Integration of true-to-mechanism (DeProF) relative permeability maps for 2-ph flow in p.m. into the COMSOL^{TM} Earth Science Module

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Abstract

- **True-to-mechanism** relative permeability maps for steady-state 2-phase flow in porous media were integrated within the COMSOL^{TM} Earth Science module, to resolve field-scale flows.
- The essential characteristic of **relative permeability dependence on local flow conditions** (capillary number and flowrate ratio) have been provided by the ***DeProF*** **model** and associated **theory**.
- The **flow dependent** **relative permeability** maps have been incorporated in the Earth Science module and an appropriate modeling strategy has been contrived to treat the actual **2-phase flow** problem as a an **equivalent "effective-phase" (1-ph)** flow problem.
- Various flow arrangements, considering gravity effects and sources/sinks have been simulated.
- The simulations showed that the integration scheme is stable, it converges and numerical instabilities are only localized in areas where flow concentration takes extremely high values
- (as expected).

- **Keywords:** Two-phase flow, porous media, relative permeability, simulators, field scale.

Objectives

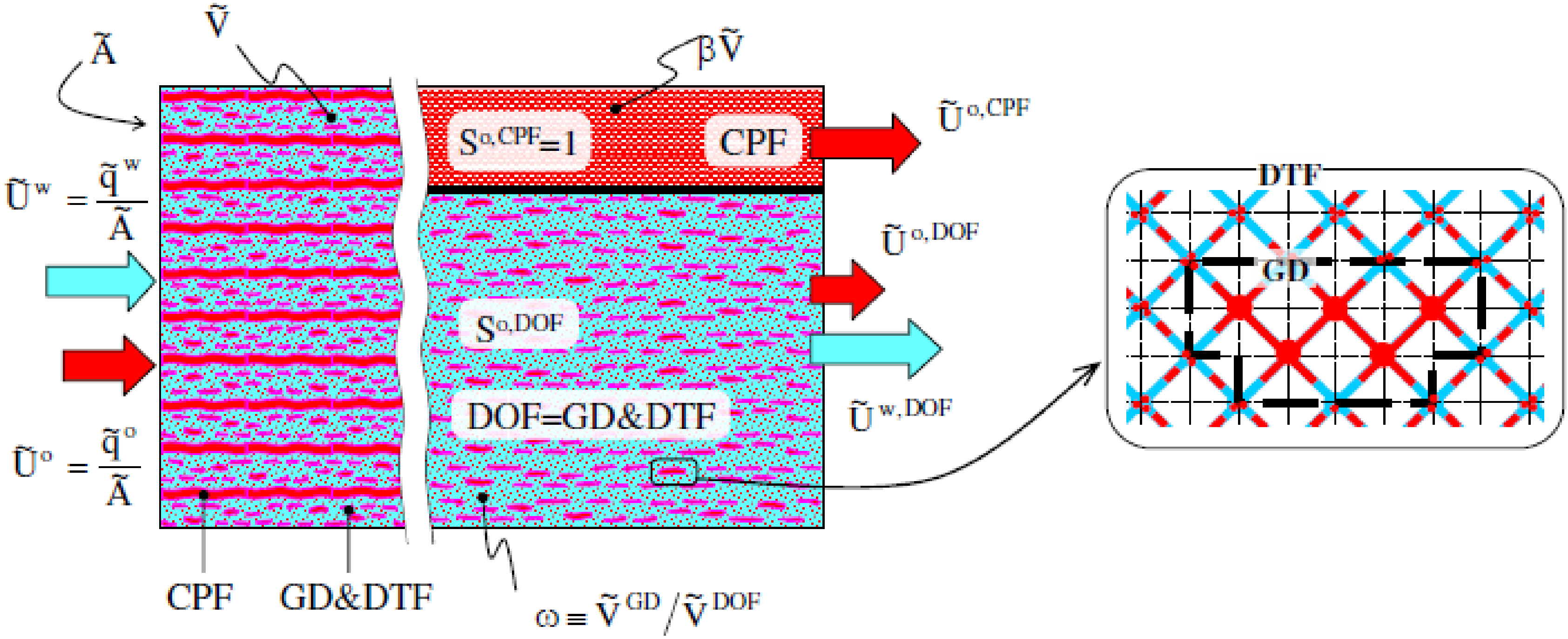
- To consider the possibility to **develop more accurate** 2-ph flow in porous media FEM **simulators**
  - Implementation of the *DeProF* model (true-to-mechanism) relative permeability maps, within state-of-the-art FEM algorithms incorporating Darcian flow modules,
  - in order to resolve field-scale 3D steady-state flows in porous media for different geometrical & flow configurations.
- The mechanistic model *DeProF* for immiscible steady-state two-phase flow in pore networks [Valavanides, *Oil & Gas Science and Technology* **67**(5) (2012)]
  - predicts the relative permeability of each phase using the concept of decomposition in prototype flows.
- It combines
  - effective medium theory with appropriate expressions for pore-to-macro scale consistency for oil and water mass transport, and
  - takes into account the pore-scale mechanisms and the network-wide cooperative effects as well as the sources of non-linearity, caused by the motion of interfaces and other complex effects.

Objectives (contnd)

- Using the ***DeProF* model**, steady-state 2-ph flow in porous media is described in terms of
  - capillary number, *Ca*
  - oil/water flowrate ratio, *r*
  - oil/water viscosity ratio, κ
  - advancing / receding contact angles
  - parameter vector, comprising dimensionless geometrical and topological parameters affecting the flow
  - including absolute permeability of porous medium, *k*
- *Extended DeProF* simulations and experimental validation provide
  - scaling law functions of *Ca* & *r*, for the reduced pressure gradient, ***x(Ca,r)*** - *and* relative permeabilities, k_{ro}, $k_{rw,}$
  - Show remarkable specificity

- These maps were integrated into general purpose FEM solvers (**COMSOL^{TM} Earth Science module**) of 2-ph flows in p.m.

