# Optimum insulation for corrugated pipes

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**Abstract**

Heat transfer to and from pipes is a major factor affecting the efficiency and operating cost of thermal systems. A large number of systems use a special type of corrugated pipes that are not easy or straightforward to insulate effectively. For one, the thickness of the pipe is not constant. In addition, the non-uniform cross-section on the inside continuously changes the internal flow characteristics, thereby affecting the internal heat transfer coefficient. Lastly, for exposed pipes, the non-uniform outer surface disturbs the flow over the pipe and changes the convection coefficient accordingly. As a result, the heat loss (or gain) through the surface of the pipe will not be uniform. The non-uniformity of heat loss from the pipe means that some of the insulation material used on the outer surface of the pipe could be put to better use.

This work investigates the external heat transfer process from a corrugated pipe. The heat flow is modeled using a thermal circuit that extends from the bulk internal fluid flow to the outer ambience. Thermal resistances include internal and external convection as well as conduction for the pipe and insulation materials. With a few assumptions, the number of variables in the overall heat balance reduces to one, the insulation thickness. Requiring a uniform heat loss from the pipe outer surface translates into having the derivative of the equation set to zero and solved for the actual insulation thickness. The resulting equation is a third-order, nonlinear, non-homogeneous ordinary differential equation. A fourth-order Runge-Kutta finite difference algorithm is used to solve the equation for the optimum thickness function. Then, the COMSOL Multiphysics Optimization Toolbox is used to optimize the insulation thickness via a spline made of a number of points, the location of which are the subject of optimization. Lastly, a closed-form solution with built-in parameters is used to draw the optimum insulation outer surface. The methodology could be easily extended to random pipe geometries and/or different thermal boundary conditions.

## 1 Background and Motivation

Pipe insulation is one of the most effective methods of reducing energy use in commercial and industrial applications. It is a prime target for those concerned with protecting and preserving the environment too. This is because heat loss from pipes exposed to the environment is a major source of inefficiency of energy systems. In certain applications, (maximization of) *heat gain* is the critical factor that determines the efficiency, efficacy and viability of the design, such as in heat exchangers and cryogenics. Whatever the use of the piping system, applying insulation to pipes is the main mechanism by which heat transfer to and from a pipe is controlled. In addition to reducing heat transfer, and the economics the process entails, optimum insulation has other implication relating to safety, maximum surface temperature and often times condensation on cold lines. Yet the simple, constant-thickness insulation used on the large majority of pipes is far from reliable or efficient [1].

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Applying insulation material *uniformly* to pipes and structures has its obvious advantages but none of them justifies the misuse of the material and the waste incurred. Optimization of pipe insulation thickness has therefore been the subject of a number of studies throughout the years. Only a small number of studies considered using variable insulation thickness for pipes. Sahin and his collaborators investigated optimum insulation thickness for a number of settings including convection radiation via a number of methods such as control theory, steepest descent, as well as simplified analytical methods [2–4]. Pipe subjected to convective heat transfer as well as external thermal radiation were studied with the aim of minimizing the heat loss. It was found that the optimum insulation thickness varies along the tube if the heat transfer is to be minimized.

Beyond efficiency, however, having too much insulation material will affect the initial and operating costs of systems and might decrease the overall return. As a result, a number of studies considered the economic side of the insulation of pipes [5,6]. Ozturk et al. considered both thermal and economic aspects of the pipe insulation problem, determining both pipe diameter and thickness. Analyses considered the first law of thermodynamics, exergy, as well as cost [7]. Kayfeci et al. used artificial neural networks to predict optimum insulation thickness of pipes, combined with life cycle cost analysis [8, 9]. The model had the capability of determining the optimum insulation thickness for any circular pipe given a set of boundary and domain conditions. Zaki and Al-Turki employed constrained optimization methods to study multilayer insulations strategies, where the objective was reducing both annual energy losses and insulation initial costs [10]. Yusuf Başoğul et al. found optimum insulation thickness as well as expected savings in energy, payback period and environmental impact for a number of pipe diameters, fuel types and pipe insulation [11, 12]. Finally, Ertřk optimized insulation thicknesses of pipes with respect to a number of insulation materials, fuels, and climate zones [13].

