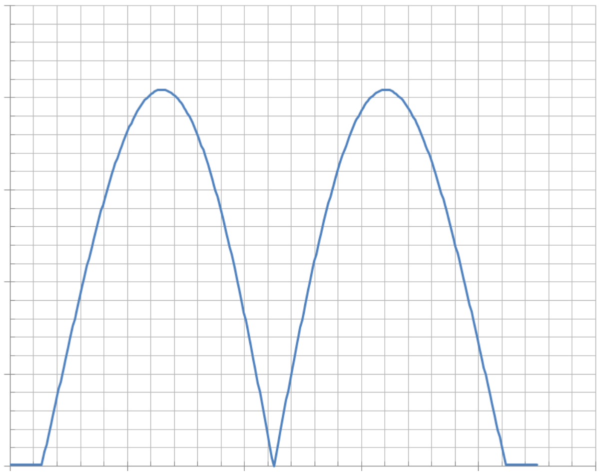
Eigenfrequency, cut-off / cut-on

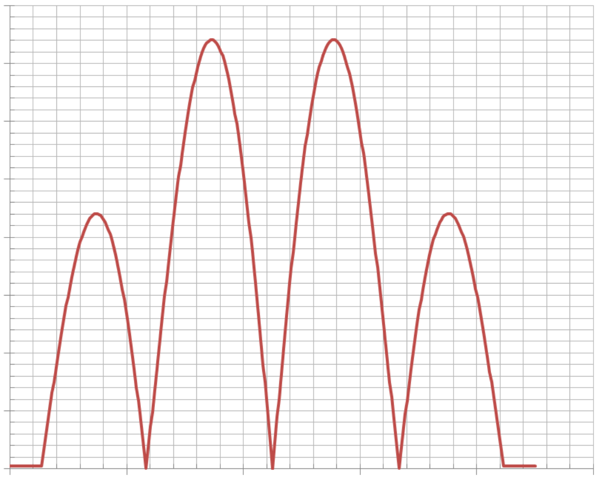


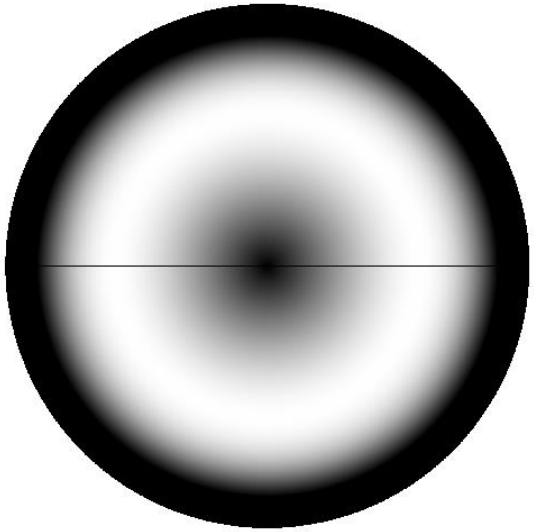
Eigenmodes, Hard Walls

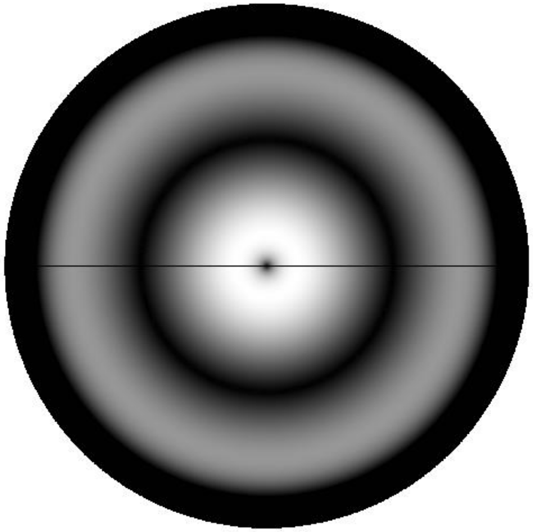
radial
displacement
amplitude

$\hat{u}_r \; (a.u.)$

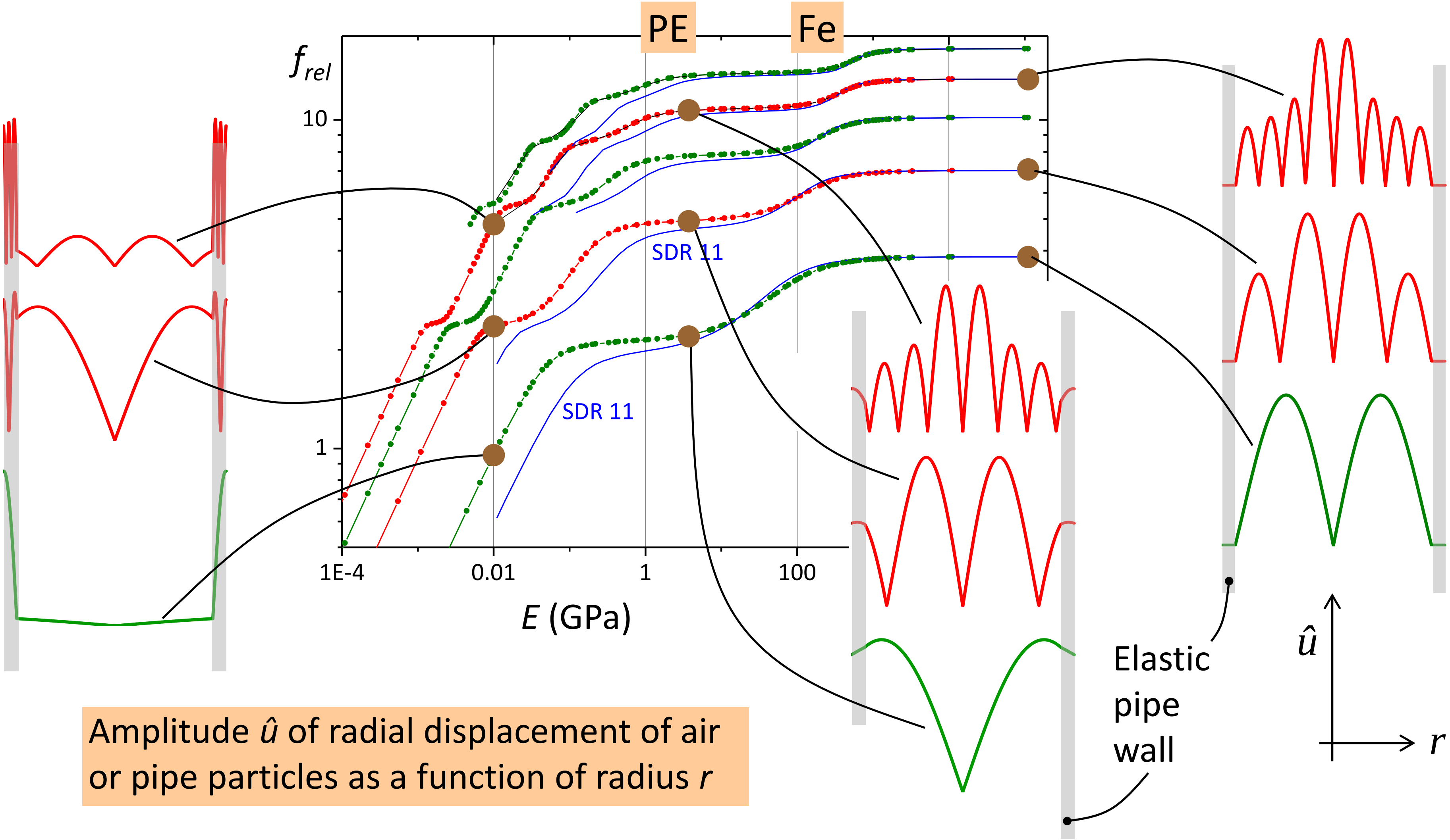






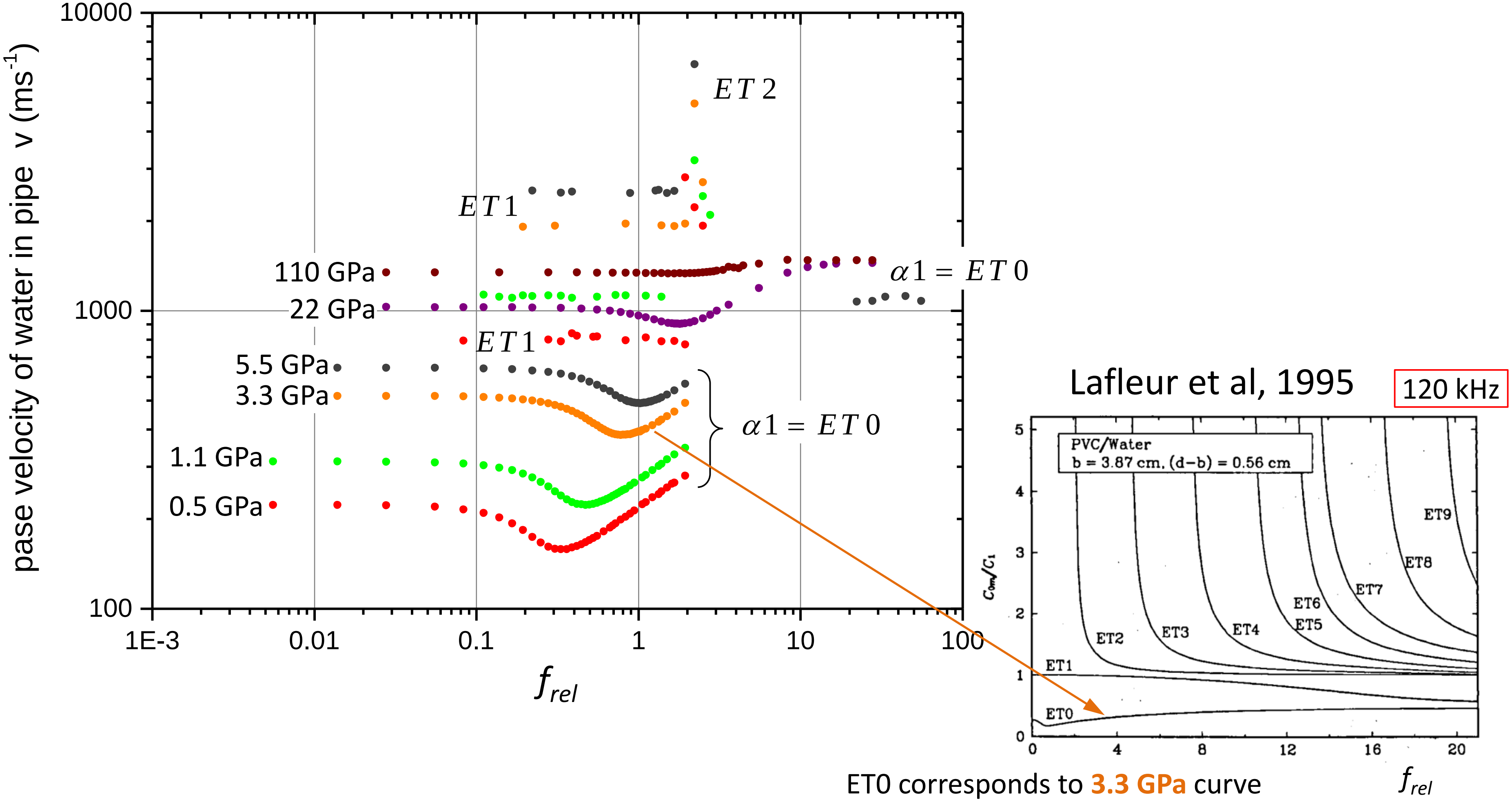


Modes, Radial Displacement