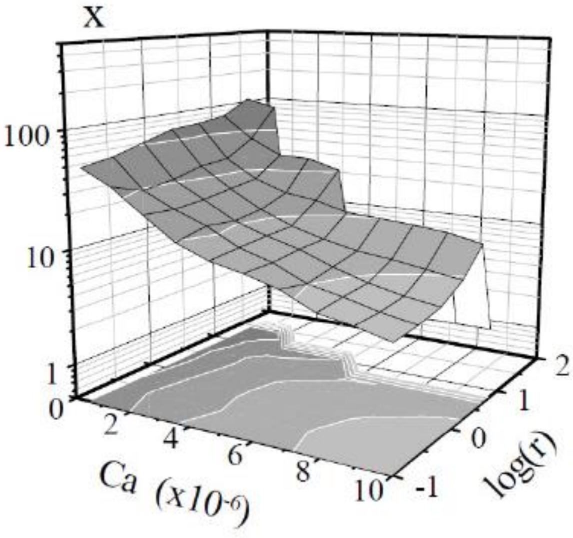
The DeProF Model for 2-ph flow in porous media



The DeProF model of the macroscopic 2ph flow and its conceptual decomposition into connected pathway flow (CPF) and disconnected-oil flow (DOF), comprising ganglion dynamics (GD) and drop traffic flow (DTF).

Functional form of the reduced pressure gradient x(CA,r)

- True-to-mechanism, flow dependence of relative permeabilities
- Functional dependence of reduced pressure gradient, on local flow conditions → universal scaling law x(Ca,r) (*DeProF* theory)



$$x(Ca,r)=\left(-\frac{\partial\tilde{p}}{\partial\tilde{z}}\right)\frac{\tilde{k}}{\tilde{\gamma}_{ow}Ca}=\begin{cases}A(r)\left(10^{6}Ca\right)^{-B(r)} & r\le r_{\lim}(Ca)\\ n/a & r>r_{\lim}(Ca)\end{cases}$$

$$A(r)=10^{\sum_{i=0}^{3}A_i(\log r)^i},\ B(r)=\sum_{i=0}^{3}B_i(\log r)^i$$

i	A_i	B_i
0	1,522	0,7215
1	0,077	-0,1356
2	0,1090	-0,0037
3	0,0332	0,0035

Oil & water relative permeabilities, k_{ro} & k_{rw}, readily computed (Valavanides, 2012):

$$k_{rw}(Ca,r)=(\tilde{\gamma}_{ow}Ca)\left[\tilde{k}(-\partial\tilde{p}/\partial\tilde{z})\right]^{-1}=\frac{1}{x(Ca,r)}$$

$$k_{ro}(Ca,r)=\kappa r k_{rw}(Ca,r)$$

$$(\kappa=\tilde{\mu}_o/\tilde{\mu}_w)$$

Implementation & Integration Scheme

- Treat **2-ph flow** problem as **equivalent 1-ph (saturated) flow** problem.
- **Virtual fluid** local effective mobility = sum of local oil & water mobilities
- FEM algorithm solves equivalent 1-ph (saturated) flow problem (potential)
- [Darcy + continuity + transport equation for the water/oil (DeProF)]
- Apparent density and viscosity = saturation weighted averages of fluid properties
- Effective hydraulic conductivity depends on local flow conditions.
- Effective (equivalent 1-ph) superficial velocity=sum of oil and water velocities.
- Local values of *Ca* & *r* are readily computed and the local value of the effective mobility is estimated as the sum of the local individual mobility of oil & water.
- Mobilities (or, equivalently, the relative permeabilities), and the reduced pressure gradient, are looked-up from the *DeProF* relative permeability map for the corresponding (*Ca*, *r* values.
- Implementation standard 1-ph Darcy velocity *vs* pressure gradient relation for equivalent phase delivers the new effective superficial velocity (equivalent 1-ph flow).
- Velocity decomposition into local oil & water superficial velocities (value flowrate ratio).
- Procedure repeated along effective flow streamlines (coincide actual 2-ph streamlines).

Integration Scheme Basic Analysis (1)

- Key variables:

oil/water flowrate ratio: $r = \frac{\tilde{q}_o}{\tilde{q}_w} = \frac{\tilde{U}_o}{\tilde{U}_w}$

Capillary number: $Ca = \frac{\tilde{\mu}_w \tilde{U}_w}{\tilde{\gamma}_{ow}}$

Viscosity ratio: $\kappa = \frac{\tilde{\mu}_o}{\tilde{\mu}_w}$

Saturation ratio: $\frac{S_o}{S_w} = \frac{\tilde{\mu}_o}{\tilde{\mu}_w} \frac{\tilde{U}_o}{\tilde{U}_w} = \kappa\, r$

Integration Scheme Basic Analysis (2)

- The non-linear problem of 2-ph flow considers
  - mass conservation of a mixture of two phases
  - Effective Darcy velocity,
  - arithmetic mean density, and
  - harmonic mean viscosity,
  - phases (species) transport: convection and diffusion (omitted here)

Mass conservation:

$$\frac{\partial \varepsilon \rho}{\partial t} + \nabla \cdot \rho \mathbf{u} = 0$$

Phase transport:

$$\frac{\partial \varepsilon s_o \rho_o}{\partial t} + \nabla \cdot s_o \rho_o \mathbf{u} = \nabla \cdot D_o \nabla \left(s_o \rho_o \right)$$

Darcy mean velocity:

$$\mathbf{u} = -\frac{\mathbf{k}}{\mu} \nabla p$$

Efective 1-ph properties:

$$\rho = s_o \rho_o + s_w \rho_w, \quad \frac{1}{\mu} = s_w \frac{k_{rw}}{\mu_w} + s_o \frac{k_{ro}}{\mu_o}$$

Integration Scheme Basic Analysis (3)

The Darcy problem is non-linear since the velocity is dependent on the relative permeabilities, which are also functions of velocity:

$$\mathbf{u}_{w/o} = -\frac{k_{\mathrm{rw/ro}}\mathbf{k}}{\mu}\nabla p$$

$$\mathbf{u} = -\frac{\mathbf{k}}{\mu}\nabla p = -\frac{\mathbf{k}}{\mu}\frac{\tilde{\gamma}_{ow}Ca}{\mathbf{k}}\,x\left(Ca, r\right) = u_w\left[-A(r)\left(10^6 Ca\right)^{-B(r)}\right]$$

Velocity can be rewritten as a function non-dependent of itself, which can be solved with direct solvers:

$$\mathbf{u} = \left(\frac{-2\mathbf{k}\nabla p}{A(r)}\right)^{\frac{1}{1+B(r)}}\frac{S_w}{\kappa}\left(\frac{\mu_w 10^6}{\gamma_{ow}}\right)^{\frac{B(r)}{1+B(r)}} \qquad \kappa = \frac{\mu_o}{\mu_w}$$

Solution is obtained with typical FEM methods, after mesh independence analysis.