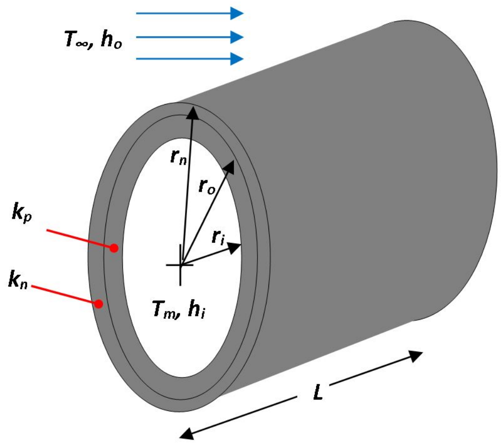
In the current work, the idea of optimizing the insulation material use is taken to the limit, by requiring that the heat loss from the outer surface of the pipe (or heat gain through the inner surface for the case of cold fluid) be uniform along the axis of the pipe. A uniform heat loss means that there is no weak spots along the pipe where heat "leaks" at a higher rate than the other locations. The insulation material in these other locations could be put to better use by shifting it to the weak spots. As a first step, a mathematical model describing the conduction through the pipe/insulation material as well as internal and external convection heat transfer is developed. The resulting ordinary differential equation is solved using a fourth-order Runge-Kutta scheme for the optimum insulation profile.

In the second phase, the heat transfer problem is modeled using the finite element method (COMSOL Multiphysics). Two approaches are used for optimization; the optimized profile describing the outer surface of the pipe (insulation) can either take a functional form or a series of points connected by a spline. The COMSOL Optimization Module is used to optimize the function parameters or the actual data points that describe the surface.

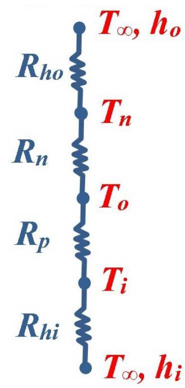
# 2 Problem Formulation

The problem considered is that of a pipe with known thickness $t$, length $L$ and mean inner radius $R$–Figure 1(a). The fluid flowing inside the pipe has a bulk temperature of $T_{m,i}$ while the pipe is placed in an environment where the ambient temperature is $T_{\infty,o}$. The pipe material has a thermal conductivity $k_p$ and the insulation material has an effective thermal conductivity $k_n$. Heat transfer coefficients of the inner pipe surface and outer insulation surface are $h_i$ and $h_o$, respectively. Radii of the inner pipe surface, outer pipe surface and insulation are given by $r_i$, $r_o$ and $r_n$, respectively. For the sake of generality, the insulation thickness is taken to be a function of the pipe axial distance, $s = s(z)$. This thickness is the subject of the optimization. Heat conduction through the pipe/insulation material is governed by the 2-D, steady, constant-properties, axisymmetric heat equation in cylindrical coordinates,

$$\frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2} + \frac{\partial^2 T}{\partial r^2} = 0 \tag{1}$$




(a) Schematic diagram of the problem with internal and external boundary conditions.




(b) Thermal circuit.

Figure 1: Problem representation.

# 3 Semi-Analytical Solution

The thermal circuit representing the flow of heat through the pipe is shown in Figure 1(b). Neglecting axial conduction, the radial heat loss from the pipe can be written, using the thermal resistance paradigm, as,

$$q_r = \frac{\Delta T}{R_{tot}} \tag{2}$$

where $\Delta T = T_{m,i} - T_{\infty,o}$ is the overall temperature difference, and $R_{tot}$ is the total resistance to heat transfer which includes convection from the fluid to the inner pipe surface, conduction through the pipe material, conduction through the insulation, and convection from the insulation surface to the environment. For a circular pipe, the total resistance can be written as,

$$R_{tot} = \frac{1}{2\pi L h_i\, r_i} + \frac{\ln\left(\frac{r_o}{r_i}\right)}{2\pi L k_p} + \frac{\ln\left(\frac{r_o+s(z)}{r_o}\right)}{2\pi L k_n} + \frac{1}{2\pi L h_o(r_o+s(z))} \tag{3}$$

As stated above, the optimum insulation layer thickness is one that lets out heat uniformly along the length of the pipe, preventing heat "leaking" spots, and thus inefficient use of insulation material. To satisfy this condition for the above problem, the derivative of the radial heat transfer, equation (2), with respect to the axial coordinate, $z$, must be set to zero, viz.,

$$\frac{dq_r}{dz} = \frac{-\Delta T}{R_{tot}^2}\frac{dR_{tot}}{dz} \tag{4}$$

For the above expression to be zero (and the insulation be optimum), either $\Delta T = 0$, which is the trivial solution, or $dR_{tot}/dz = 0$, which is pursued further like so,

$$\frac{dR_{tot}}{dz} = 0 + 0 + \frac{(ds/dz)}{2\pi L k_n(r_o+s(z))} + \frac{-(ds/dz)}{2\pi L h_o(r_o+s(z))^2} \tag{5}$$

Clearly, the above expression is zero when $ds/dz = 0$, in other words, when the insulation thickness $s$ is not a function of $z$, which is known *a priori*.

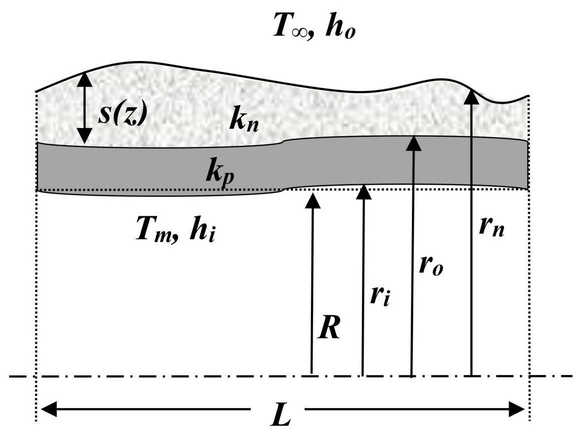



Figure 2: Cross-sectional view showing internal domains and boundary conditions.

When the radius pipe is described by a sinusoidal function of its axial distance, a few more variables enter the equation for optimum insulation thickness. First, all the radii except of the mean inner radius, $R$, are now functions of $z$. Considering one period of length $L$ of the pipe shown in Figure 2, the new variables now are,

$$r_i \longrightarrow r_i(z) = R + a\sin(2\pi z/L) := P(z) \tag{6}$$
$$r_o \longrightarrow r_o(z) = r_i(z) + t = t + R + a\sin(2\pi z/L) := Q(z) \tag{7}$$
$$r_n \longrightarrow r_n(z) = r_o(z) + s(z) = t + R + a\sin(2\pi z/L) + s(z) = Q(z) + s(z) \tag{8}$$

where $R$ is the pipe mean inner radius, $a$ is the amplitude of the pipe corrugation, $t$ is the (constant) pipe thickness, and $s(z)$ is the insulation thickness to be optimized. Rewriting the equation for the derivative of the total thermal resistance, equation (5), with the new variables,

$$R_{tot} = \frac{1}{2\pi L h_i\, r_i(z)} + \frac{\ln\left(\frac{r_o(z)}{r_i(z)}\right)}{2\pi L k_p} + \frac{\ln\left(\frac{r_n(z)}{r_o(z)}\right)}{2\pi L k_n} + \frac{1}{2\pi L h_o\, r_n(z)} \tag{9}$$

Furthermore, the derivatives of the various radii, equations (6)-(8), with respect to $z$ are,

$$\frac{dr_i}{dz} = \frac{2a\pi}{L}\cos(2z\pi/L) := F(z) \tag{10}$$
$$\frac{dr_o}{dz} = \frac{2a\pi}{L}\cos(2z\pi/L) = F(z) \tag{11}$$
$$\frac{dr_n}{dz} = \frac{2a\pi}{L}\cos(2z\pi/L) + \frac{ds(z)}{dz} = F(z) + \frac{ds(z)}{dz} \tag{12}$$

