Effect of Length and Porosity on the Acoustic Performance of Concentric Tube Resonators

David Neihguk*[1], and Abhinav Prasad[1]
[1]Mahindra Research Valley, Mahindra & Mahindra Ltd.
*Corresponding author: Mahindra Research Valley, Mahindra World City, Anjur P.O, Chennai – 603 004, neihguk.david@mahindra.com

Abstract: The acoustic performance of acoustically short Concentric Tube Resonator (CTR) is analyzed in terms of the length to diameter (L/D) ratio and the porosity. A three dimensional model is built using the acoustic module of COMSOL Multiphysics. The accuracy of the predicted Transmission Loss (TL) result is validated with experimental results reported in the literature. Finally, an empirical formula is given to estimate the optimum porosity for a given L/D ratio to achieve wide band TL performance.

Keywords: Concentric tube resonator, porosity, optimum porosity, acoustically short, wide band Transmission Loss, back pressure.

1. Introduction

In normal vehicle development program, design of engine, powertrain, exhaust muffler and vehicle body happens simultaneously. This approach provides sufficient flexibility to work out tradeoffs strategy to address various conflicting requirements of exhaust system performance. However, sometimes due to a strategic goal of common product platform, new mufflers have to be designed for a production vehicle. In such situations, the freedom for installation space is inherently limited which often leads to either a compromise on performance and cost or both. This is because the muffler or CTR must fit into the available installation space by conforming its profile according to the available package space. In general, the installation space is bounded by the ground clearance, ramp over angle, departure angle, BIW floor and propeller shaft in case of four wheel drive and all-wheel drive vehicles.

The common practice to meet NVH targets under tight package space is to have mufflers and resonators cascaded in series or in parallel along the exhaust system. In many cases, the permissible muffler or CTR dimensions are such that the higher order modes will propagate un-attenuated [1]. As a result, the Transmission Loss (TL) performance predicted by one dimensional acoustic analysis which does not take into account the effect of higher order modes will not be adequate to initiate the design of an acoustically short efficient CTRs. The criteria for efficient design of acoustically long extended CTR have been presented by Ramya and Munjal [2].

Therefore, it is desirable to have some guideline to assess the acoustic performance of acoustically short CTRs from the knowledge of the dimensions and porosity before a full three dimensional acoustic analysis is initiated. The acoustic module of COMSOL Multiphysics is used without mean flow to perform parametric studies based on TL in a set of CTR involving higher order mode propagation.

2. Theory

It is well known that the TL performance of CTR is a strong function of the porosity (σ) and length (L) [1-5] as shown in Figure 1. Moreover, the TL characteristics also depends on whether the resonator is short or long. For short resonators, the primary resonance frequency f_r is less than the first axial mode frequency f_1 of the cavity ($f_1 = c/2L$) where c is the speed of sound. The resonator is considered to be long if $f_r > f_1$ [5].

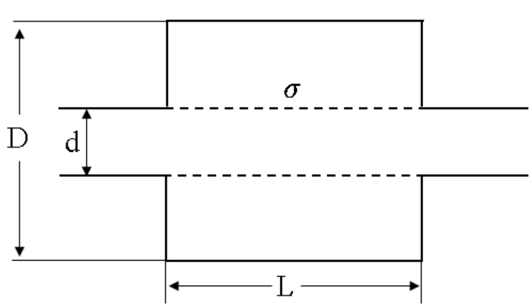

Figure 1. Schematic sketch of a CTR.

The primary resonance frequency f_r and the first axial mode frequency f_1 can be made to merge by changing the non-dimensional parameter k_0L, where k_0 is the wave number

($k_0 = 2\pi f/c$) and L is the length of the resonator [3].

3. Governing Equations

By using the acoustic pressure as the independent variable, the wave propagation in the resonator is solved in the frequency domain using the time harmonic pressure acoustic mode in COMSOL. The governing equation is the modified version of the 3-D Helmholtz equation:

$$\nabla.\left(-\frac{\nabla p}{\rho}\right) - \frac{\omega^2 p}{\rho c^2} = 0 \qquad (1)$$

where ρ, c and ω are the density, speed of sound and angular frequency, respectively. The following boundary conditions are used to simulate the system:

1. $p = 1$ Pa is the standing wave pressure at entrance to the inlet pipe.