Indicative Applications

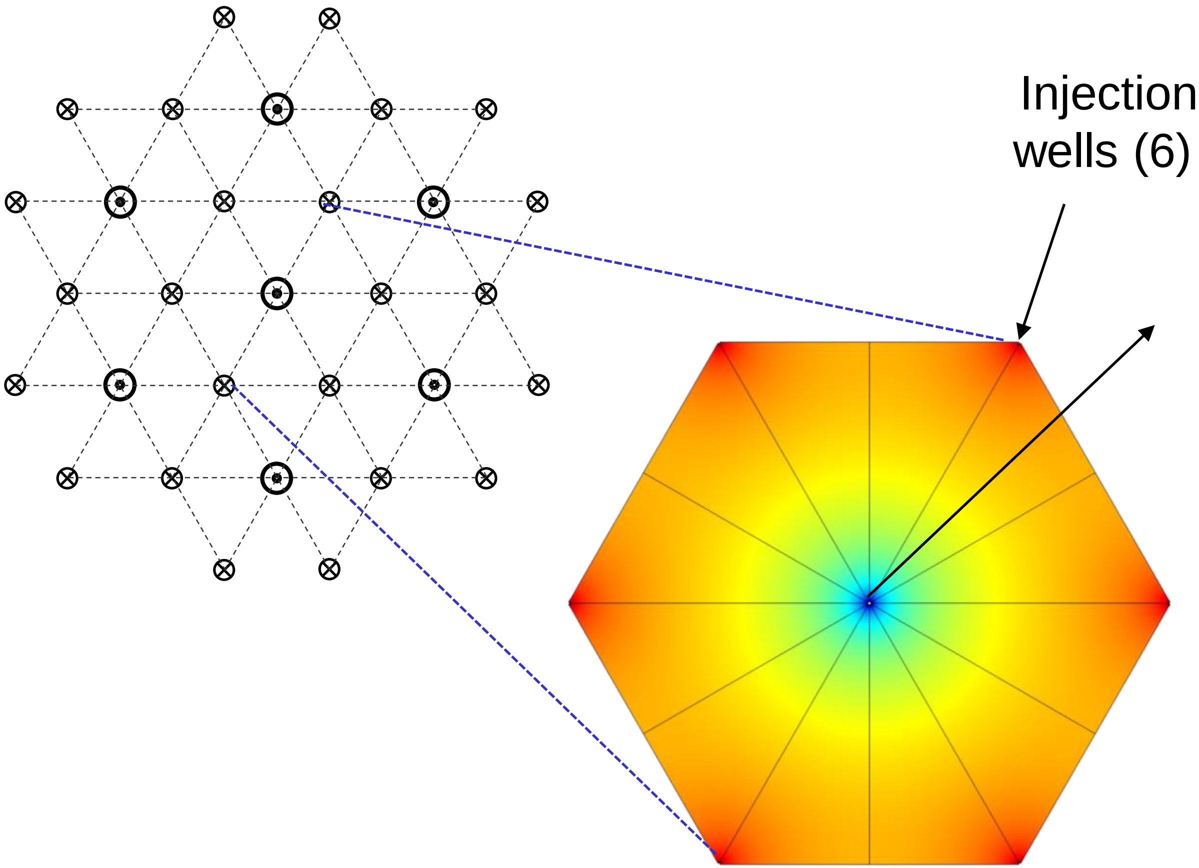
Integration scheme within the **COMSOL^{TM} Earth Science** module has been applied

- to a variety of injection/production patterns
- solitary wells
- direct and staggered line drives
- 5-, **7-** and 9-**spot water flooding arrangements*** (**exhibit 1**)
- **annular sleeve water drives** (**exhibit 2**)