The derivatives of the terms of $R_{tot}$ with respect to $z$ are,

$$\frac{d}{dz}\left[\frac{1}{2\pi L h_i}\frac{1}{r_i(z)}\right] = \frac{1}{2\pi L h_i}\left[\frac{-1}{r_i^2(z)}\right]\frac{d}{dz}\Big[r_i(z)\Big] = \frac{1}{2\pi L h_i}\left[\frac{-F(z)}{r_i^2(z)}\right] \tag{13}$$
$$\frac{d}{dz}\left[\frac{1}{2\pi L k_p}\ln\left(\frac{r_o(z)}{r_i(z)}\right)\right] = \frac{t}{2\pi L k_p}\left[\frac{-F(z)}{r_i(z) r_o(z)}\right] \tag{14}$$
$$\frac{d}{dz}\left[\frac{1}{2\pi L k_n}\ln\left(\frac{r_n(z)}{r_o(z)}\right)\right] = \frac{1}{2\pi L k_n}\left[\frac{1}{r_n(z)}\frac{ds(z)}{dz} - \frac{F(z)s(z)}{r_o(z)r_n(z)}\right] \tag{15}$$
$$\frac{d}{dz}\left[\frac{1}{2\pi L h_o}\frac{1}{r_n(z)}\right] = \frac{1}{2\pi L h_o}\left[\frac{-F(z)}{r_n^2(z)} - \frac{1}{r_n^2(z)}\frac{ds(z)}{dz}\right] \tag{16}$$

After heavy simplification, the equation can be written as,

$$\frac{dR_{tot}}{dz} = \left[\frac{-F(z)}{H_i P^2(z)}\right] + \left[\frac{-F(z)}{K_p P(z) Q(z)}\right] + \left[\frac{1}{K_n[Q(z)+s(z)]}\frac{ds(z)}{dz} - \frac{F(z)s(z)}{K_n Q(z)[Q(z)+s(z)]}\right]$$
$$+ \left[\frac{-F(z)}{H_o[Q(z)+s(z)]^2} - \frac{1}{H_o[Q(z)+s(z)]^2}\frac{ds(z)}{dz}\right] \tag{17}$$

where $H_i = 2\pi L h_i$, $H_o = 2\pi L h_o$, $K_p = 2\pi L k_p/t$, $K_n = 2\pi L k_n$, and $P(z)$, $Q(z)$ and $F(z)$ are as defined above. We note that all terms in the above equation are known functions of $z$ with the exception of the last four, which have terms like $s(z)$ and $s' = ds/dz$.The question now becomes, can the whole expression be set to zero as planned, to get a closed-form mathematical expression for the function $s(z)$, providing the optimum insulation thickness? The fact that the above equation is a first-order, nonlinear, ordinary differential equation in $s(z)$ suggests that this is a formidable task. However, as a first step, let us recast the equation (17) in terms of a few more simplified coefficients. Setting the equation equal to zero, taking primes to be differentiation with respect to $z$, and dropping the $(z)$ functional dependency labels for $F(z)$, $P(z)$, $Q(z)$, and $s$, we can write,

$$Ass' + Bs' + Cs^2 + Ds + E = 0 \tag{18}$$

where,

$$A = H_i K_p P^3 Q \tag{19}$$
$$B = H_i K_p P^3 Q^2 - H_i K_p K_n P^3 Q \tag{20}$$
$$C = -H_o K_p K_n FPQ - H_i H_o K_n F P^2 - H_i K_p F P^3 \tag{21}$$
$$D = -2H_o K_p K_n FPQ^2 - 2H_i H_o K_n F P^2 Q - H_i K_p F P^3 Q \tag{22}$$
$$E = -H_i K_p K_n F P^3 Q - H_i H_o K_n F P^2 Q^2 - H_o K_p K_n FPQ^3 \tag{23}$$

In preparation for numerical solution, we rewrite the equation in a more convectional ODE form by multiplying by the denominators and collecting terms,

$$\begin{aligned}&(H_o BC - K_n BC + K_n H_o AC^2)\\ &\quad + (K_n H_o AC + 2K_n H_o AC^2 + 2H_o BC - K_n B)s\\ &\qquad + (3K_n H_o AC + H_o B)s^2 + (K_n H_o A)s^3 - K_n Cs' - K_n ss' = 0\end{aligned} \tag{24}$$

# 4 Numerical Results

The problem of optimizing the thickness of the insulation layer over the corrugated pipe is solved via two methods: A semi-analytical method that simplifies the heat equation governing the flow of heat through the pipe and insulation material and solves the resulting ODE, and a pure numerical method which uses COMSOL Optimization Module to solve the problem numerically. In all cases, the target is to achieve uniform heat loss through the outer surface of the pipe. The two methods' results are discussed in turn.

## 4.1 Semi-Analytical Results

The ODE can be solved via a number of methods, most efficient of which is the fourth-order Runge-Kutta method (RK4). Runge-Kutta methods are a family of implicit and explicit numerical methods designed to solve ordinary differential equations iteratively. The current problem can be thought of as an initial value problem where an initial thickness of the insulation is given and the solution progresses in a way that, for this formulation, keeps the heat transfer constant along the $z$-direction by changing the thickness of the insulation, $s(z)$.

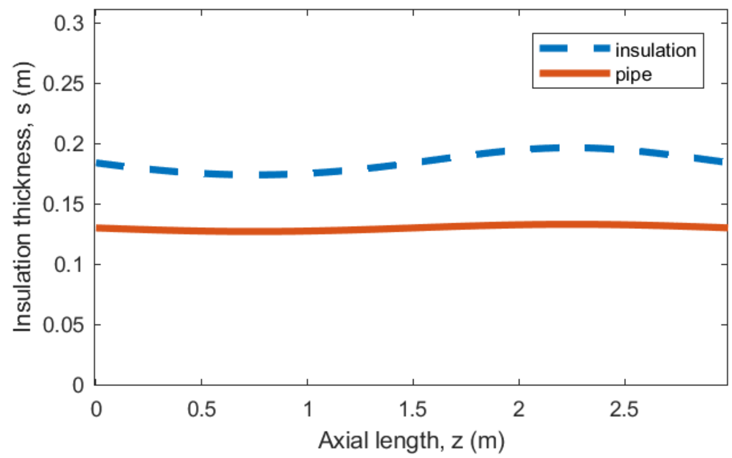



Figure 3: RK4 results for the optimum insulation profile over the sinusoidal pipe.

If the initial value problem is described as,

$$s' = f(z, s), \quad s(z_0) = s_0 \tag{25}$$

then some sort of linearization is necessary to make our problem amenable to the RK4 general form. Isolating the slope, $s'$, the unknown function, $f(z, s)$ can be written as,

$$s' = \frac{-C(z)s^2 - D(z)s - E(z)}{A(z)s + B(z)} \tag{26}$$

For a step size $dz > 0$, the RK4 approximation of $s(z_{n+1})$, referred to as $s_{n+1}$ is defined as,

$$s_{n+1} = s_n + \frac{1}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right) \tag{27}$$

$$z_{n+1} = z_n + dz \tag{28}$$

for $n = 0, 1, 2, \ldots$, where $k_1$, $k_2$, $\ldots$, are defined in the usual manner [14].

The distribution of the insulation material can best be deduced from a plot showing the thickness of the insulation layer on top of the pipe outer surface, Figure 3. It is noted that when the pipe diameter is largest along the sine wave, the insulation thickness is largest, and vice versa. It is reasoned that the heat loss area is more exposed when the pipe is convex, whereas the concavity of the pipe outer surface at smaller diameters shields the outer surface from the external convection effects, thereby requiring slightly less insulation material. Qualitative comparison with the COMSOL optimization results are given below.

## 4.2 Optimization Module Results

For the Optimization Module Specifying the shape of the insulation layer can be done in a number of ways. One can, for example, specify a number of points connected by straight segments, cubic spline, or anything in between. The nodes defining the insulation surface can further be crowded near areas where the geometry is complicated. On the other hand, if enough insight is available into the behavior of the heat conduction in this pipe, one might be able to guess an analytical expression for a function defining the thickness of the insulation given all other properties of the pipe and flow.

In the first part of this geometric optimization, a series of points are used to define the outer surface of the insulation. The investigation started with 11 points, equally-spaced along the $z$-axis of the pipe. A minimum of $2t$ (twice the pipe thickness) and a maximum of $4t$ was used for the insulation radius at

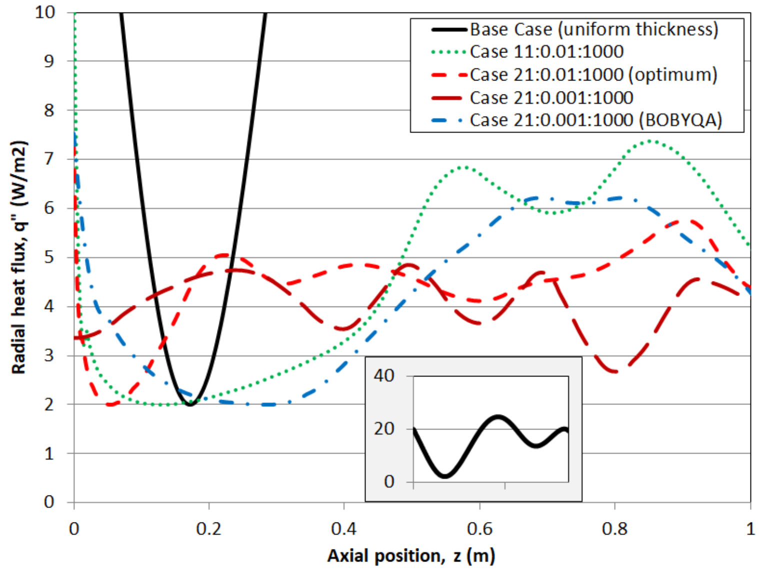



Figure 4: Radial heat flux as a function of the axial position for a number of optimization settings. Insert is for the uniform thickness case–note the $y$-axis scale.

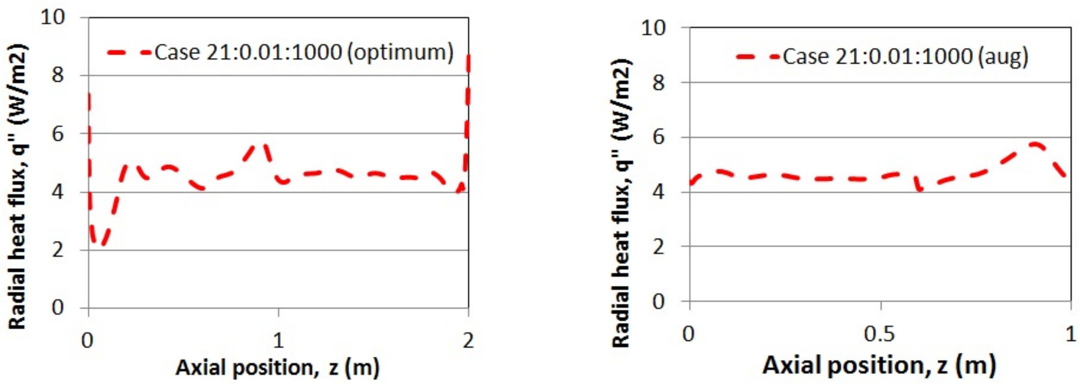
each $z$ station. A 3rd-order spline was used to construct the surface using the points $r_1 \dots r_{11}$. Figure 4 shows the radial heat flux as a function of the axial position. The optimality tolerance and the maximum number of model evaluations for this case were 0.01 and 1000, respectively (the default values of the module.) This will be referred to as the *base case* for this part of the problem. When the number of nodes was increased from 11 to 21, better heat transfer characteristics were observed–optimum case in Figure 4.