2. Plane wave radiation at the outlet pipe, this boundary condition gives a vanishing reflection coefficient for normally incident waves.

The Transmission Loss (TL) is expressed as the difference between the incident acoustic power at the inlet, W_i using the applied pressure, p and the transmitted acoustic power at the outlet, W_o using the computed pressure, p_c

$$TL = 10\log_{10}\left(\frac{W_i}{W_o}\right)\ dB \qquad (2)$$

where $W_i = \oint \frac{p^2}{2\rho c}$ and $W_o = \oint \frac{{p_c}^2}{2\rho c}$

The acoustic impedance of the inner perforated tube is modeled as a user input interior impedance boundary. The empirical formula suggested by Sullivan and Crocker is used as given in Equation (3).

$$\varsigma_p = [0.006 + jk_0(t + 0.75d_h)]/\sigma \qquad (3)$$

where, ς_p, k_0, t, d_h, σ are the non-dimensional impedance, wave number, tube thickness, perforated hole diameter and porosity, respectively.

4. Numerical Model

The models are created in COMSOL by using the geometry builder tools as shown in Figure 2. Free tetrahedral mesh with maximum mesh size of one-sixth the minimum wavelength ($\lambda_{min}/6$) is used as recommended in COMSOL user guide as shown in Figure 3. Fluid medium of air at a temperature of 294 K is used from the inbuilt materials library in COMSOL.

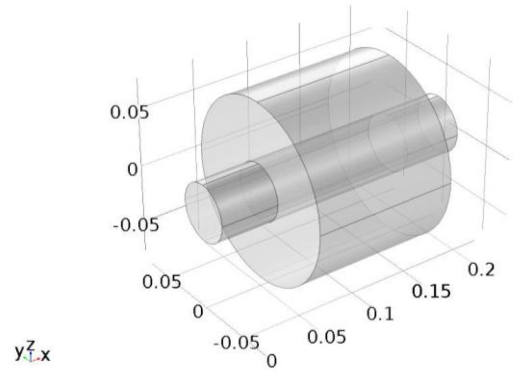

Figure 2. COMSOL model of CTR.

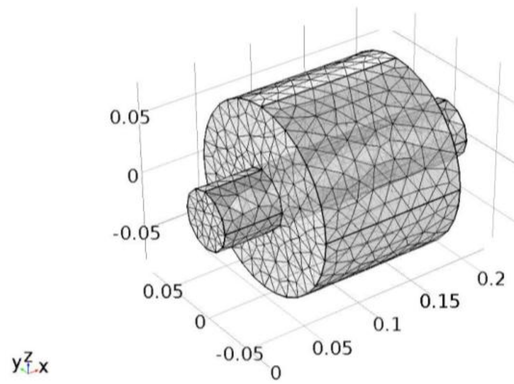

Figure 3. COMSOL model of meshed CTR.

5. Validation

The COMSOL model is validated with the experimental results from Ref. [6] as shown in Figure 4.

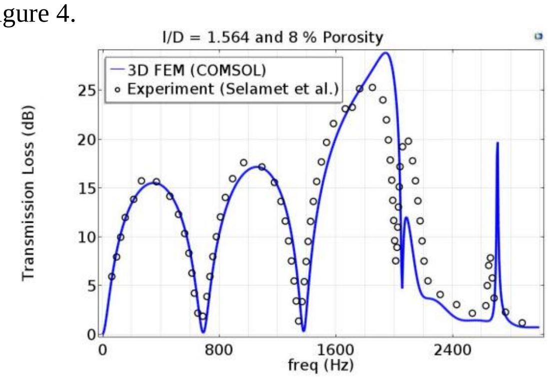

Figure 4. TL of CTR validated with experiment

The dimensions of the CTR that was used for validating the COMSOL model are length L=25.72 cm, uniform perforated duct and annular chamber of diameters d=4.9 cm and D=16.44 cm, respectively, perforate hole diameter d_h=0.249 cm, thickness of perforated tube t=0.09 cm and porosity of 8 %. Air as medium at temperature of 305.54 K. The TL results in Figure 4 amply validates the COMSOL model.

6. Results and Discussion

The acoustic performance in terms of the Transmission Loss (TL) and the back pressure performance in Pa is presented in the following sections.

6.1 Transmission Loss (TL)

The Simple Expansion Chamber (SEC) with geometries ranging from L/D of 0.205 to 3.525 adopted from Ref. [6] are converted to CTR by the addition of a perforated inner tube. The other dimensions include uniform perforated duct and outer chamber diameters are 4.859 cm (d) and 15.318 cm (D) respectively.

It is observed that the L/D ratio of 0.205 behave as short resonator for all possible porosities and the L/D ratio of 0.612 behave as short resonator for porosities of 6% and below. It is also observed that as the L/D ratio increases, the optimum porosity decreases as shown in Table 1. Consequently, L/D greater than 0.8 and above are not included in this study.

Table 1: L/D and optimum porosity σ_o for wide band TL performance

Model	L/D	σ_o (%)	Frequency bandwidth of TL above 20 dB (Hz)
1	0.450	27	1198
2	0.500	13	1125
3	0.612	6	930
4	0.700	4	820
5	0.800	3	667

At low porosities, the primary and secondary resonance peaks are clearly distinct and wide apart as seen in Figures 5-9. The primary and secondary peaks tends to merge as the porosity increases and ultimately merge at a particular porosity resulting into a wide band TL that has a practical application only for the short resonator due to the absence of the repeating domes. The coupling of the primary and secondary peak frequencies occur at relatively high porosities for small L/D ratio as compared to larger L/D ratio within the short resonator regime. Moreover, the coupling become very sensitive to porosity as the L/D ratio increases as shown in Figures 8 and 9.

It can be seen from Figures 5 to 9 that for a given L/D ratio, there exist an optimum porosity that provides wide band TL attenuation above 20dB in the low frequency range provided the porosity is manufacturable. It may also be noted that the optimum porosity decreases as the L/D ratio increases. The general expression for the optimum porosity for a given L/D ratio is given by Equation (4).

$$\sigma = 0.03739(L/D)^{-8.075} + 3.288 \quad (4)$$

Therefore, beyond a certain L/D (0.800 in this case), the optimum porosity predicted by Equation (4) is not practical. For such L/D and higher, the criteria proposed by Ramya and Munjal [2] may be used.

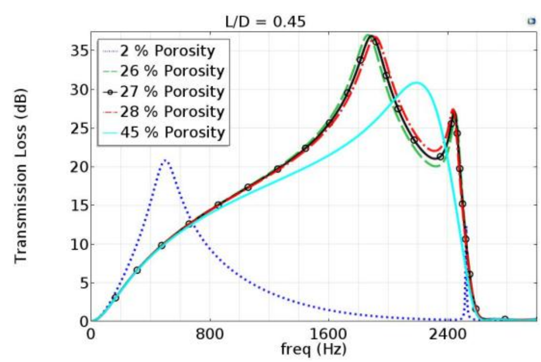

Figure 5. TL of CTR with $L/D = 0.45$

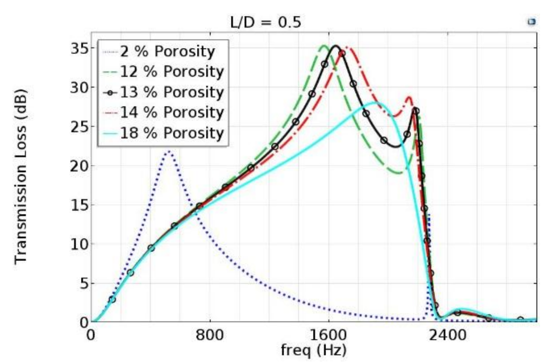

Figure 6. TL of CTR with $L/D = 0.5$

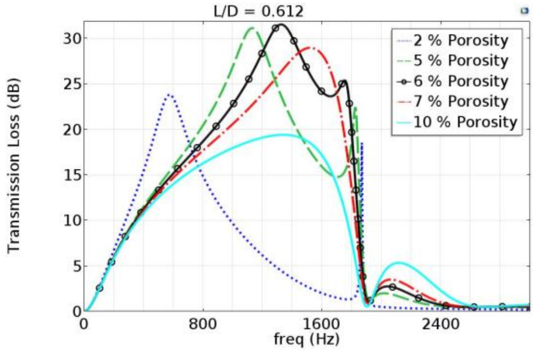

Figure 7. TL of CTR with $L/D = 0.612$

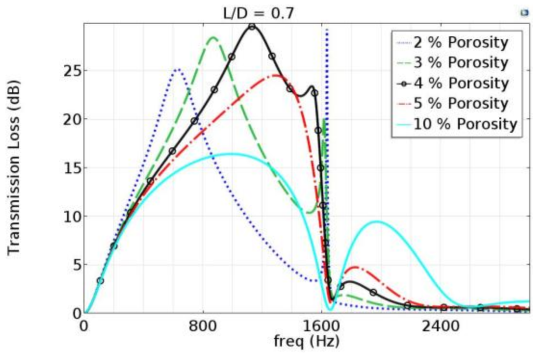

Figure 8. TL of CTR with $L/D = 0.7$

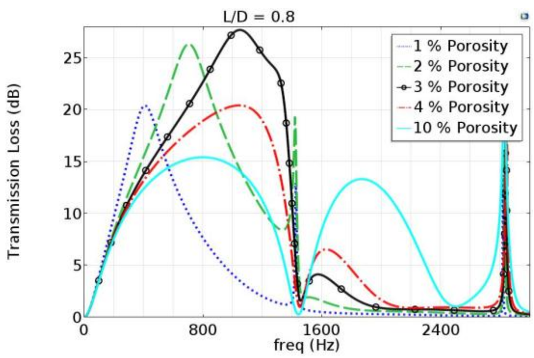

Figure 9. TL of CTR with $L/D = 0.8$

6.2 Back Pressure

The back pressure of the CTRs given in Table 1 are predicted in terms of the friction factor (f) for perforated ducts [7] which is given as

$$f = 0.0304 + 0.15\sigma \quad (5)$$

For a density ρ, of 1.19 Kg/m^3 and at an average velocity v, of 5 m/s, the back pressure of the CTRs is calculated from equation (6) and the results are given in Table 2.

$$\Delta p = f \frac{L}{d} \frac{\rho v^2}{2} \quad (6)$$

Table 2: Back pressure of acoustically short CTRs with optimum porosity

Model	L/d	σ_o (%)	f	Δp (Pa)
1	1.419	27	0.0709	1.496
2	1.576	13	0.0499	1.170
3	1.929	6	0.0394	1.131
4	2.207	4	0.0364	1.195
5	2.522	3	0.0349	1.309

7. Conclusions

The effect of length on the acoustic performance of SEC is widely known. However, the effect of length and porosity on the acoustic performance of CTR for short resonators is not quantitatively reported in the literature. The results presented in this study shed some light on the optimum porosity for a given L/D ratio for short resonators. An empirical formula is presented to estimate the optimum porosity as a function of the L/D ratio. Thus, efficient, light weight and cost effective resonators can be designed to suite stringent package space in automotive intake and exhaust systems.

8. References

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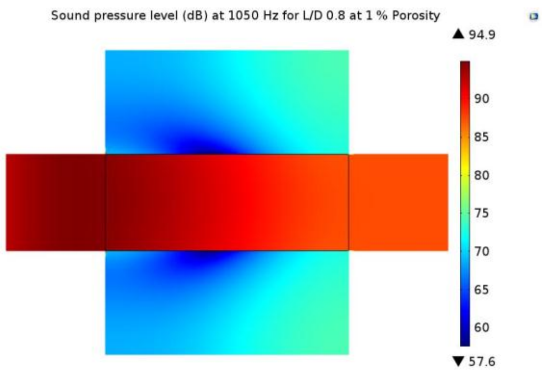
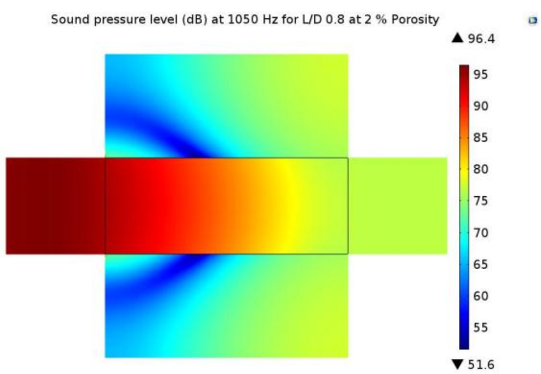
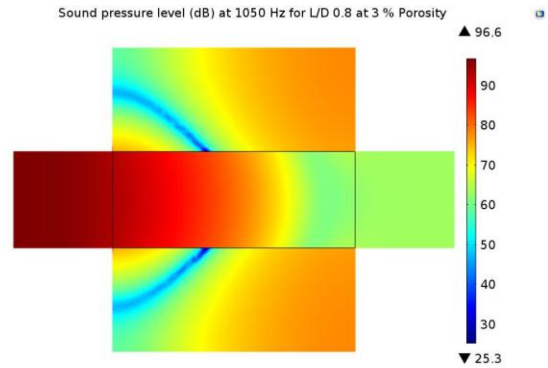
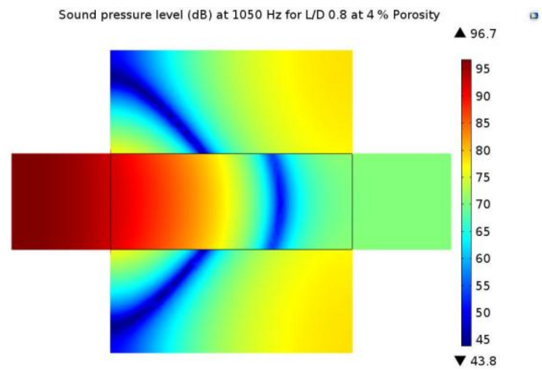
9. Appendix

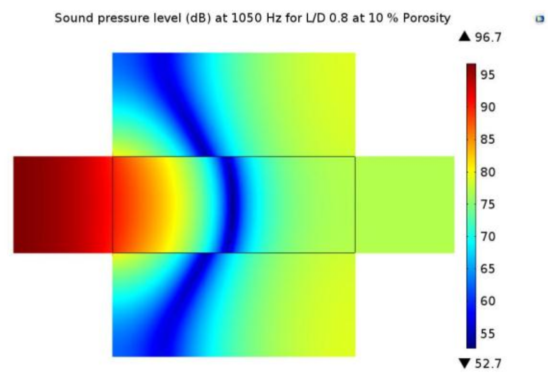
The Sound Pressure Level (SPL) and the total acoustic pressure contour (Pa) is given for a CTR with L/D ratio of 0.8. The pattern of the figures agrees well with the TL results of Figure 9.