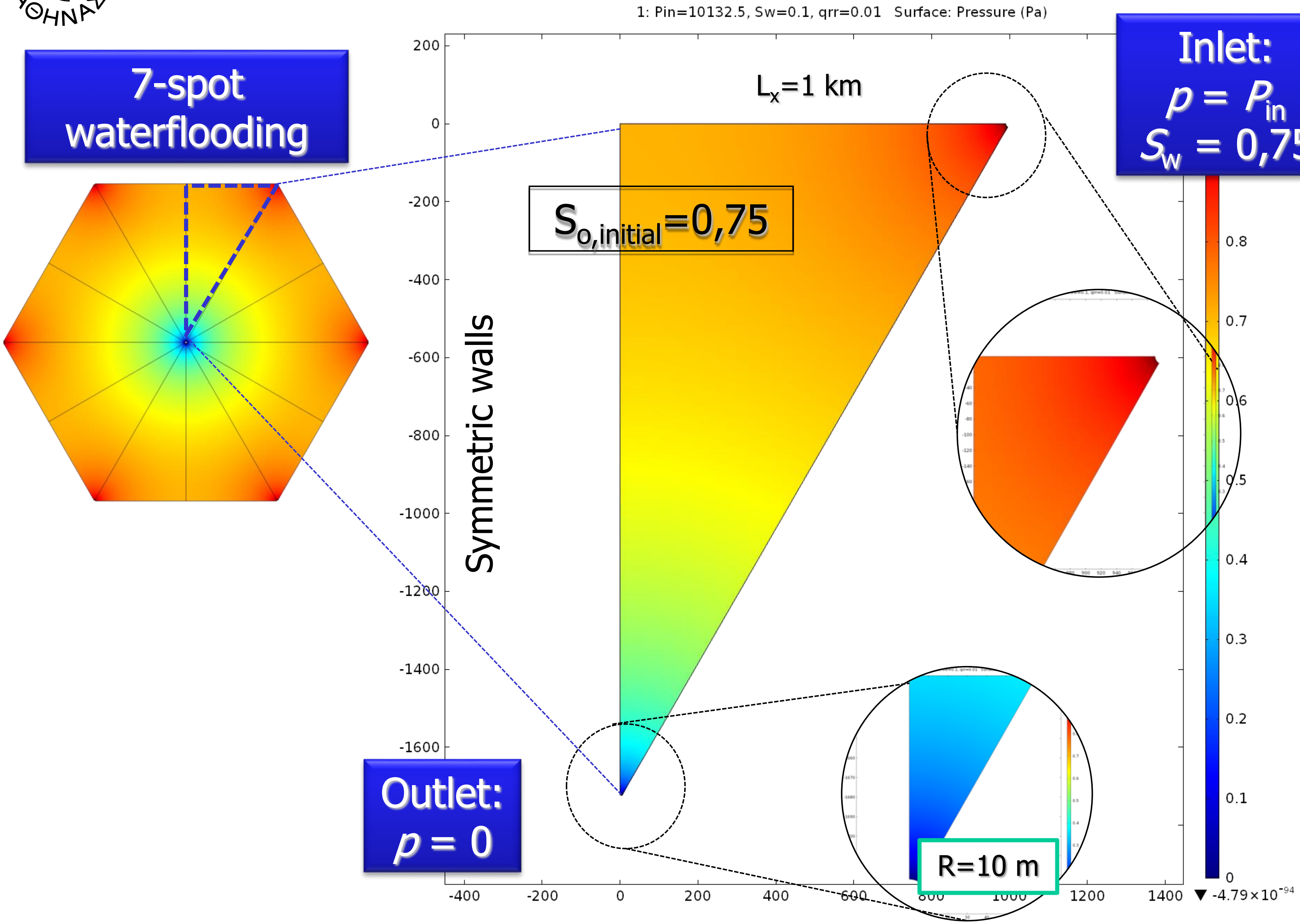
for different orientations including gravity effects

*Comparisons between direct prediction of the energy utilization index (process efficiency) [*Valavanides & Skouras, Fresenius Environmental Bulletin 23(11) 2014*] and FEM/DeProF simulations, are made.