On the numerics side, most of the computational work was done using the Nelder-Mead algorithm. The BOBYQA algorithm was tried for the 21-node case and did not fare well–as shown in the figure. However, when the optimality tolerance was decreased by a factor of 10 (with the Nelder-Mead algorithm), to become 0.001, the results were better. No noticeable gains were observed, however, when it was set to 0.00025. Finally, 31 nodes were tried with 0.001 optimality tolerance and 10,000 maximum evaluations but the optimization algorithm was not able to map out a smooth insulation surface and the results were abysmal–also shown in the figure.

It is worth noting that the $r$ values along all $z$ points of the axis were a subject of optimization. The resulting shape was optimum to the degree that COMSOL was able to deduce the "periodicity" of the shape–the continuity of the sine wave from one period to the next. When two periods were simulated and optimized, continuity of the insulation thickness was clearly visible. Ideally, the value of the first and last $r$ point as well as their slopes must match. This could have been done using some kind of constraint (but wasn't.) The optimum case from the above results (Case 21:0.01:1000) was ran for two periods–Figure 5(a). Then, the thickness values from half of the second period were used on the original case of one period. The result is shown in Figure 5(b). This hints at a problem with the thermal boundary condition used at the two end surfaces, which were set to `periodic`. The result demonstrates, however, the ability of the module to arrive at a near perfect distribution of the insulation material, one that makes the heat loss nearly uniform along the pipe.

Alternatively, one could reason that the shape of the outer layer of the insulation must follow some sort of a general sine wave that may or may not be modified. With a few embedded parameters, a sine wave is able to represent a large number of shapes. A few possibilities are:

- A sine wave that follows the pipe (*i.e.*, constant thickness insulation).




(a) $q''$ vs. $z$ for optimum case for two periods.

(b) One period with augmented first part form second period.

Figure 5: Use of interior insulation description to get around end point effects.

- A sine wave that increases or decreases the insulation thickness at the peak of the pipe diameter.
- A sine wave that includes a phase shift that prevents the insulation maximum thickness from coinciding with the pipe's.
- A sine wave that mimics a constant outer diameter of the insulation.

The outer surface of the pipe was described by Equation 7. With some insight into the physical problem and the direction of the heat flow, a functional form of the outer surface of the insulation is proposed as,

$$R + 2t + t\sin\left(\frac{2\pi z}{L}\right) f_1 \tag{29}$$

where $f_1$ is the target of the optimization. To define the objective function, an integration operator is set up on the outer boundary, and a variable, $Qtotal$, is defined using this operator to integrate the total, normal heat flux on the boundary, $Qtotal = intop1(ht.ntflux)$[1]. In the `Optimization` node of the study, the objective function becomes $comp1.Qtotal$ and the Control Variable is $f_1$, with appropriate initial value, scale and bounds. The same settings for the Nelder-Mead algorithm were used, namely, 0.01 optimality tolerance and 1000 maximum model evaluations.

The result of the optimization of this parameter, $f_1$, is shown in Figure 6, where the total, normal heat flux as a function of $z$ for one period of the pipe for a constant insulation thickness. It is shown that whatever gains are achieved on one end of the figure is lost on the other end, *i.e.*, the *spread* of the heat flux values are practically the same–which is the goal of the optimization. This might be due to the simple nature of the geometric function of the insulation outer surface–Equation 29.

A slightly more elaborate outer surface function,

$$R + 3t + t\left[\sin\left(\frac{2z\pi}{L} + f1\right) - \cos\left(\frac{2\pi z}{L} + f_2\right)\right] f_1 \tag{30}$$

was tested and gave the result shown in the same figure. It is noticed that adding what amounts to a phase shift to the sine wave of the insulation surface function, via adding the $f_2$ factor, better controls

[1]One could just as easily use the magnitude of the heat flux on the boundary, $Qtotal = intop1(ht.tfluxmag)$, to define the variable, but we reasoned that the normal flux makes better sense theoretically.

