Weakly non-linear analysis of azimuthal thermoacoustic modes in annular combustion chambers

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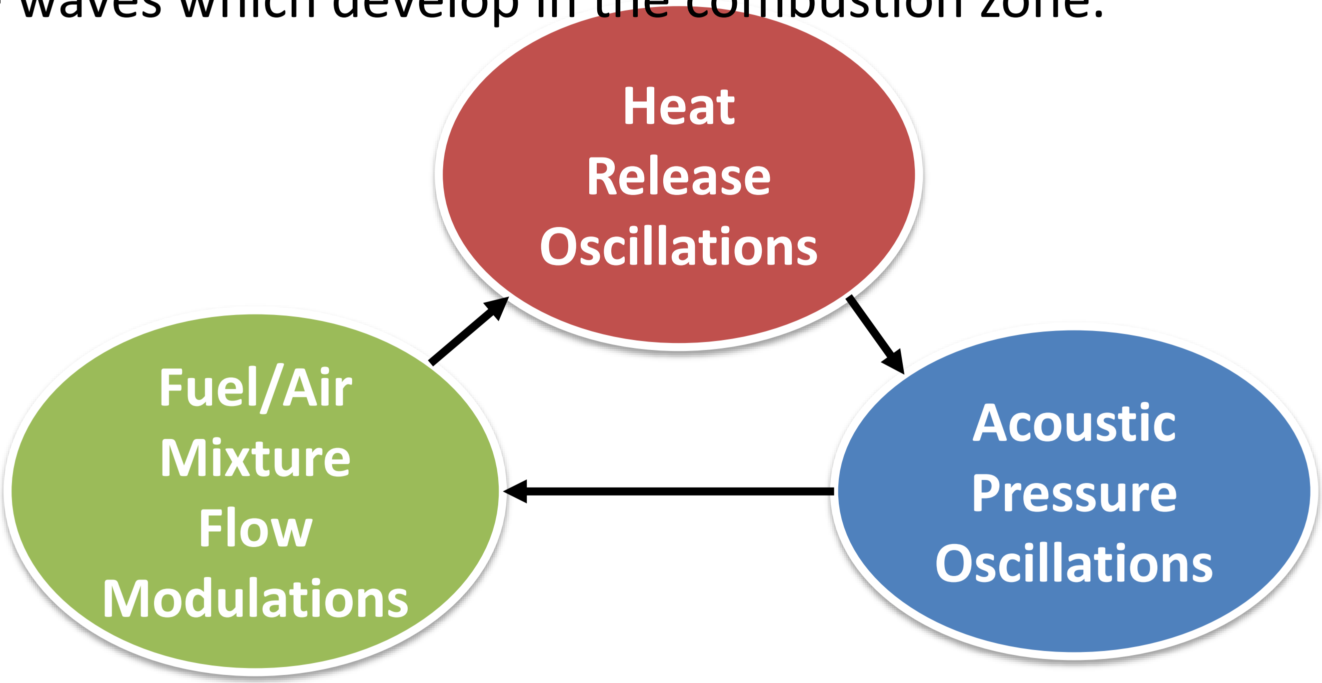
Origin of Thermoacoustic Instability

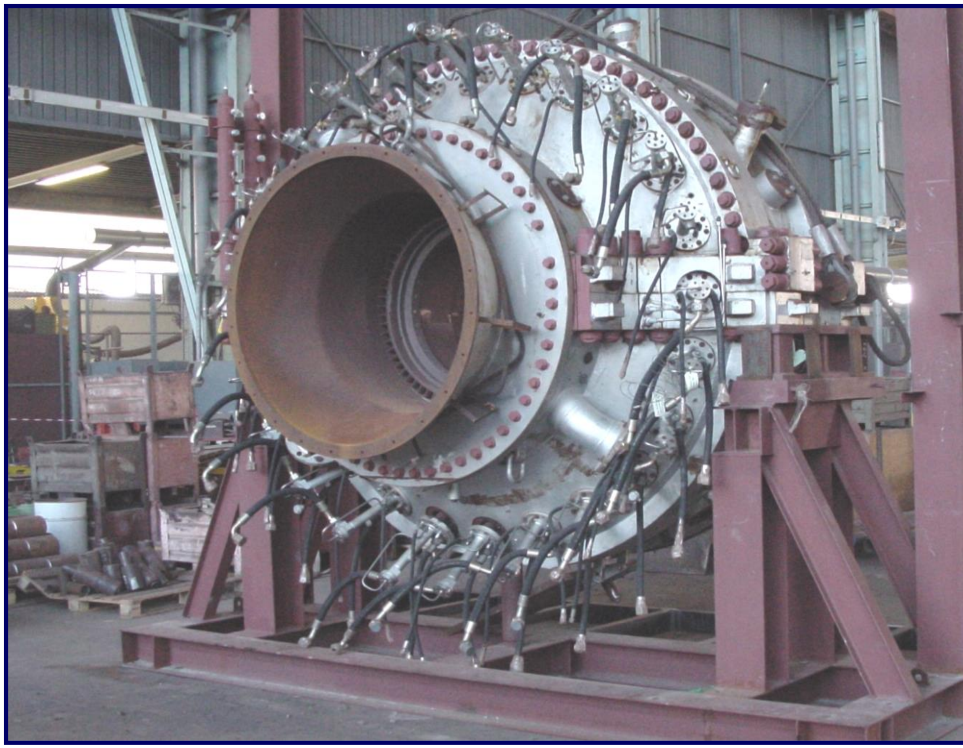
Thermoacoustic instabilities are **self-excited** pressure oscillations generated by a mutual interaction between heat release fluctuation and pressure waves which develop in the combustion zone.

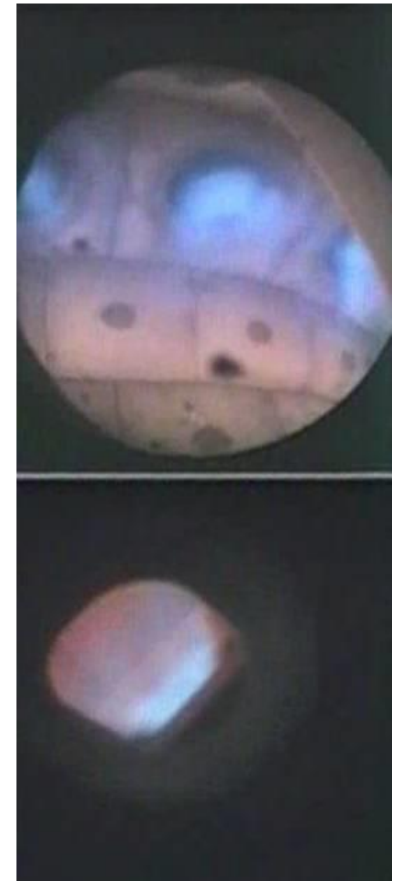


Combustion dynamics → Acoustics

In general, the thermoacoustic oscillations are associated with one natural pure acoustic modes of the combustion chamber of the system (bulk, axial and transverse modes).