Exhibit 1
7-spot arrangement
Waterflooding a confined oil reservoir



Oil-Water pressure profile



7-spot waterflooding

L_x=1 km

$S_{o,initial}$=0,75

Symmetric walls

Inlet:
$p = P_{in}$
S_w = 0,75

Outlet:
p = 0

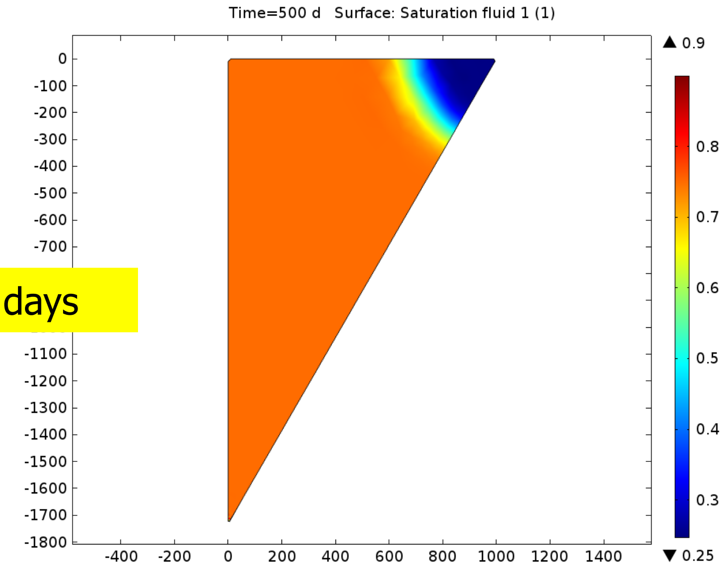
R=10 m

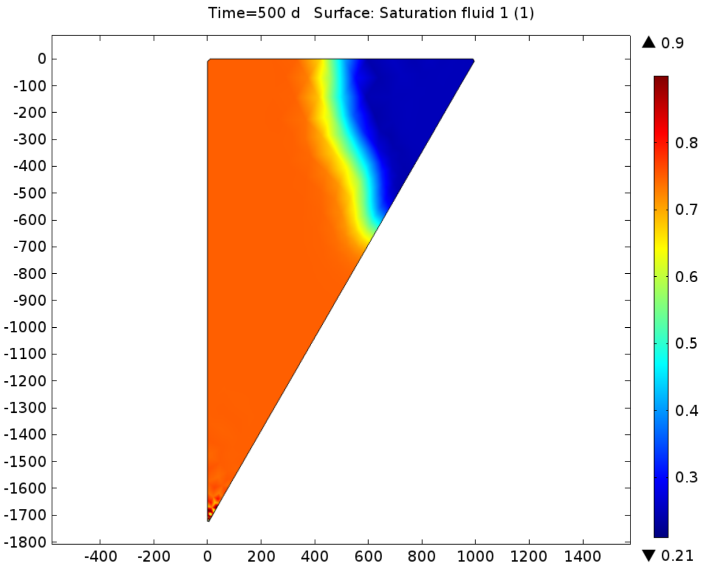
Oil saturation profiles

P_{in}=0.1atm

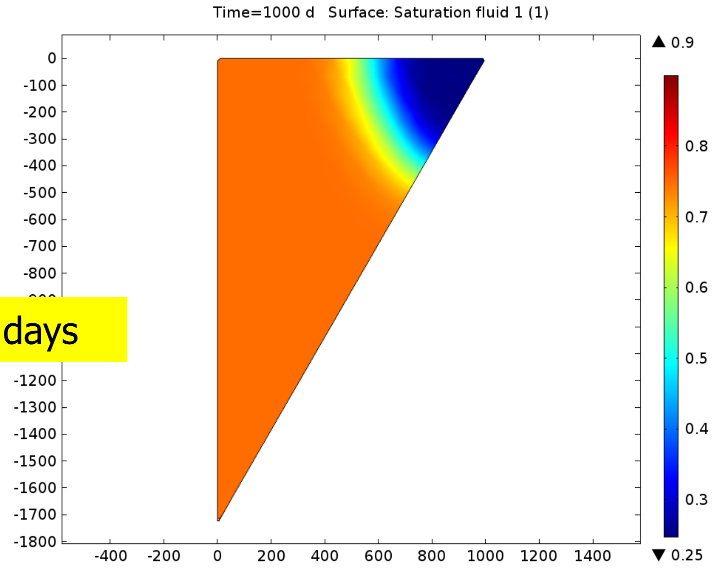
P_{in}=1atm

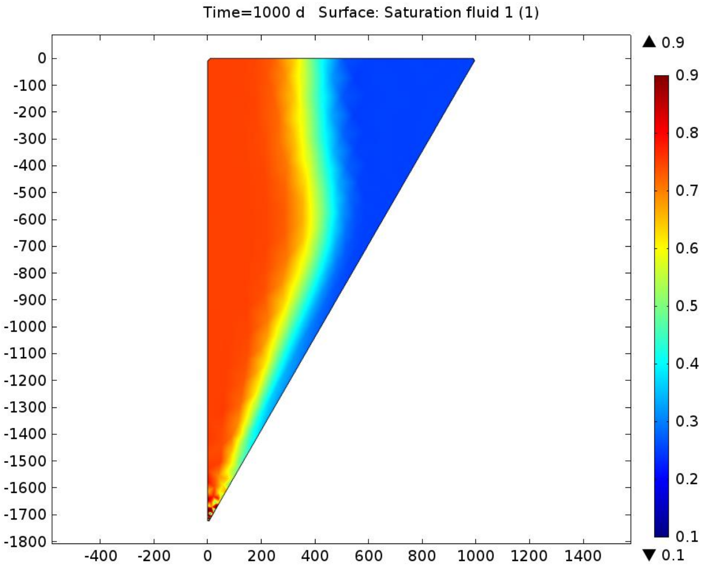
t = 500 days





t = 1000 days





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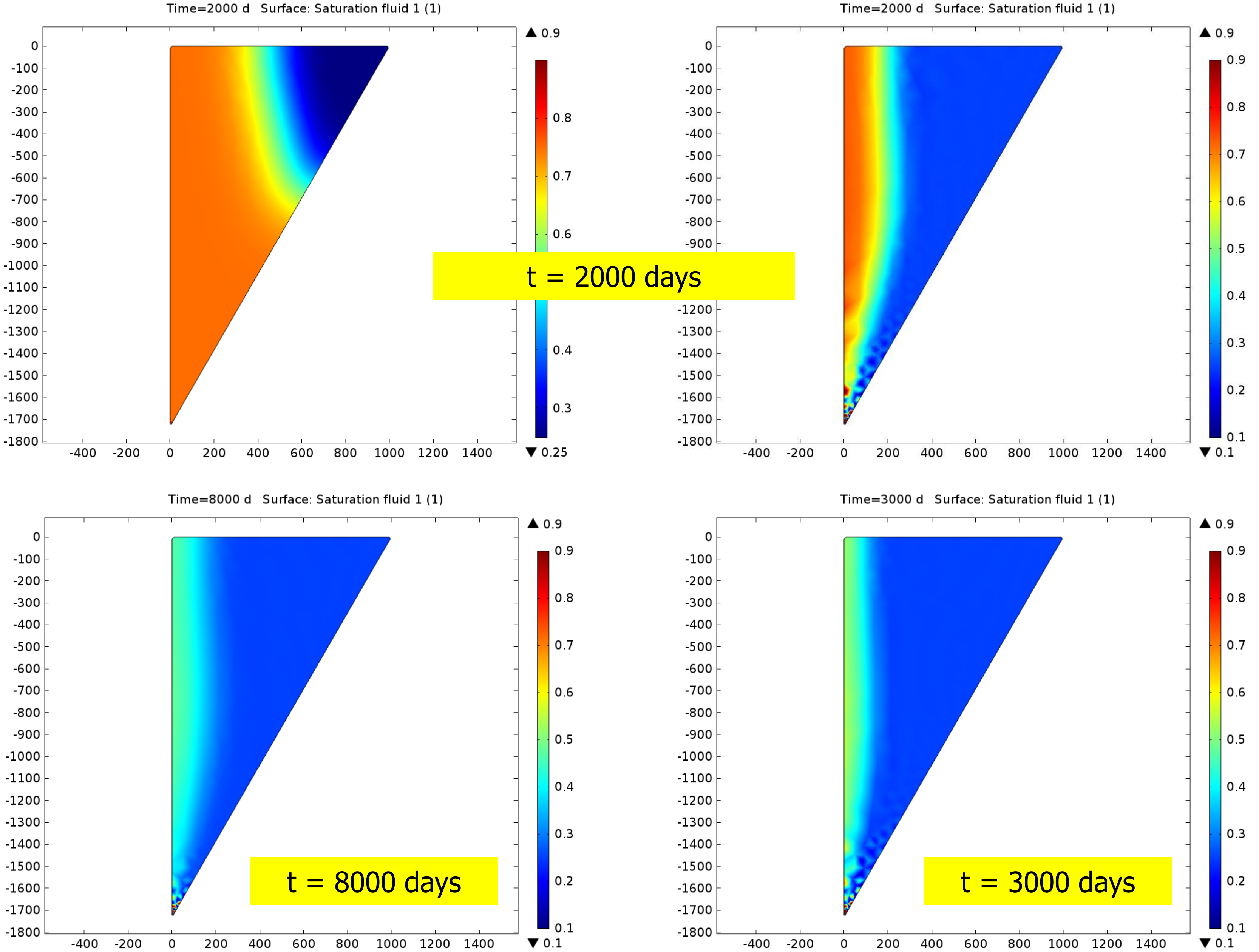
Evolution of saturation profiles

P_{in}=0.1atm

P_{in}=1atm

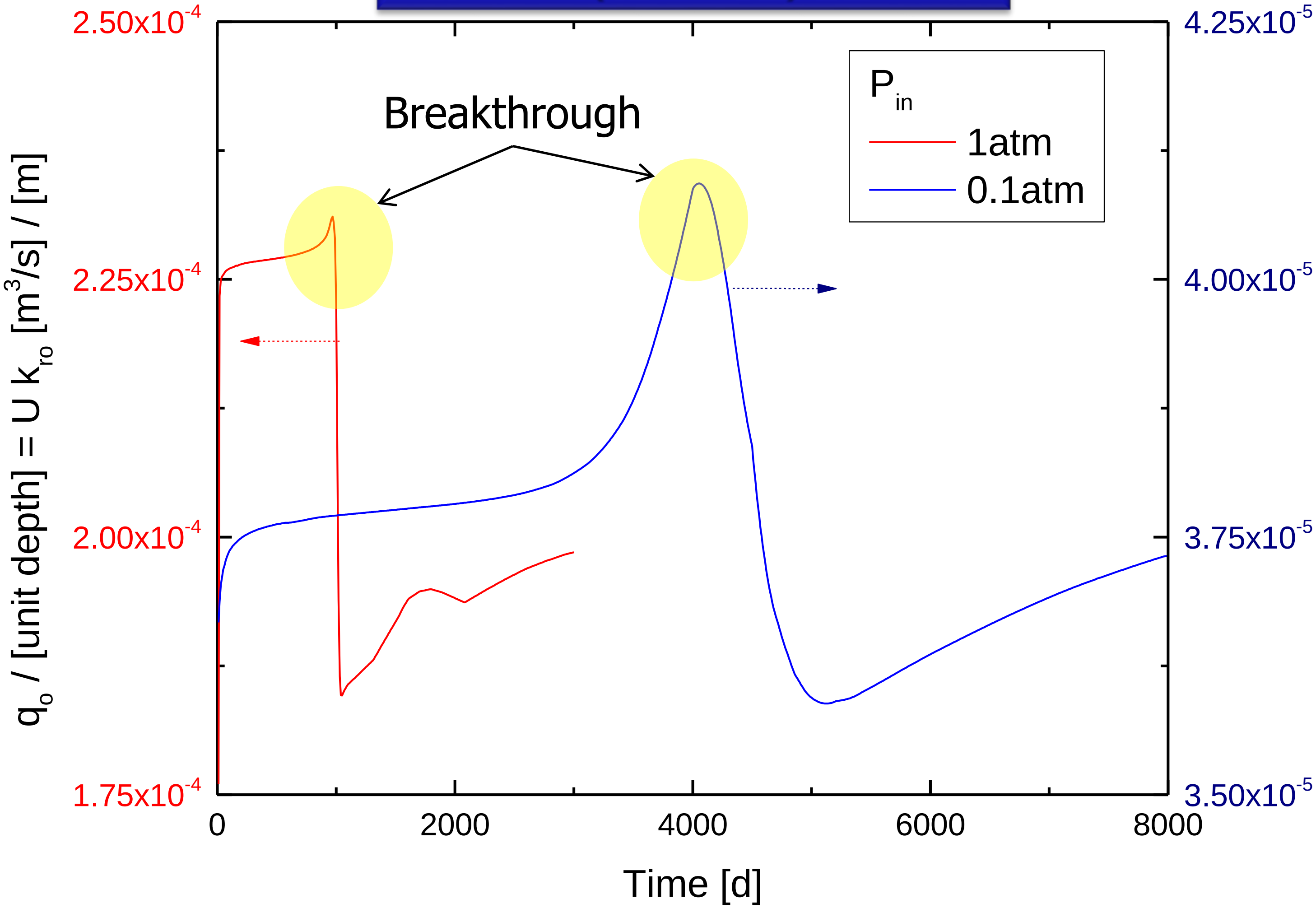


t = 2000 days

t = 8000 days

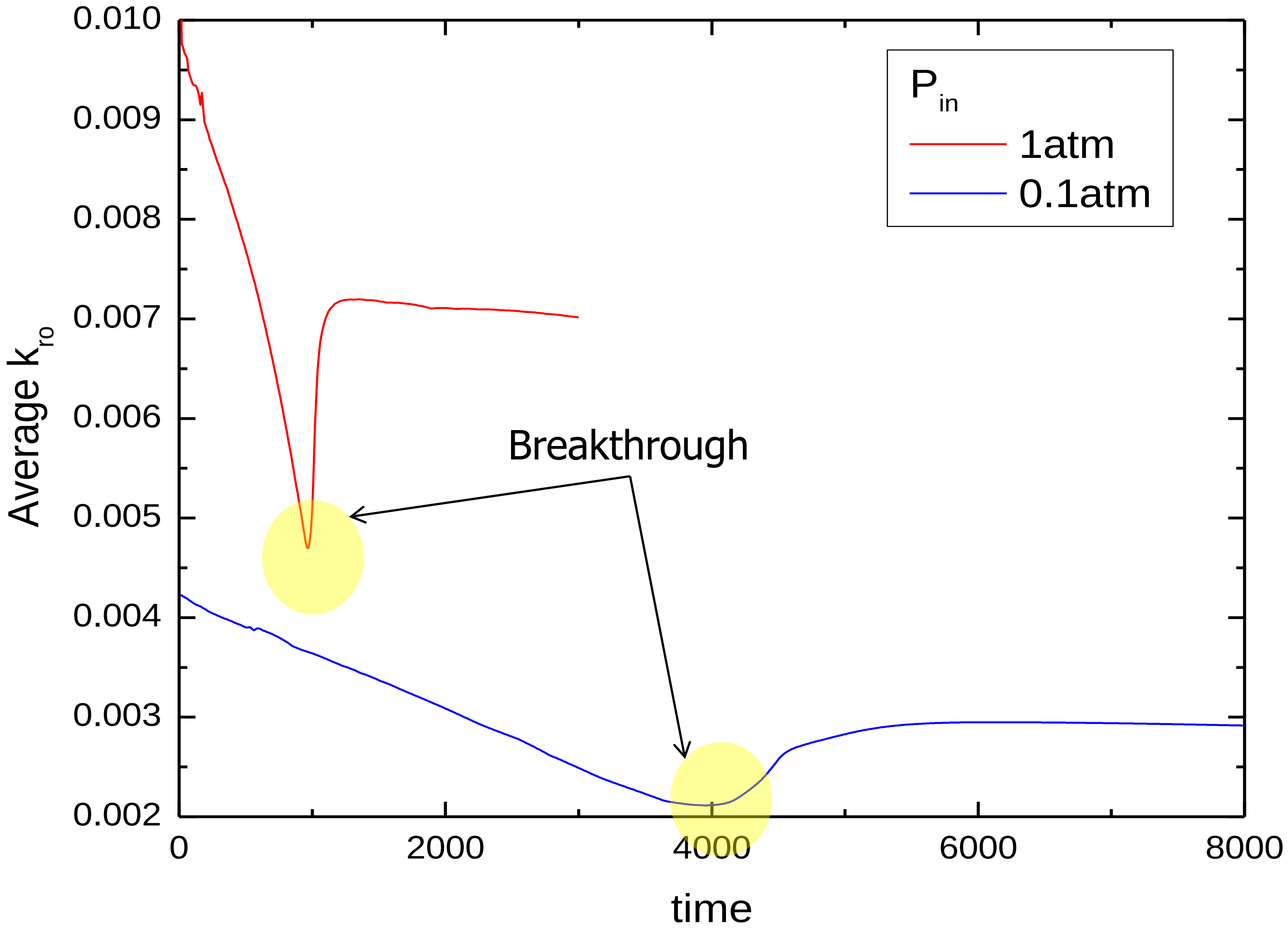
t = 3000 days

Oil extraction rate (at outlet)



Cumulative oil extraction at 3000 days:
- P_{in}=0,1 atm: 9832.32 m^3/m
- P_{in}=1,0 atm: 53066.88 m^3/m

Volume-averaged oil relative permeability, k_{ro}



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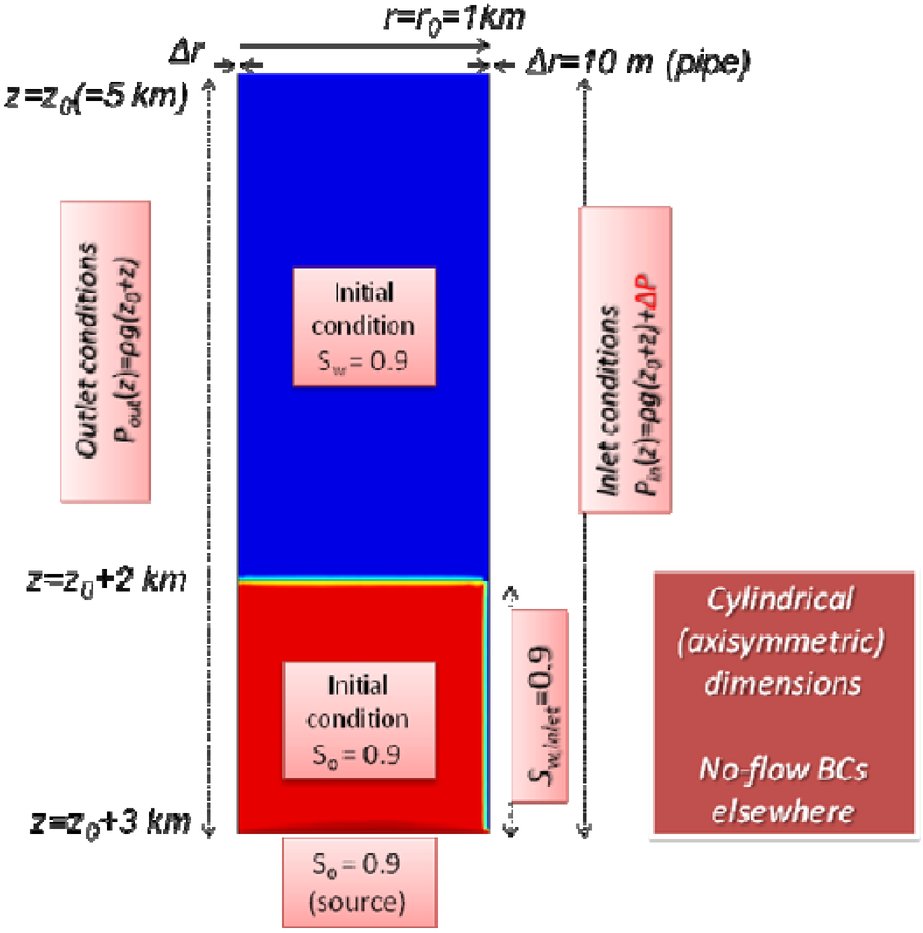
Application 2
Bottom, annular sleeve, water-drive arrangement