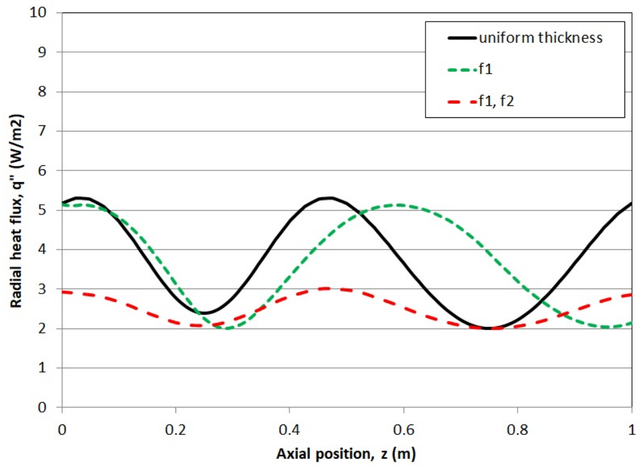



Figure 6: Radial heat flux as a function of the axial position for the closed-form optimization.

the distribution of the insulation material. In the end, both methods, the array of points and the closed-form function, showed promise in optimizing the insulation material distribution. The 2D temperature contour plot of this optimum case and the 3D-rotated, axisymmetric, 2-period version of it are shown in Figures 7.

# 5 Final Remarks

The problem of heat transfer from a corrugated pipe with internal and external convection was modeled via COMSOL Multiphysics. The profile of the applied insulation to the exterior of the pipe was optimized via a number of methods. The main difference between the semi-analytical methods attempted here and the Optimization Module approach was the practically unlimited range of physical problems the module can accommodate. We believe that the simplicity of the pipe shape did not give the optimization module the chance to shine, so to speak. With more complicated shapes of the pipe, internally and/or externally, due to fouling on the inside, dirt on the outside, asymmetric heating conditions, etc., the optimizer could really make a huge difference and result in huge savings compared to the simple, constant-thickness insulation.

Like most optimization problems, what it comes down to is the insight into the physical problem. For example, one might be able to "guess" the functional form of the insulation surface if one knows what parts of the sine wave incurs more heat transfer than others. Conversely, nodal points (of the first method) could be graded in areas of large heat transfer gradients to better control the shape of the outer surface of the insulation. In all cases, the methods presented herein do prove useful in finding some sort of an optimum insulation profile and could be further developed to negotiate more complicated geometries and physical phenomena. Future work in this research will incorporate the effect of fluid flowing inside the pipe, with local Nusselt number changes, secondary flows, recirculation zones in grooves of the sine wave, as well as axial conduction.

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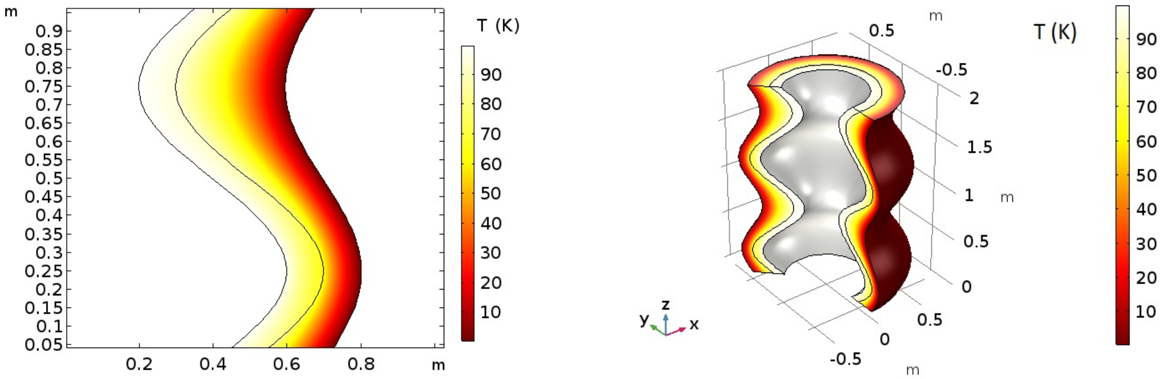


(a)

(b)

Figure 7: (a) 2D temperature contours and (b) its 3D-axisymmetric rotated, 2-period version of the optimum case..

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