Thermoacoustic Instability





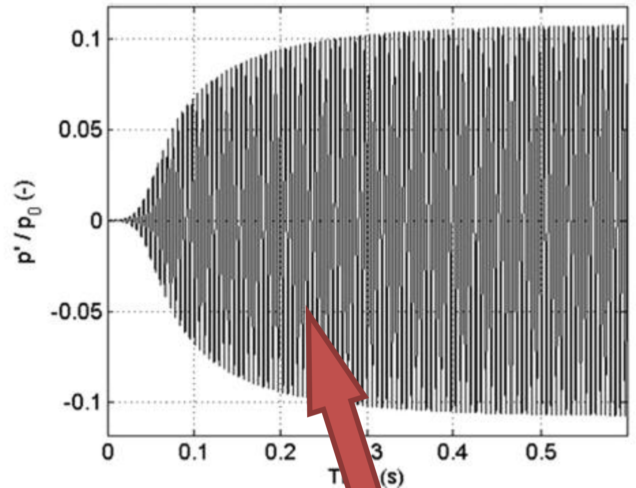
Thermoacoustic Instabilities

Gain of Acoustic

$\Rightarrow$ *Instability*

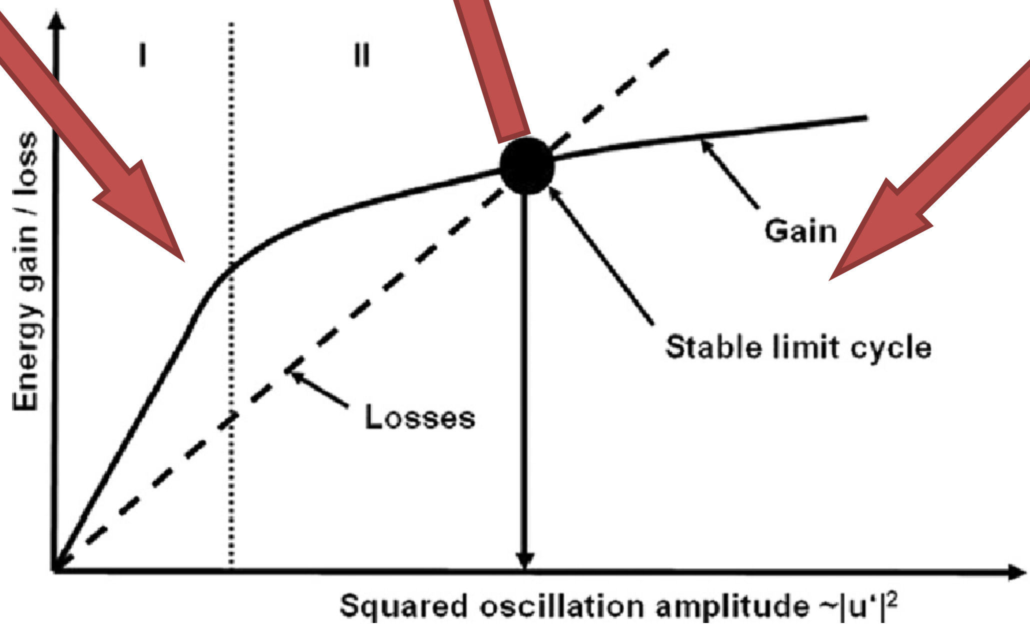
$p' \longrightarrow$ acoustic press

$q' \longrightarrow$ heat release f



Linear Zone

Nonlinear Zone

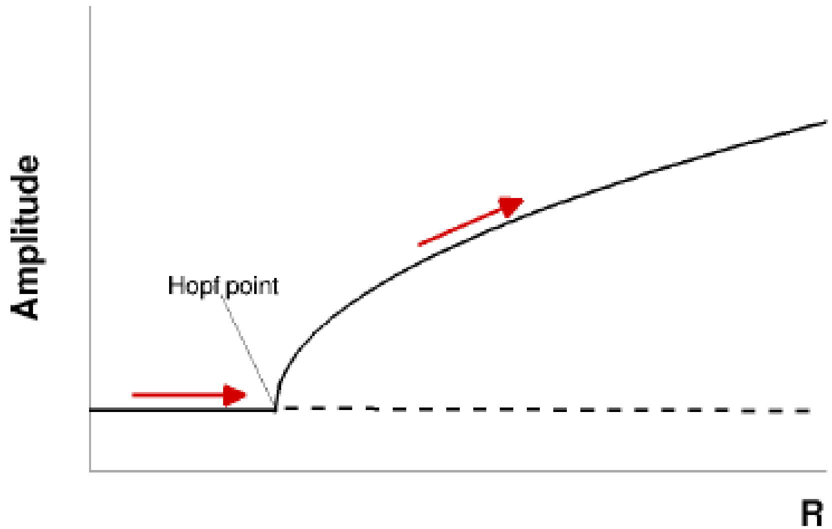


Krebs et al., ***Journal of Engineering for Gas Turbines and Power***, 135 (8), 081 503.

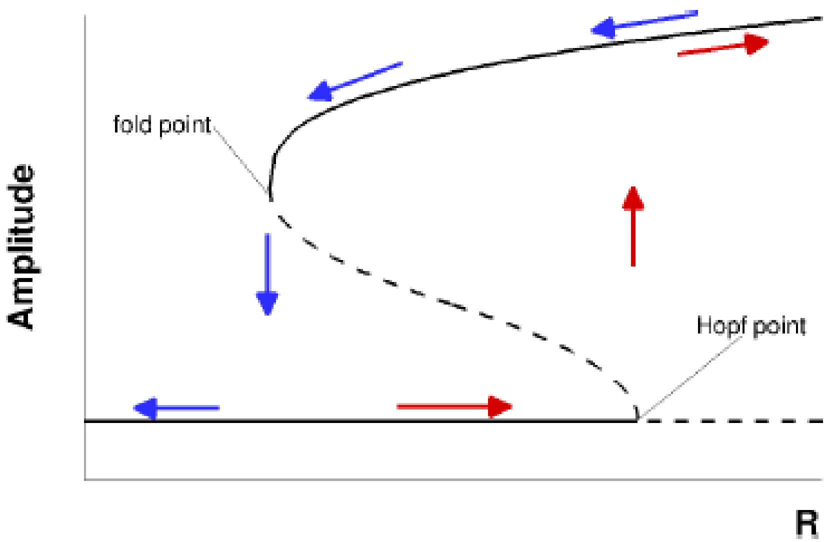
ThermoAcoustic Instabilities: Hopf Bifurcation

NonLinear Flame Models are able to

- Get the limit cycle amplitude
- say if the system is subject to hysteretic phenomena
- provide information about the range of stability for certain parameters