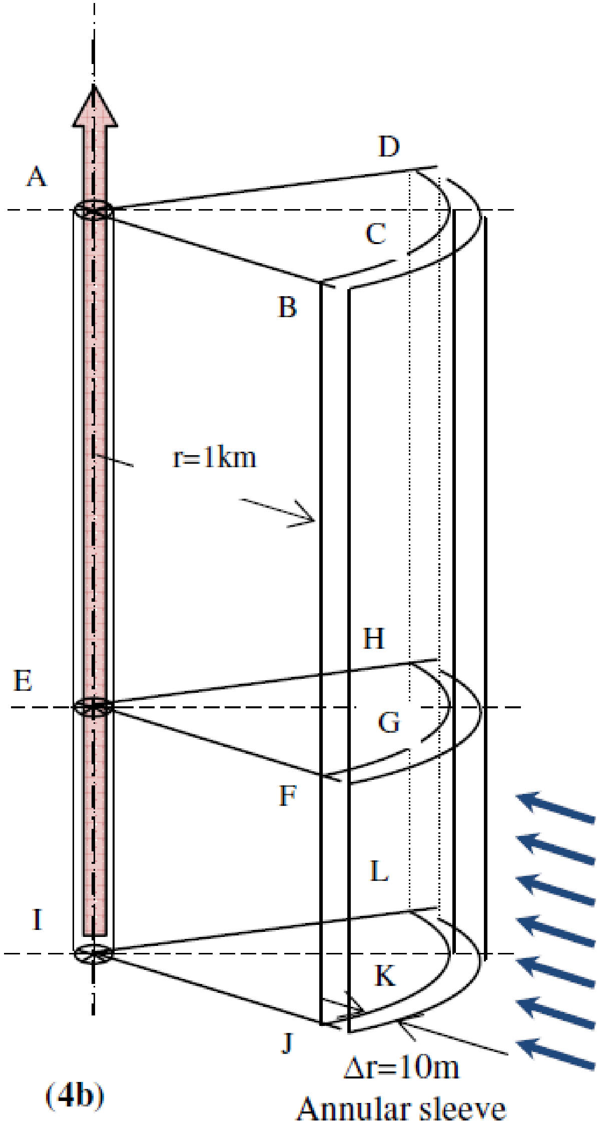
Depth well: Setup



Cylindrical (axisymmetric) dimensions

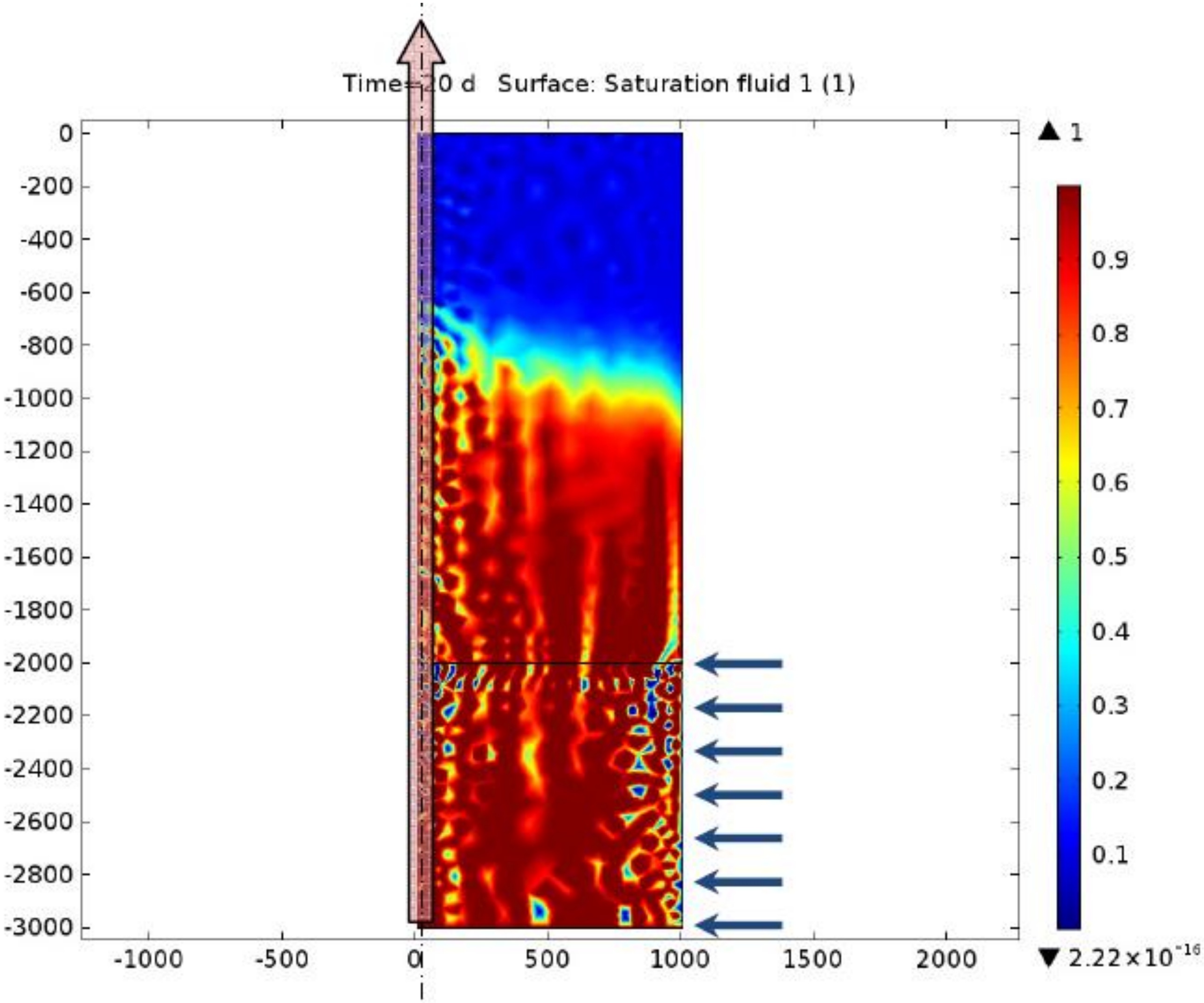
No-flow BCs elsewhere

(4a)



(4b)

Exhibit 2
Bottom, annular sleeve, water-drive arrangement



Conclusions

- Functional forms of 2-ph flow in p.m. true-to-mechanism relative permeability maps have been used to simulate waterflooding arrangements.

- The essential characteristic of **relative permeability dependence on the local flow conditions** have been provided by the **DeProF model**.

- Local flow conditions pertain to superficial velocities of oil and water or, equivalently, capillary number and flowrate ratio (true-to-mechanism).

- Flow-dependent relative permeability maps have been integrated in the **COMSOL^{TM} Earth Science** module.

- **Actual 2-ph flow** problem → **equivalent “effective-phase” (1-ph) flow** problem.

- Stable, converging integration scheme, numerical instabilities are only localized in areas where flow concentration takes extremely high values.

- Development of efficient & more reliable simulators incorporating the actual physics of two-phase flow in porous media processes.

Acknowledgements

ImproDeProF project →
http:/users.teiath.gr/marval/ArchIII_en.html

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ImproDeProF project → http:/users.teiath.gr/marval/ArchIII_en.html

(...just google → "ImproDeProF")