Introduction: The acoustic performance of an acoustically short Concentric Tube Resonator (CTR) is investigated to find an optimum porosity for a given length to diameter ratio. The study is motivated by the following practical difficulties encountered in automotive exhaust noise control.
1. Increasing trend of downsized engine and increase in power to weight ratio of engine for better fuel economy.
2. The need to reduce system weight (vehicle weight) for improving mileage.
3. Emergence of compact cars leads to reduction in package space and requires compact mufflers and CTRs for intake and exhaust noise control.
4. Limitations of 1D tools to analyze compact acoustic mufflers and CTRs.

![Figure 1. Schematic of a CTR](image)

Computational Methods: The 3D Helmholtz equation is solved by using the acoustic module of COMSOL Multiphysics with a unit pressure inlet and plane wave radiation with anechoic termination outlet.

\[ \nabla \left( -\frac{\nabla p}{\rho} \right) - \frac{\omega^2 p}{\rho c^2} = 0 \]

The Transmission Loss (TL) is computed as the logarithm of the incident acoustic energy \( W_i \) to the transmitted acoustic energy \( W_t \) as

\[ TL = 10 \log_{10} \left( \frac{W_i}{W_o} \right) \text{ dB} \]

Acoustic impedance \( \zeta_p = [0.006 + jk_0(t + 0.75d_h)]/\sigma \) is used as a boundary condition for the perforated pipe.

![Figure 2. COMSOL model of a CTR](image)

Results: The COMSOL model is first validated with experimental results of Selamet et al. [6]. The following observations are made:
1. Increase in L/D leads to decrease in optimum porosity.
2. For a given L/D, there exist an optimum porosity that provides wide band TL above 20 dB in the low frequency range.
3. The optimum porosity for a given L/D is given by

\[ \sigma = 0.03739(L/D)^{-0.075} + 3.288 \]

![Figure 3. Verification](image)

![Figure 4. L/D 0.45](image)

![Figure 5. L/D 0.612](image)

![Figure 6. L/D 0.8](image)

![Figure 7. Contour of acoustic pressure](image)

<table>
<thead>
<tr>
<th>L/D</th>
<th>( \sigma )</th>
<th>( \Delta p \text{ TL}&gt;20 \text{ dB} )</th>
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Table 1. Optimum porosity and back pressure

Conclusions: The effect of length and porosity on the acoustic performance of acoustically short CTR is quantitatively discussed. An empirical formula is presented to estimate the optimum porosity as a function of the L/D ratio. The back pressure is predicted using the friction factor for perforated ducts.

References: