Analysis of Sound Propagation in Lined Ducts by means of a Finite Element Model

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Abstract: The present paper describes the results of a Finite Element Model used to analize sound propagation in lined ducts. By means of a numerical model it was possible to predict the insertion loss inside rectangular lined ducts in a frequency range from 250 Hz to 4000 Hz. The model was validated by a comparison with experimental data obtained in accordance to ISO 11691 and ISO 7235 standards. The sound absorbing material for the lining was mineral wool, whose thickness was varied from 25 mm to 150 mm. Comparisons showed a good accordance between experimental and numerical data, suggesting that FEM can be an accurate and inexpensive way to predict sound attenuation in lined ducts.

Keywords: acoustics, sound propagation, lined ducts, insertion loss.

1. Introduction

Increasingly strict standards regulating sound emissions in outdoor environments and the demand for indoor noise abatement have led to a wide use of lined ducts with the aim of reducing the transmission of sound along the ducts.

Lined ducts (Fig.1) are a kind of dissipative mufflers obtained mounting along two or more sides of a duct a sound-absorbing material confined in with sound-transparent layers; the lining material is usually mounted inside of the duct or in external symmetrical side pockets.

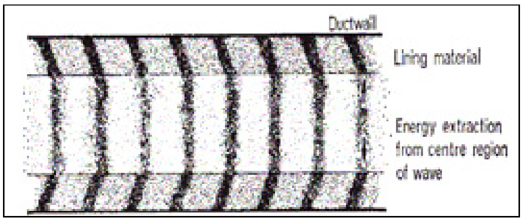

Figure 1. Dissipation of energy at ends of plane waves in a lined duct [1].

Applications range from those of a more traditional kind, as seen in air conditioning plants, to sectors such as the control of noise emitted by the induction and exhaust ducts of boilers or power plants.

Mineral wool is the material normally used for the lining because of its good sound-absorbing efficiency, with an acoustic behaviour typical of porous materials [2]. The absorption mechanism is based on viscous effects in the air contained inside the interconnecting pores which separate the fibers; the solid structure can generally be regarded as ideally rigid and stationary, and the acoustic absorption as produced by the viscosity of the air, which is subjected to a process of alternate compression and rarefaction as a sound wave passes through it [3, 4].

Using an experimental apparatus to predict sound attenuation in lined ducts usually requires a certain amount of time and money. Ingard method [5], Cremer method [6] and Kurze method [7] were therefore used in order to theoretically evaluate sound attenuation and to avoid expensive and time consuming experiments, but comparisons with experimental data revealed that predictions were not so accurate as design requires. Then, propagation of sound along rigid ducts lined with sound absorbing material was modelized (Fig. 2) by means of a commercial FEM software (COMSOL Multiphysics).

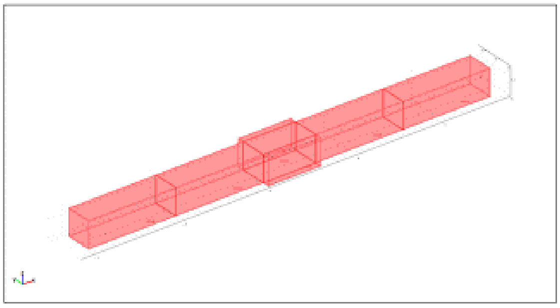

Figure 2. Geometric model for a lining thickness of 50 mm.

Mineral wool was assumed as sound absorbing material and its acoustical impedance was calculated by means of the Delany-Bazley correlation [8].

To validate the FEM model accuracy, the numerical results were compared with data measured by using an experimental set-up based on standards ISO 11691 and ISO 7235; the apparatus was used to measure the insertion loss in straight ducts with square cross section lined with mineral wool.

The Finite Element Model was finally utilized to analyze the effect of the density of the lining material on the sound propagation inside lined ducts.

2. Governing Equations

In a lossless medium sound waves are governed by the following equation:

$$\frac{1}{\rho_0 c_s^2}\frac{\partial^2 p}{\partial t^2}+\nabla\cdot\left(-\frac{1}{\rho_0}(\nabla p-q)\right)=Q \qquad (1)$$

where p is the acoustic pressure (whose SI unit is Pa), ρ_0 (kg/m^3) is the density, c_s (m/s) is the speed of sound and q (N/m^3) and Q (1/s^2) are respectively the dipole source and the monopole source, both optional.

In the frequency domain, the above wave equation corresponds to the equation:

$$\nabla\cdot\left(-\frac{1}{\rho_0}(\nabla p-q)\right)-\frac{\omega^2}{\rho_0 c_s^2}p=Q \qquad (2)$$

which is an inhomogeneous Helmholtz equation, where p=p(x,ω). This formulation makes possible to compute frequency response using the parametric solver to sweep over a frequency range using a harmonic load, specifying the angular frequency ω through the frequency f.

Must be noted that for spaces as rectangular lined ducts, a plane wave only would propagate (with all higher modes, even if present, being cut-off) if a frequency is small enough so that:

$$f<\frac{c_s}{2h}$$

where h is the larger of the two transverse dimensions of the rectangular duct [9]. In the geometry of the problem discussed in this paper, this condition is sastisfied for frequencies up to about 600 Hz.

3. Experiments

3.1 Experimental set-up

The experimental apparatus, compliant with the standards above mentioned, was used to assess the insertion loss in straight ducts with a square cross section (300x300 mm).

The apparatus comprised the following elements (Fig. 3): a loudspeaker, test ducts, a muffler or a substitution duct, a transition element and a reverberation room [10].

The loudspeaker is fitted inside a duct 1,70 m long, made of 20 mm thick chipboard panels, and is enclosed in a sealed cabinet filled with mineral wool.

The ducts and the transition element were made up by 1 mm thick galvanised steel sheets. The length of the test duct must be at least half of the wavelength corresponding to the minimum frequency in the test range and not less than four times the duct cross-section diagonal.

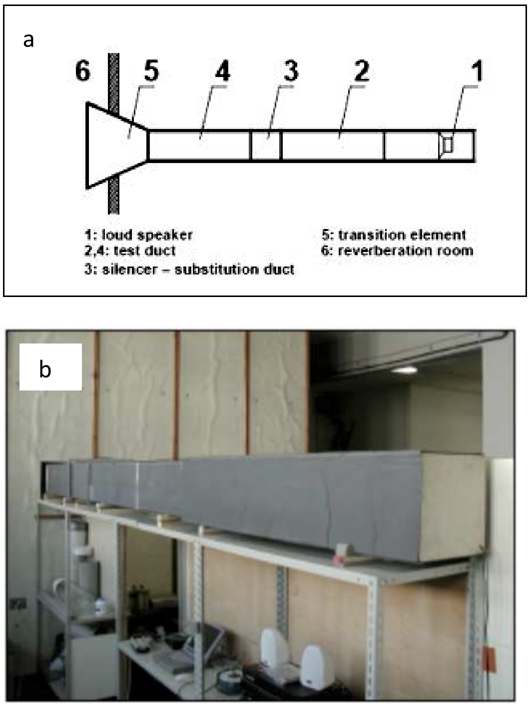

Figure 3. Experimental setup: a) sketch; b) picture.

The substitution duct, which was designed and built in the same way as the test ducts, was used in the measurement procedure to define a reference condition.

The transition element, which enables the apparatus to be connected to the reverberation room, must have a low reflection coefficient and dissipative losses close to zero; moreover, it must not produce any endogenous noise that might influence measurements in the test room. The transition element constructed for the present apparatus is shaped like a truncated pyramid with an internal angle of 30° between the walls, terminating in the reverberation room.

In order to limit flanking transmissions as far as possible, particular steps were taken. With regard to transmission paths through solid media, rubber washers were inserted between flanges; to limit transmission through air, the entire test section was enclosed in sound-proofing panels made of expanded polyurethane with a 1 mm thick lead sheath inserted in between.

The muffler consists in a duct equipped with two replaceable symmetrical side pockets of various thickness (Fig. 4). Sound transparent layers were used for confining the lining materials: this way, it is possible to vary both the thickness of the lining and the absorbing material, and therefore to analyze the effect that thickness and absorbing material have on insertion loss, that is, on the effectiveness of the dissipative muffler.

The sound source consisted of a random noise generator, an amplifier and a loudspeaker; the power of the sound produced must be sufficient to ensure that the sound pressure level in the reverberation room is at least 10 dB greater than the background noise.

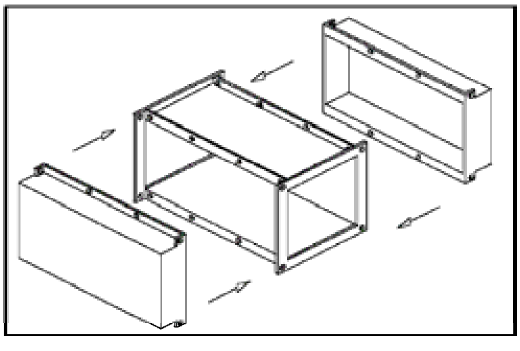

Figure 4. Substitution duct with replaceable symmetrical side pockets.

3.2 Experimental procedure

A pink noise was generated at the closed end of the channel by means of a broad-band loudspeaker; then the generated signal was picked up by a microphone positioned in three different points inside the reverberation room, according to ISO standards.

By means of a sound level meter and its data analysis software, the signal was elaborated to determine the spectrum of the sound pressure level inside the reverberation room, which constituted the reference environment for the measurements.

To determine the spectral insertion loss, IL, provided by the muffler under examination, measurements were taken by alternatively mounting on the experimental set-up either the muffler itself or the substitution duct. The difference, in decibels, between the mean sound pressure level inside the reverberation room before (SPL_1) and after (SPL_2) the insertion of the dissipative muffler along the test duct returned insertion loss values:

$$IL = SPL_1 - SPL_2 \qquad (3)$$

3.3 Experimental results

Insertion loss was determined for four different lined ducts characterised by a thickness t of the absorbing lining equal to 25, 50, 100 and 150 mm. Insertion loss was determined for the elements described before when lined with bulk mineral wool (density 50 kg/m^3) and was measured in the range 250-4000 Hz, for one-third octave band.

In the frequency range examined, for lining thicknesses greater than 100 mm, the results obtained showed a substantial independance from the thickness of the lining. On the contrary, for lower lining thickness, a selective behaviour was observed, with a peak in the insertion loss close to the frequency of 1000 Hz.

These results were used to validate the numerical model.

4. Use of COMSOL Multiphysics

4.1 Modelling

To numerically solve Eq. (2), and to determine the sound attenuation inside the lined duct the *Time-harmonic analysis* model type has been used in Comsol Multiphysics.

The elements are of the *Lagrange-quadratic* type, and boundary and subdomain conditions were set as follows:

- the *sound-hard boundary* condition has been used to model rigid surfaces;
- the *sound-soft boundary* condition has been used as an appropriate approximation for the ending section of the duct;
- the *radiation* condition has been used in the entrance section of the duct to specify the acoustic pressure at the boundary.

To model the sound damping by means of the rock-wool set inside the simmetrical side pockets of the simulated lined duct, the *Delany-Bazley model* for porous media [8] has been used: in this way, we have that the speed of sound and the density are frequency dependant complex quantities, denoted by ρ_c and c_c respectively:

$$\rho_c = \frac{Z_c k_c}{\omega} \qquad c_c = \frac{\omega}{k_c}$$

The Helmholtz equation becomes:

$$\nabla \cdot \left(-\frac{1}{\rho_c} (\nabla p - q) \right) - \frac{\omega^2}{\rho_c c_c^2} p = Q \qquad (2')$$

and, according to the model of Delany-Bazley, we have:

$$k_c = \frac{\omega}{c_s} \left[1 + C_1 \left(\frac{\rho_0 f}{R_f} \right)^{-C_2} - iC_3 \left(\frac{\rho_0 f}{R_f} \right)^{-C_4} \right] \qquad (4)$$

$$Z_c = \rho_0 c_s \left[1 + C_5 \left(\frac{\rho_0 f}{R_f} \right)^{-C_6} - iC_7 \left(\frac{\rho_0 f}{R_f} \right)^{-C_8} \right] \qquad (5)$$

here, the flow resistivity R_f is calculated as a function of apparent density ρ_{rw} and mean fiber diameter d_{rw} of rock-wool [11]:

$$R_f = \frac{\rho_{rw}^{1.53} K}{d_{rw}^2} \qquad (6)$$

In Eq. (4) and (5) C_i are coefficients specific for the material used; in Eq. (5) K is an empirical coefficient equal to $3.18*10^{-9}$ [Ns/m^4].

The mesh used for the simulations is a *brick mesh* with cuboid shaped elements, and the number of *Lagrangian* elements per wavelenght has been kept between six to ten in order to obtain the desired accuracy [12] and to validate the model.

The hardware configuration used to run the simulations was a Macintosh computer with a 2.16 GHz dual-core Intel processor and 4 GB of RAM.

The model has been at first solved using one of the most commonly employed algorithms for matrices arising from finite element formulations for the Helmholtz equation [13]; such solvers are commonly used with preconditioners that intend to improve the overall conditioning of the system matrix to obtain a good performance of the solver itself. The chosen solver has been the Generalized Minimal RESidual algorithm for solving nonsymmetric linear systems (GMRES) [14]; the preconditioner used in the simulations has been the Geometric Multigrid one, which allows to accelerate the convergence of the base iterative method by correcting, from time to time, the solution globally by solving a coarse problem.

This way it has been possible to obtain correct results for frequencies less or equal to 2000 Hz; from then on, the software couldn’t give any results giving errors due to lack of RAM, or continued endlessly in the iteratig steps to solve the problem without coming to converge.

The problem was completely resolved with the PARDISO solver, available with the 3.5 version of the software, which made possible to analize frequencies up to 4000 Hz in a very faster way than using traditional solvers.

As an example, figure 5 shows the calculated pressure distribution for a lining thickness of 50 mm and a frequency of 400 Hz: comparison of figure 5c with figure 1 suggests that the model correctly simulates the behaviour of a lined duct with dissipation of energy at ends of plane waves.

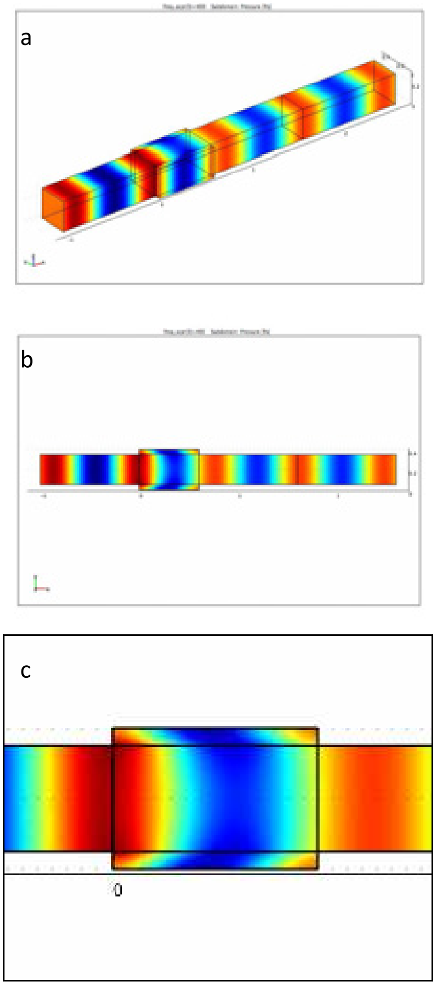

Figure 5. Calculated pressure distribuition: a) isometric view; b) x-y plane; c) x-y plane detail.

4.2 Model validation

To validate the accuracy of the numerical model the experimental data and the simulated IL curves were compared along the whole frequency range.

Fig. 6 reports the comparison for each lining thickness. Figure 6 shows a good accordance between experimental and simulated data. For lining thickness of 25 and 50 mm the Finite Element Model exactly identifies the frequency of maximum insertion loss and predicts IL on the whole range with a mean absolute error of 1.0 dB.

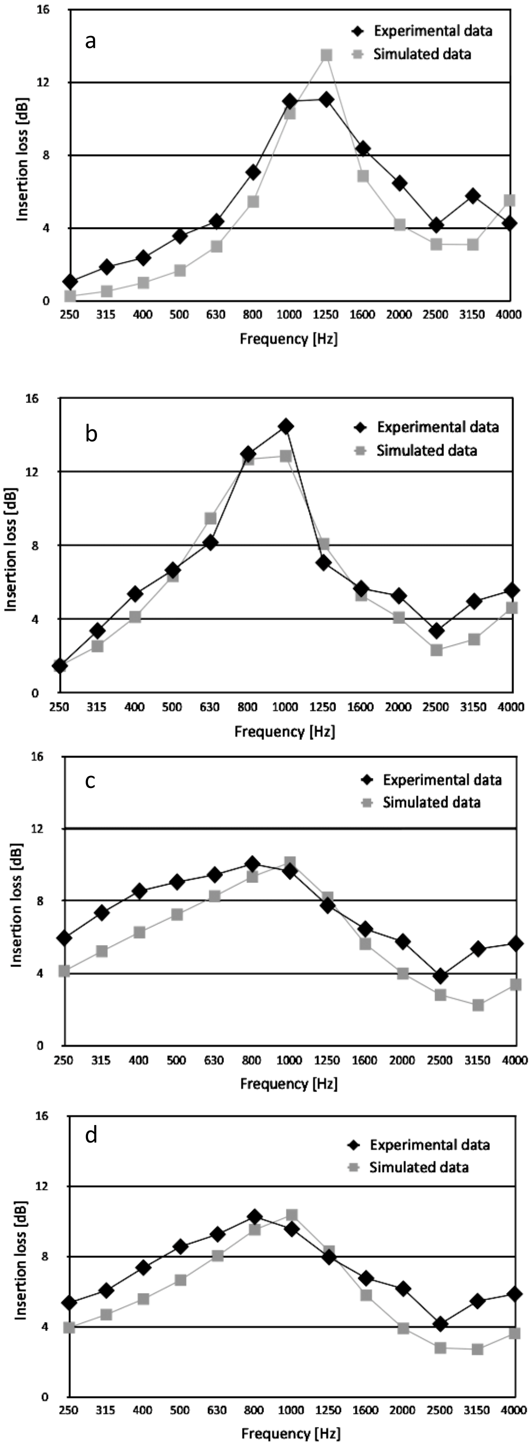

Figure 6. Comparison between experimental and simulated data for different thickness: a) 25 mm, b) 50 mm , c) 100 mm and d) 150 mm.