Amplitude $\hat{u}$ of radial displacement of air or pipe particles as a function of radius r

Higher Wave Modes



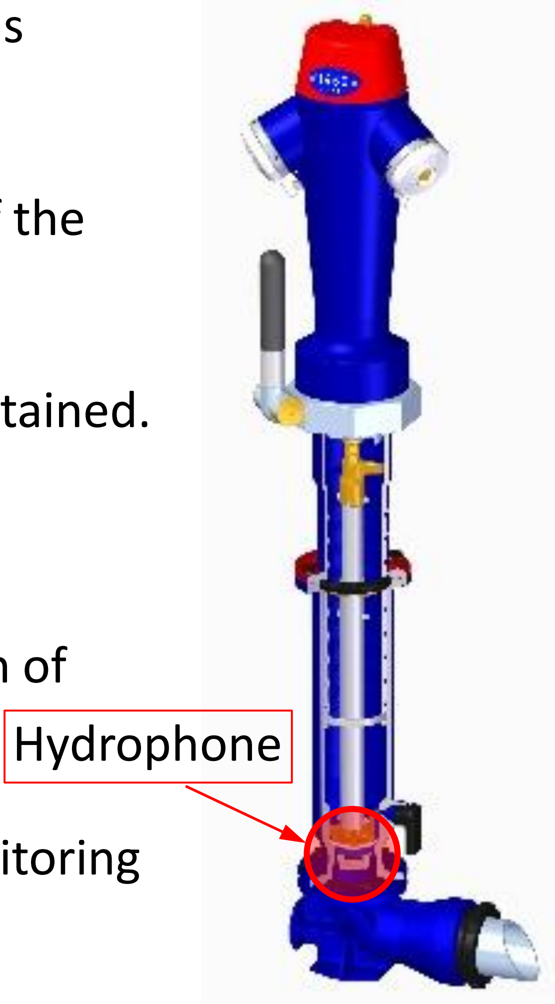
Lafleur et al, 1995

120 kHz

ET0 corresponds to **3.3 GPa** curve

Summary

- The propagation of leak noise in polyethylene pipes was investigated.
- The axisymmetric $\alpha 1$ mode is the major component of the leak noise.
- Universal dispersion relations of the $\alpha 1$ mode were obtained.
- There exists little dispersion at low frequency.
- The 2D eigenfrequencies increase in steps as a function of wall elasticity.
- Hinni hydrants are equipped with hydrophones. A monitoring algorithm shall locate the leak in the pipe network.



Thank you for your attention !

Eigenfrequency, cut-off / cut-on