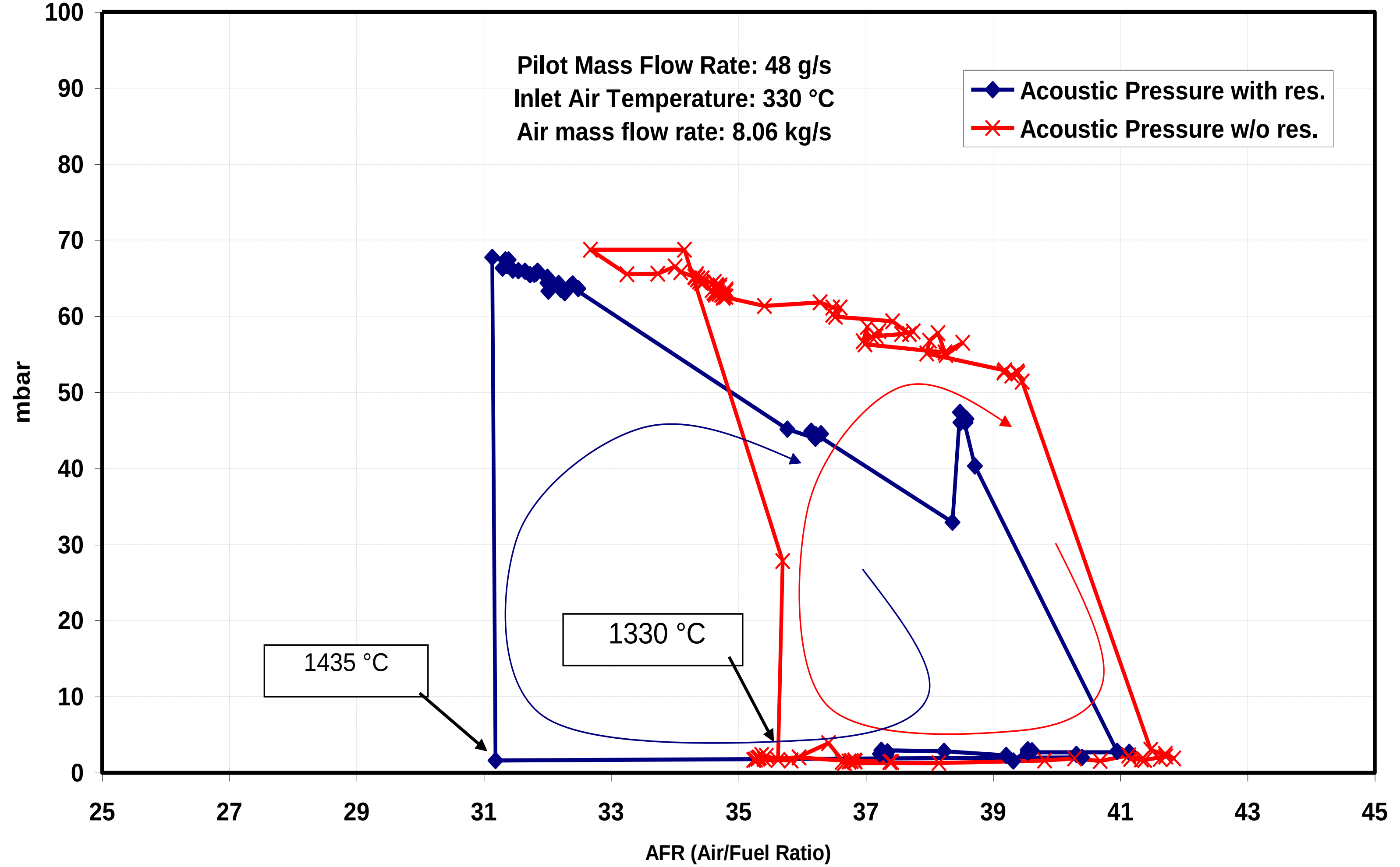


Supercritical Bifurcation



Subcritical Bifurcation

Real Subcritical Bifurcation Process



Objective

Develop a simplified 3D FEM tool able to study the nonlinear behavior (limit cycles amplitudes, hysteresis cycles, triggering) of annular combustion chambers

- Introduction of non linearities in the Flame Transfer Function model

- Determine the bifurcation diagrams for these flame models by means of a weakly non-linear analysis.

Mathematical Model

Wave equations with damping and acoustic source

Eigenfrequency Analysis

$$\frac{1}{\bar{c}^2}\frac{\partial^2 p'}{\partial t^2} + \frac{\zeta}{\bar{c}R}\frac{\partial p'}{\partial t} - \bar{\rho}\,\nabla\cdot\left(\frac{1}{\bar{\rho}}\nabla p'\right) = \frac{\gamma-1}{\bar{c}^2}\frac{\partial q'}{\partial t}$$

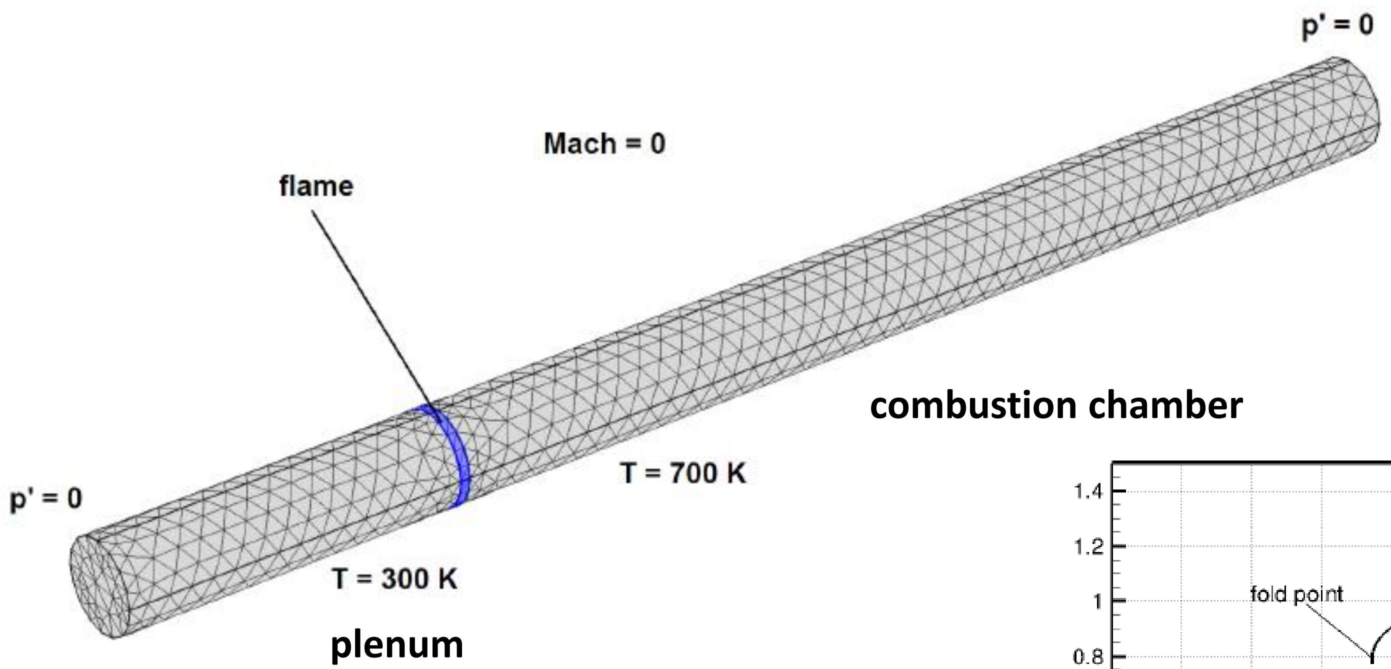
$$\frac{\lambda^2}{\bar{c}^2}\hat{p} + \frac{\lambda\zeta}{\bar{c}R}\hat{p} - \bar{\rho}\,\nabla\cdot\left(\frac{1}{\bar{\rho}}\nabla \hat{p}\right) = \frac{\gamma-1}{\bar{c}^2}\lambda\hat{q}$$

where $\lambda=-i\omega$ is the complex eigenvalue of the system:

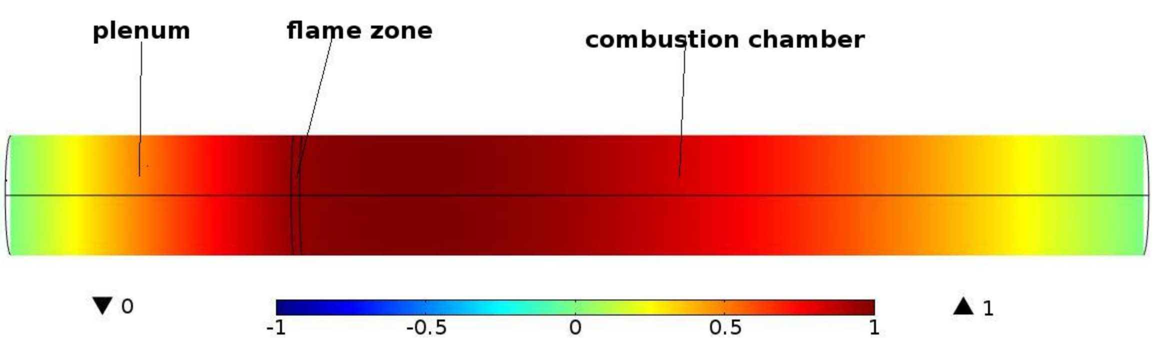
- Real part --> Eigenfrequency;
- Imaginary part --> Growth Rate:
  - Growth Rate Positive --> Unstable Mode;
  - Growth Rate Negative --> Stable Mode.

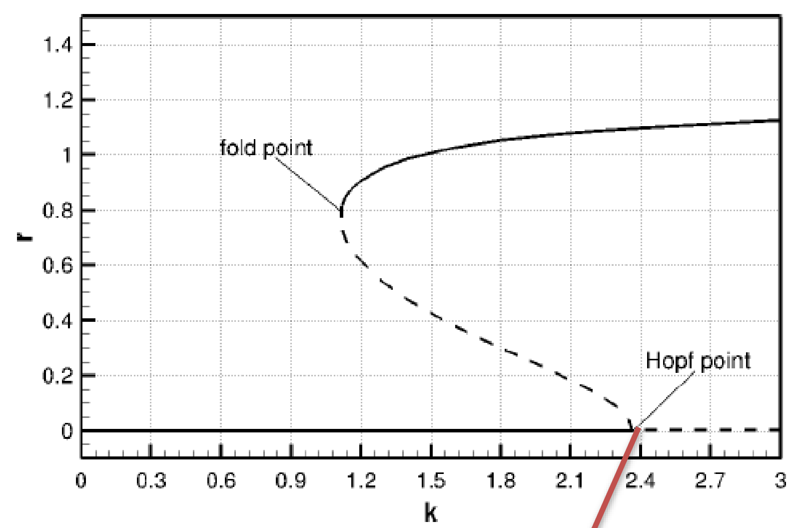
Numerical method used --> Arnoldi ARPACK
Simulation Code --> COMSOL Multiphysics

Application



$$\frac{q'(t)}{\overline{q}} = -k\left[\mu_4\left(\frac{u_i'(t-\tau)}{\overline{u}_i}\right)^5 + \mu_2\left(\frac{u_i'(t-\tau)}{\overline{u}_i}\right)^3 + \mu_0\frac{u_i'(t-\tau)}{\overline{u}_i}\right]$$





***f* = 66.7 Hz**

Weakly non linear analysis: algorithm procedure

Steps to get the bifurcation diagram:

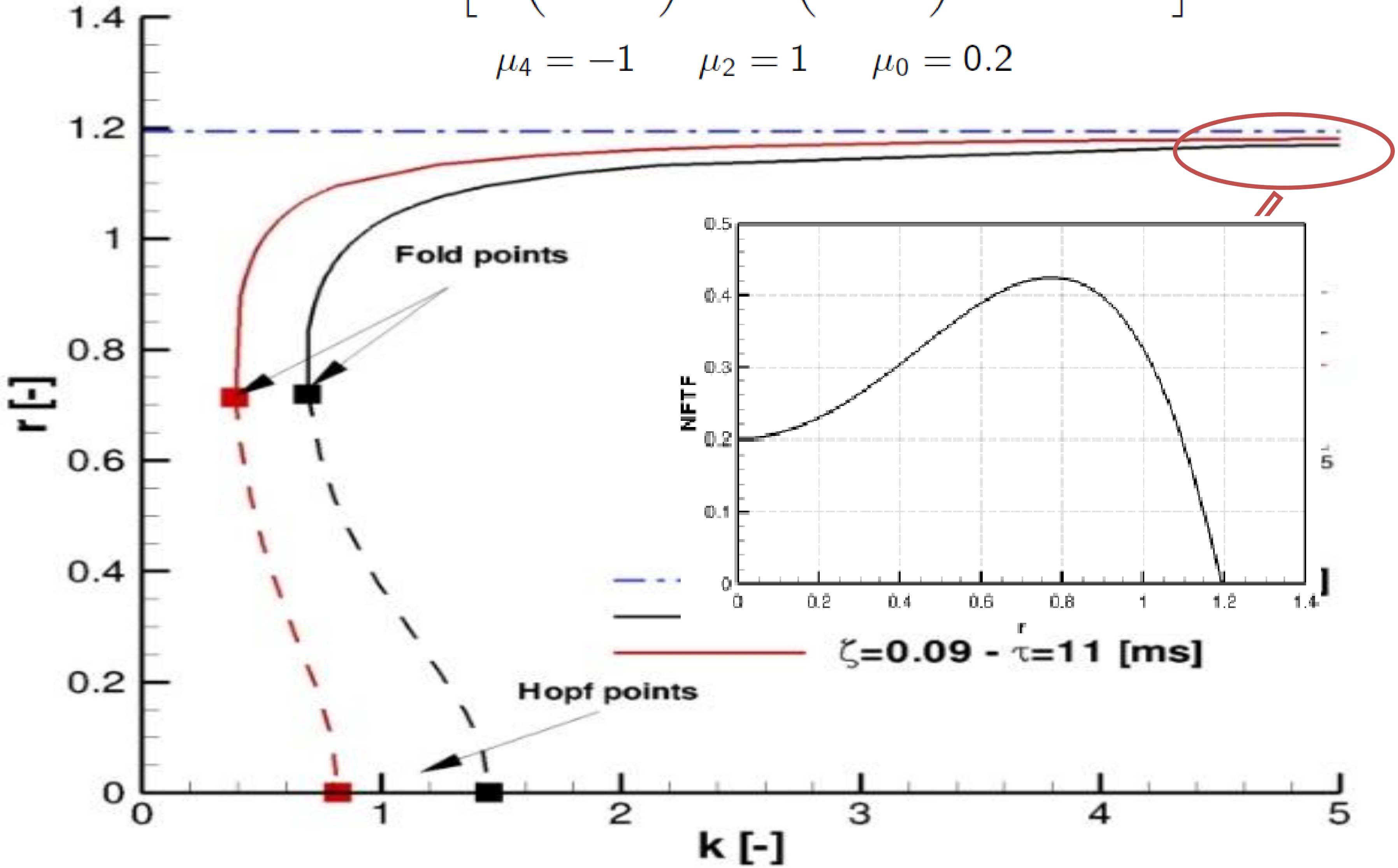
1. The interaction index k is the control parameter;
2. Define the amplitude r ($= |\hat{u}/\overline{u}|$) by guess or starting from zero;
3. For each value of r the eigenvalue problem is solved and the complex eigenfrequency is detected;
4. Vary the amplitude r until the growth rate is zero;
5. The corresponding amplitude r identifies a limit cycle solution;
6. Change the control parameter and start again.

