Finite Element Modeling of Dispersion and Thermal Noise in Whispering-Gallery Mode Cavities

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Abstract

Whispering-gallery mode (WGM) resonators are promising elements for future photonic devices, as they combine ultra-high quality factor with small size and mode volume. Many device schemes, such as electro-optic modulators [4, 7], receivers [6] and optoelectronic generators [8] and optical combs [2], has been suggested. They also found applications in science like optomechanics [3] and quantum cooling [10]. All these applications either need high frequency stability of the modes or reliable knowledge of the noise level and origin. The dispersion properties of such cavities are also of great importance for frequency combs. This work is dedicated to exact numerical modeling of modal dispersion and thermal noise in arbitrary cylindrical symmetric WGM structure.

The modal dispersion depends of the frequency on the azimuthal number. It can be easily calculated with standard Electromagnetic Waves, Frequency Domain user interface in cylindrically symmetric geometry, sweeping the azimuthal mode number. The post-processing (input data conversion, taking derivatives over sweep parameter and data preparation for the second step) is done with COMSOL Multiphysics® Application Java operators. Having mode field distribution got, noise estimations can be done following the Fluctuation-Dissipation theorem [5]. According to it we should use this field (some expression of it actually) as a harmonic pump for thermal (temperature-related noises) or mechanical (displacement-related noises) problem and the noise spectral density is then proportional to the energy loss during one pump period.

Unfortunately, standard COMSOL Heat Transfer Module are not able to do frequency domain studies and presume the solutions to have zero azimuthal number only, so Coefficient Form PDE user interface was used to calculate thermorefractive noise. Solid Mechanics Module was also found inappropriate for Brownian noise computation being written in assumption of zero torsional displacements and zero azimuthal numbers. So a suitable interface was made with Physics Builder to reflect the features of mechanical modes in cylindrical geometry.

The COMSOL Application consists of four tabs. First allows for quick geometry and material change and is made for launching the first step (mode and dispersion calculation). It also allows a user-defined material with custom material dispersion. Second tab allows to choose the mode for noise calculation and input noise determining parameters. The others are for dispersion diagrams and spectral density results.

We show that the real form of the resonator makes small influence on the thermorefractive noise spectrum and old infinite-space formula is quite universal (Fig. 1 left).

The Brownian part is also in a good agreement with the experiment (Fig. 1 right). Not all the resonances have correct height, which can be due to loss angle difference for different mode families.

An efficient and precise method of dispersion and noise calculation is developed and verified using experimental data. The COMSOL Application based on the method allows for more automation then simple COMSOL model with easy ways of modifying initial parameters.

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Figures used in the abstract

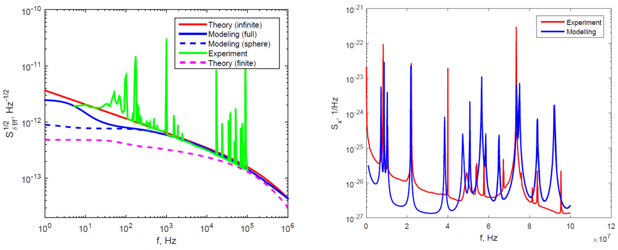

Figure 1: Left: comparison of theory [1], modeling (with and without environment), experiment [1] and spherical decomposition formula [1, formula A15]. Right: comparison of modelling and experiment [9] for micro-toroid.