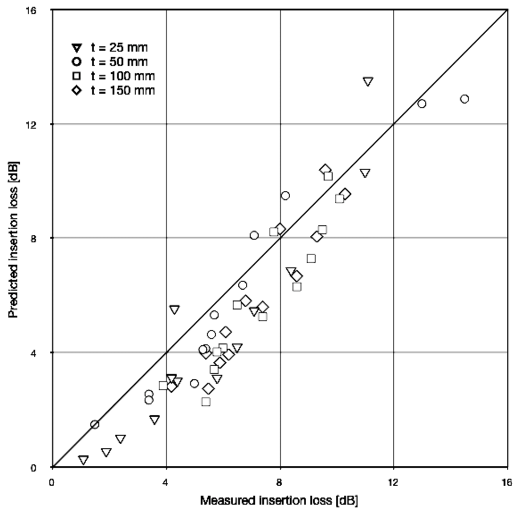

Figure 7. Comparison between measured and predicted insertion loss for the different lining thicknesses.

Again, for the thicknesses of 100 and 150 mm, it can be properly predicted that the insertion loss is almost thickness-independent.

Figure 7 presents comparison between measured and predicted insertion loss for the various lining thicknesses. The mean absolute error for all thicknesses results equal to 1.4 dB.

4.3 Effects of the density of the sound absorbing material

After the validation of the model, other simulations have been developed, by changing the sound absorbent material density, which was set to be equal alternatively to 50-70-100 kg/m^3.

Figure 8 shows, for both geometries considered, a progressive shift of the insertion loss peak towards higher frequencies, with a simultaneous lowering of the peak itself.

Besides, the model indicates that the mineral wool density influences the performance of the lined ducts up to the frequency of 1000 Hz, whereas for higher frequency values the insertion loss is not affected by.

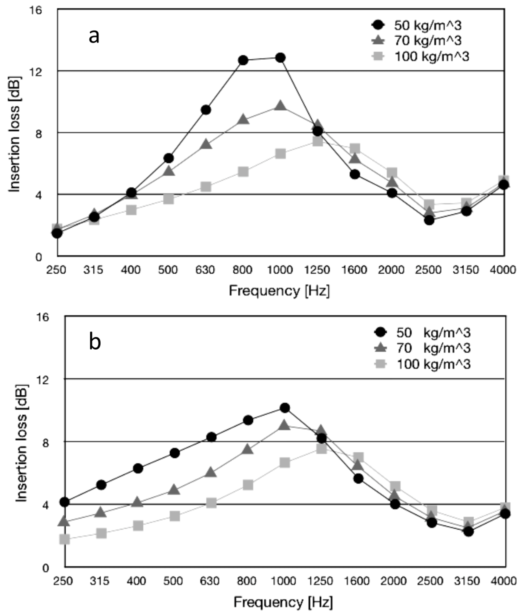

Figure 8. Effect of the density of the mineral-wool for a thickness of a) 50 mm and b) 100 mm.

5. Conclusions

In the present paper a Finite Element Model for the analysis of sound propagation in rectangular lined ducts has been described and analyzed.

A noticeable fact is the performance of the solvers: for the same problem, with same conditions and same mesh (and, of course, the same hardware configuration), the PARDISO solver made possible to reach correct solutions for higher frequencies in lesser time than using GMRES solver with a Geometric Multigrid preconditioning.

The numerical model was validated by comparing predicted data with experimental results obtained for various lining thicknesses. The comparison indicates a good accordance, with an overall mean absolute error of 1.4 dB.

Finite Element Model correctly described the effect of lining thickness on sound propagation. Besides, numerical modelling allows to analize the effects of density of the sound absorbing material on the sound propagation inside the lined ducts.

The results confirm a strong effect of lining thickness on sound propagation, with a noticeable variation of the insertion loss for geometric changes.

The next step will be to include fluidodynamics into the numerical model, so that also pressure drop inside the duct could be evaluated. This implementation is necessary to extend the modelling also to the mufflers with absorbing baffles set inside the duct.

6. References

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7. Acknowledgements

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