Simulation Code --> COMSOL Multiphysics
Physics --> “Pressure Acoustics” of Module “Acoustics”

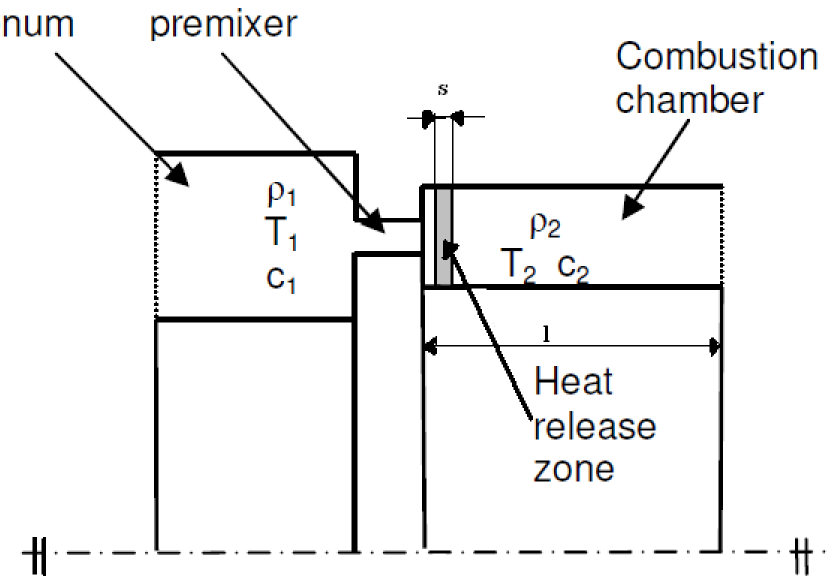
Cannular Configuration: subcritical bifurcation

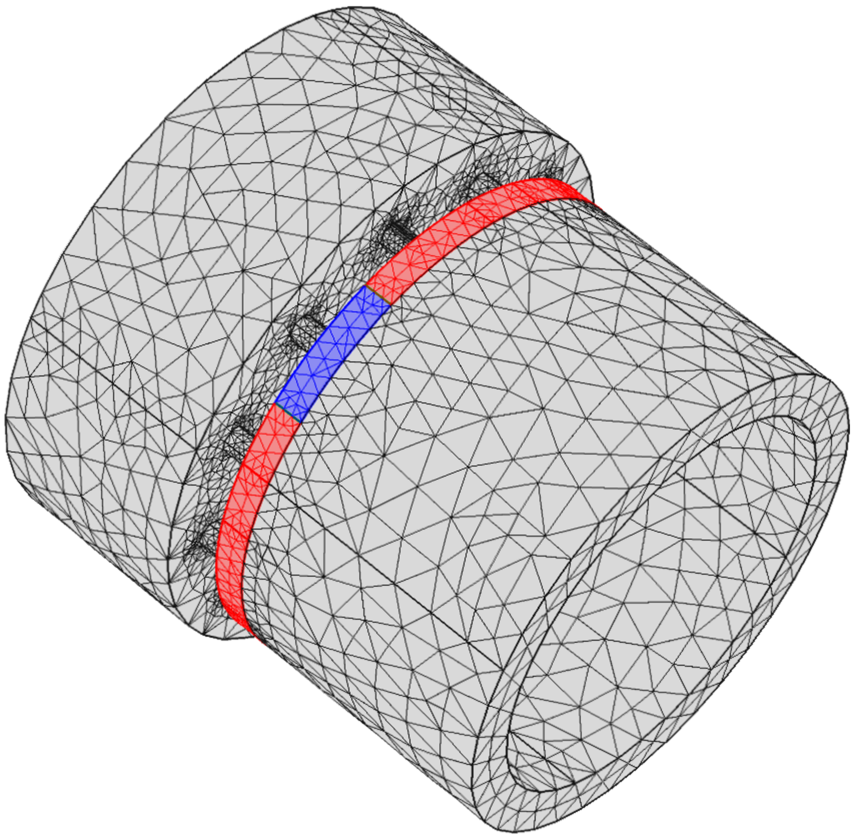
$$\frac{q'}{\bar{q}} = -k\left[\mu_4\left(\frac{u'(t-\tau)}{\bar{u}}\right)^5 + \mu_2\left(\frac{u'(t-\tau)}{\bar{u}}\right)^3 + \mu_0\frac{u'(t-\tau)}{\bar{u}}\right]$$

$$\mu_4 = -1 \qquad \mu_2 = 1 \qquad \mu_0 = 0.2$$



Annular Configuration: Model





- Temperature increases from 774 K to 2350 K across the flame
- Closed-end inlet and outlet boundary conditions, *u'* = 0

Weakly non linear analysis: algorithm procedure

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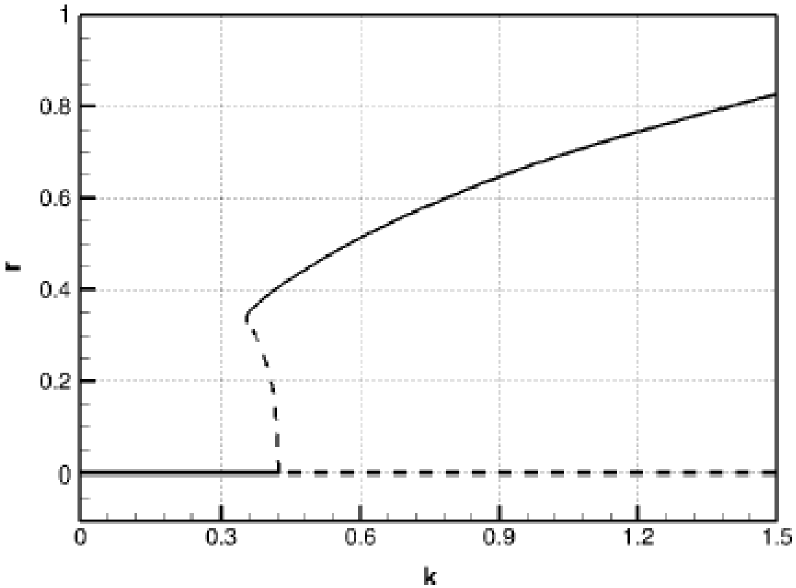
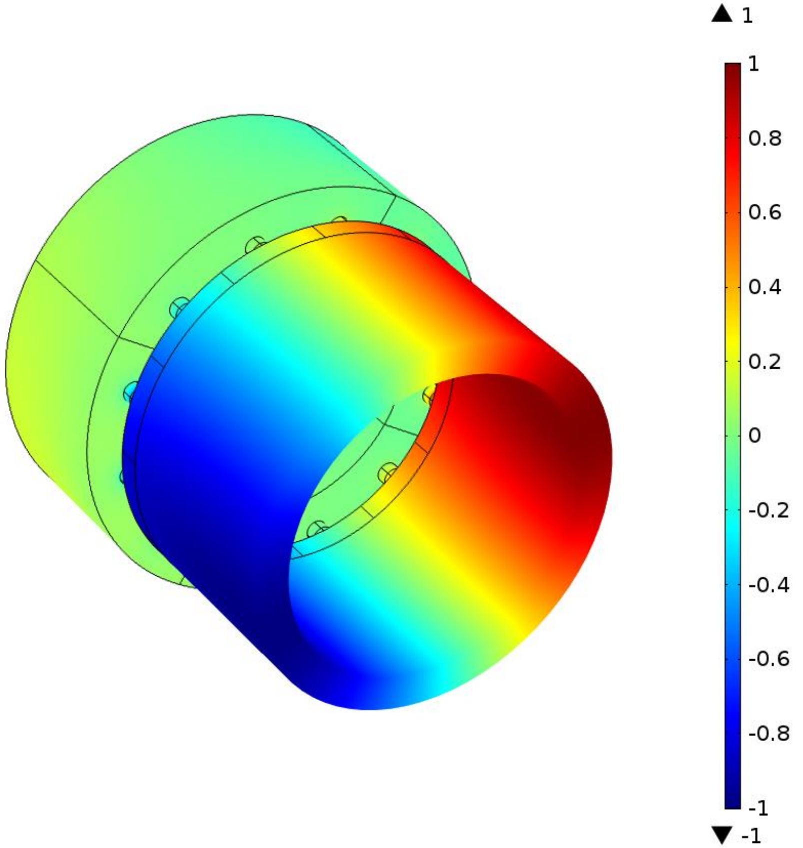
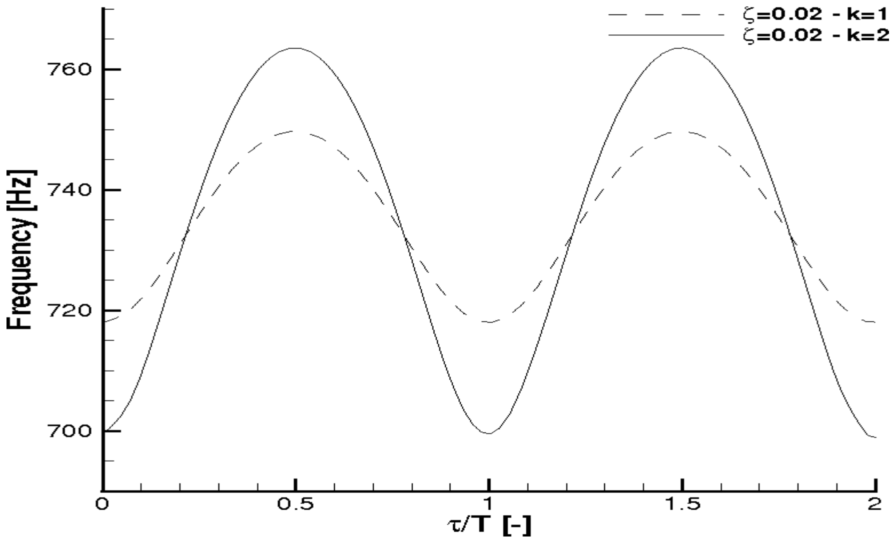
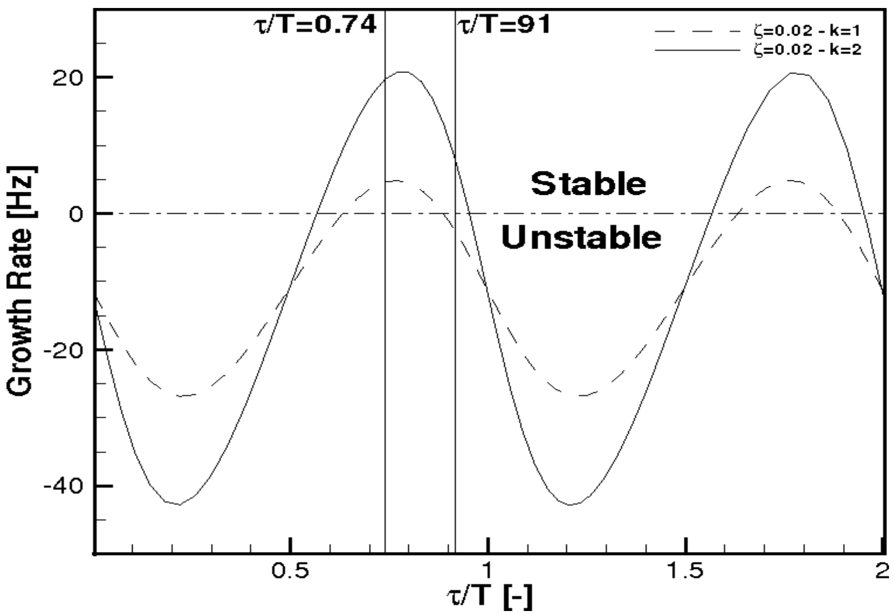


1st Azimuthal Mode (735 Hz)



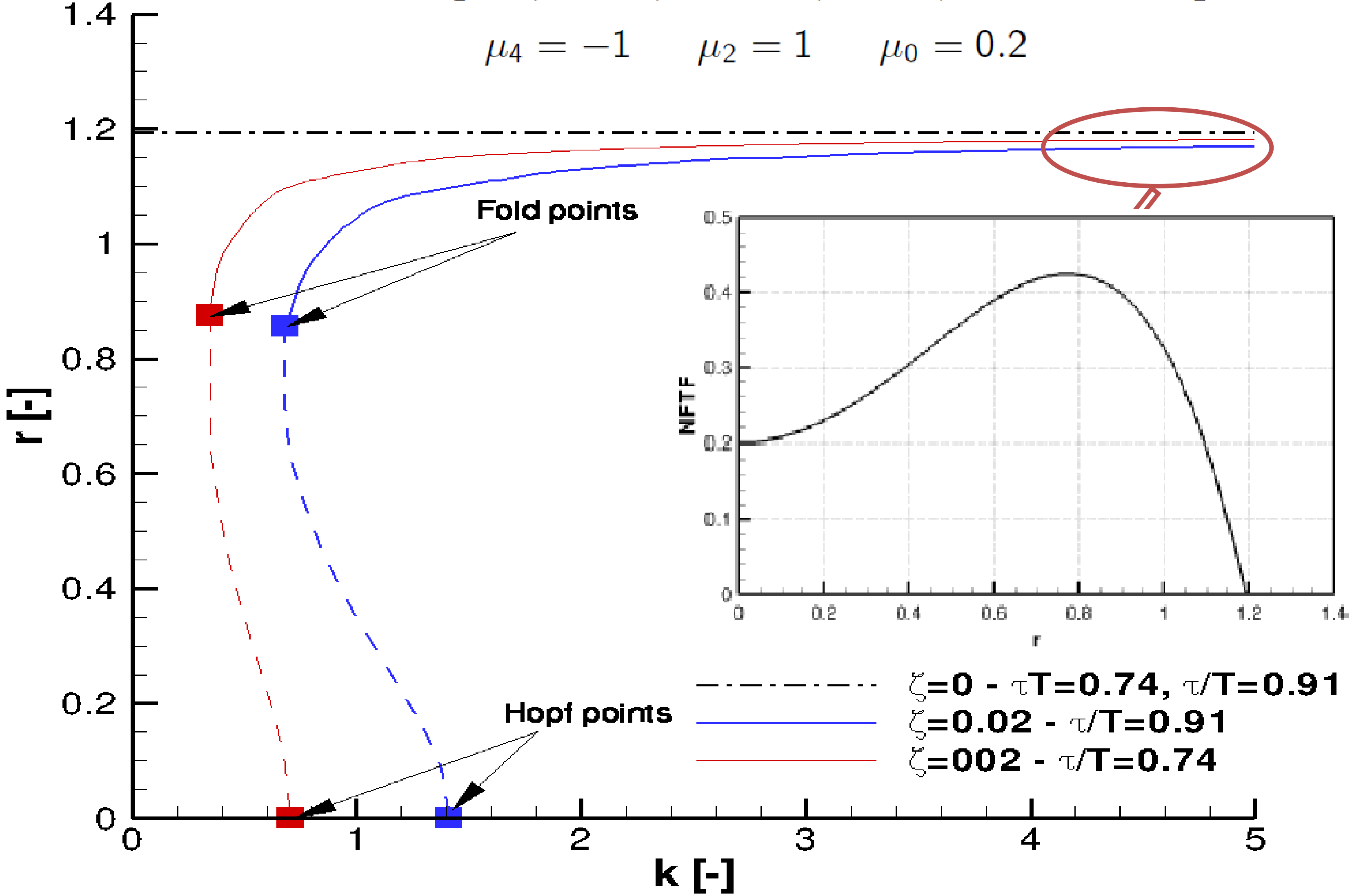




Annular Configuration: subcritical bifurcation

$$\frac{q'}{\overline{q}} = -k\left[\mu_4\left(\frac{u'(t-\tau)}{\overline{u}}\right)^5 + \mu_2\left(\frac{u'(t-\tau)}{\overline{u}}\right)^3 + \mu_0\frac{u'(t-\tau)}{\overline{u}}\right]$$

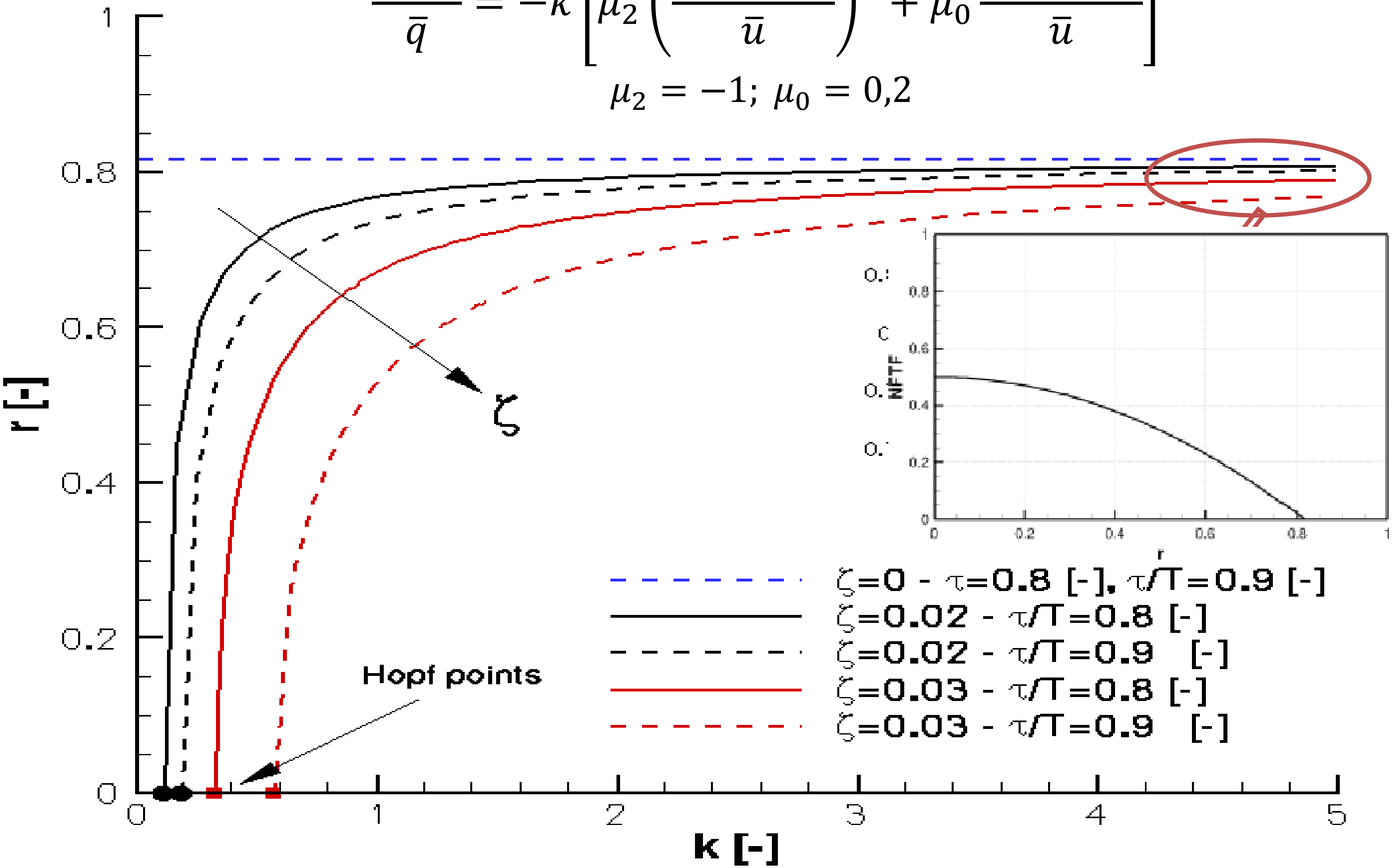
$$\mu_4 = -1 \qquad \mu_2 = 1 \qquad \mu_0 = 0.2$$