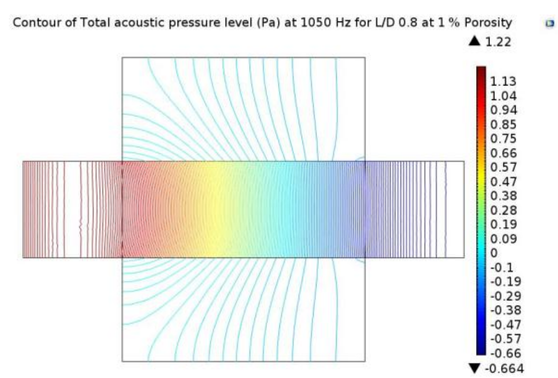


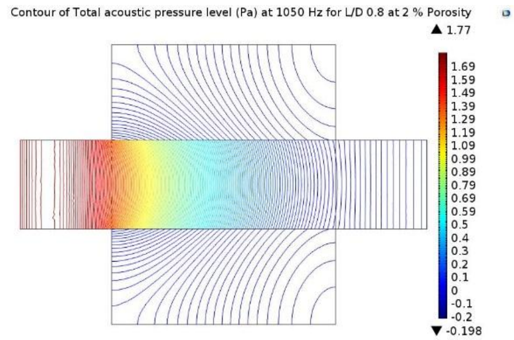


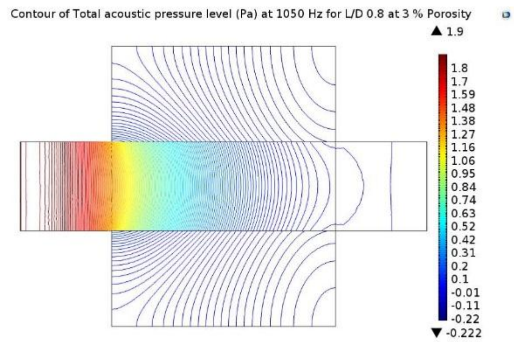


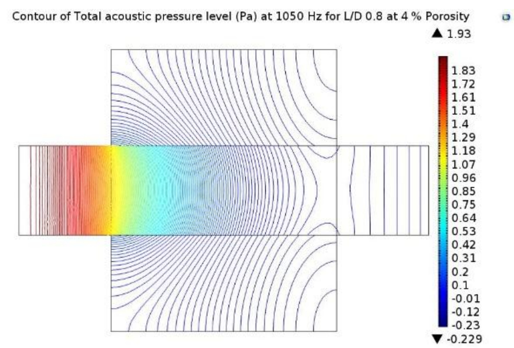


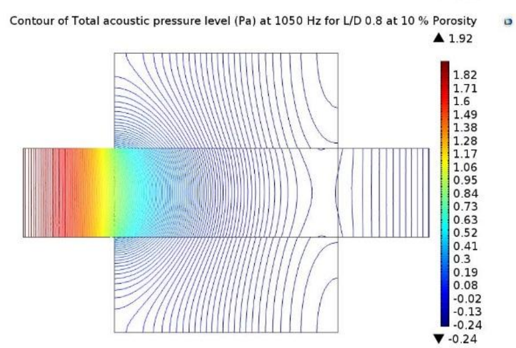





Contour of Total acoustic pressure level (Pa) at 1050 Hz for L/D 0.8 at 2 % Porosity
▲ 1.77
1.69
1.59
1.49
1.39
1.29
1.19
1.09
0.99
0.89
0.79
0.69
0.59
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
▼ -0.198


Contour of Total acoustic pressure level (Pa) at 1050 Hz for L/D 0.8 at 3 % Porosity
▲ 1.9
1.8
1.7
1.59
1.48
1.38
1.27
1.16
1.06
0.95
0.84
0.74
0.63
0.52
0.42
0.31
0.2
0.1
-0.01
-0.11
-0.22
▼ -0.222


Contour of Total acoustic pressure level (Pa) at 1050 Hz for L/D 0.8 at 4 % Porosity
▲ 1.93
1.83
1.72
1.61
1.51
1.4
1.29
1.18
1.07
0.96
0.85
0.75
0.64
0.53
0.42
0.31
0.2
0.1
-0.01
-0.12
-0.23
▼ -0.229


Contour of Total acoustic pressure level (Pa) at 1050 Hz for L/D 0.8 at 10 % Porosity
▲ 1.92
1.82
1.71
1.6
1.49
1.38
1.28
1.17
1.06
0.95
0.84
0.73
0.63
0.52
0.41
0.3
0.19
0.08
-0.02
-0.13
-0.24
▼ -0.24