Annular Configuration: supercritical bifurcation

$$\frac{q'(t)}{\bar{q}} = -\kappa\left[\mu_2\left(\frac{u'(t-\tau)}{\bar{u}}\right)^3 + \mu_0\frac{u'(t-\tau)}{\bar{u}}\right]$$

$$\mu_2 = -1;\ \mu_0 = 0{,}2$$



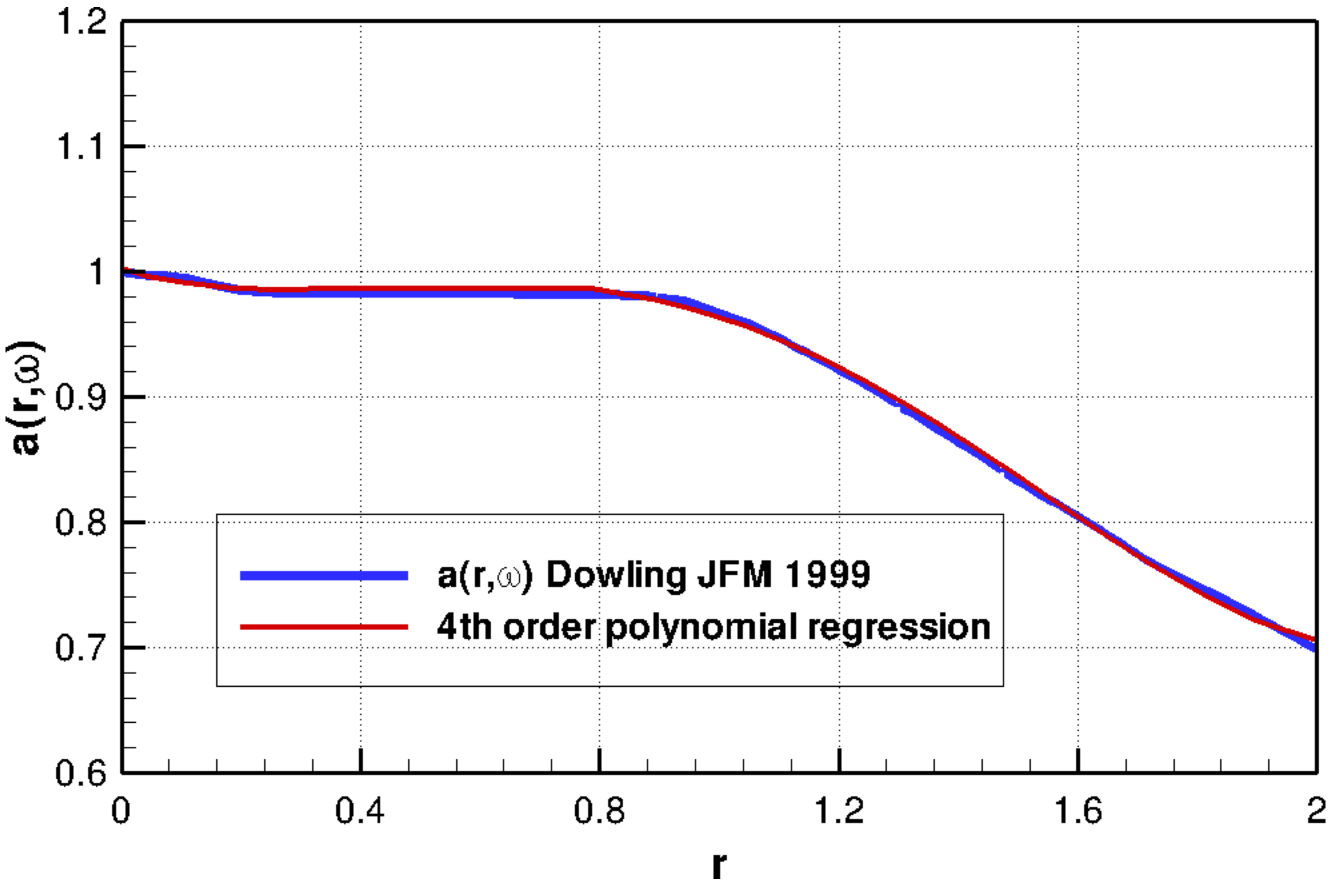
Experimental FDF – Polynomial Fitting

Ref. Dowling, JFM 1999

$$\frac{\widehat{q}}{\overline{q}} = \frac{\widehat{u}}{\overline{u}} a(r,\omega) F(\omega) \exp(-i\omega\tau)$$

Fourth order polynomial regression of the normalized gain a(r,ω)

$$NFTF_{fit} = 0.12r^4 - 0.48r^3 + 0.47r^2 - 0.16r + 1.00$$



Conclusion

- A weakly non-linear analysis procedure has been implemented in an FEM 3D Helmholtz solver for a cannular configuration
- Two different analytical non linear flame transfer function have been used.
- Bifurcation diagrams of the first unstable mode have been computed using the flame interaction index κ as control parameter
- Depending on the FTF used, the system has manifested a subcritical and a supercritical bifurcation

Future Work

- Introduction of a more realistic flame transfer function or flame describing function

- Investigation on the influence of spatial non homogeneities of flame

- Comparison with experimental data on practical machine

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Mathematical Model: NonLinear Flame Model

$$T^{L}_{flame}(\omega) = \frac{\widehat{q}/\overline{q}}{\widehat{u}_i/\overline{u}_i} = -ke^{-i\omega\tau}$$

$$\widehat{q}^{L} = T^{L}_{flame}(\omega)\frac{\widehat{u}_i}{\overline{u}_i}\overline{q}$$

$$T^{NL}_{flame}(\omega, r) = T^{L}_{flame}(\omega) \cdot NFTF(r)$$

$$\widehat{q}^{NL} = \widehat{q}^{L} \cdot NFTF(r) = T^{L}_{flame}(\omega)\frac{\widehat{u}_i}{\overline{u}_i}\overline{q} \cdot NFTF(r)$$

$$\frac{q'(t)}{\overline{q}} = -k\left[\mu_4\left(\frac{u_i'(t-\tau)}{\overline{u}_i}\right)^5 + \mu_2\left(\frac{u_i'(t-\tau)}{\overline{u}_i}\right)^3 + \mu_0\frac{u_i'(t-\tau)}{\overline{u}_i}\right]$$

L --> linear
NL --> nonlinear
r --> amplitude
NFTF --> Nonlinear FTF

$$NFTF = \frac{5}{8}\mu_4 r^4 + \frac{3}{4}\mu_2 r^2 + \mu_0$$