COMSOL Implementation for Upscaling of Two-Phase Immiscible Flows in Communicating Layered Reservoir

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Abstract: Waterflooding is widely used in secondary oil recovery. The physics is described by the model of two-phase flow in porous media. The aim of the present work is to implement this model in COMSOL and to simulate the process of waterflooding. It is analyzed in two dimentions. We use layered reservoir in our study. And we assume each layer is homogeneous and the layers are well communicating, which means between adjasent layers there exits very fast mass transfer due to pressure gradient. Saturation of water and pressure are two independent variables in the constitutive equations. They are coupled by Darcy's law, which excludes gravity and capillary force in our study. The numerical implementation is validated, comparing with analytical solution based on asymptotic derivation, in terms of average saturation profile, pseudo-fractional flow and oil revoery rate.

Keywords: two-phase flow, waterflooding, upscaling, porous media, oil recovery

1. Introduction

Many oil reservoirs are of layered structure. The properties of each layer may often be considered to be homogeneous. See Figure 1.

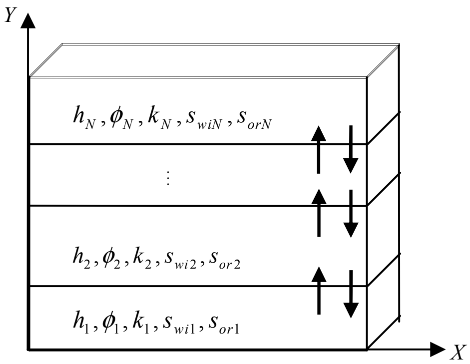


Figure 1. The layered reservoir model. The top and bottom of the reservoir are assumed to be impermeable.

Two extreme cases result from the stratified reservoir model. The first extreme case is that the barriers between layers are absolutely impermeable and the crossflow is negligible. Alternatively, this is the case when the permeability across the layers is much lower than that along the layers.

The second extreme case corresponds to perfect communication between the layers, where the pressure gradient driving exchange between layers may be considered to be instantaneous. This case is usually attributed to the viscous dominant regime of displacement, where viscous forces prevail over capillary and gravity forces (see complete asymptotic analysis in [1] and [2]). The Dykstra-Parsons upscaling method [3] is one of the widely applied upscaling methods for the first case. The Hearn-Kurbanov method (for brevity often termed the Hearn method) has been developed for the second case ([4]; [5]; [6]). Both methods have been designed for manual calculations and sacrifice accuracy in favor of simplicity.

The aim of our work is to simulate waterflooding in multi-layer reservoir in two dimentions, without the strong assumptions, for example piston-like front. Saturation of water and pressure are two independent variables in the constitutive equations. They are coupled by Darcy's law, which excludes gravity and capillary force in our study.

2. Theory

The equations for 2D two-phase flow, neglecting gravity and capillary forces in the present work, are as follows. The coordinate X is along the layers and Y across the layers of the reservoir. The mass conservation law for incompressible water and oil phases can be written in dimensionless form of

$$\Phi \frac{\partial s_w}{\partial T} + \frac{\partial}{\partial X}\left(F\bar{U}_X\right) + \frac{\partial}{\partial Y}\left(F\bar{U}_Y\right) = 0 \qquad (1)$$

$$\frac{\partial \bar{U}_X}{\partial X}+\frac{\partial \bar{U}_Y}{\partial Y}=0 \qquad (2)$$

Here s_w, Φ, $\bar{U}_i$ ($i=X,Y$), F represent saturation of water, porosity, dimensionless total velocity of oil and water,. fractional flow of water.

Referring to Darcy's law

$$\bar{U}_X=-\Lambda_X\frac{\partial P}{\partial X} \qquad (3)$$

$$\bar{U}_Y=-E\Lambda_Y\frac{\partial P}{\partial Y} \qquad (4)$$

where $E=\dfrac{x_0^2k_y}{y_0^2k_x}$ defined as the anisotropy ratio of the reservoir, which is dependent on length x_0, height y_0, permeabilities k_x, k_y of the reservoir. P , Λ_i ($i=X,Y$) represent dimensionless pressure difference and dimensionless mobility, respectively.

According to the definition of fractional flow and mobility, F and Λ_i can be expressed in terms of relative permeabilities.

$$F=\frac{kr_w}{kr_w+kr_o\left(\mu_w/\mu_o\right)} \qquad (5)$$

$$\Lambda_i=K_i\left[kr_w+kr_o\left(\mu_w/\mu_o\right)\right] \qquad (6)$$

Here $kr_\alpha(\alpha=w,o)$ is relative permeability. K_i is the dimensionless absolute permeabilities.

In equation (6), because viscosity of water is included in dimensional mobilities when we derive the dimensionless form of all equations, we only need the viscosity ratio of water to oil.

In this work, we apply the Corey power law for relative permeabilities [7].

$$\begin{aligned} kr_w&=kr_{wor}\left(1-s_{or}-s_{wi}\right)^{-2}\left(s_w-s_{wi}\right)^2\\ kr_o&=kr_{owi}\left(1-s_{or}-s_{wi}\right)^{-2}\left(1-s_w-s_{or}\right)^2\end{aligned} \qquad (7)$$

s_{or}, s_{wi} are residual oil saturation and initial water saturation, respectively. kr_{wor}, kr_{owi} are end point relative permeability of water and oil.

3. Use of COMSOL Multiphysics

In COMSOL multiphysics, PDE mode for time dependent analysis in the coefficent form is used for euqation (1) and PDE time dependent mode in general form is used for equation (2). Geometry should be 2D.

Anisotropy ratio E, viscosity ratio μ_w/μ_o, dimensionless height of each layer H_n and dimensionless total injection rate Q are defined in Contants.

s_{or}, s_{wi}, kr_{wor}, kr_{owi}, Φ and K_i are defined in Scalar expression, because they may be different in different layers. Equations (3)- (7) are also implemented in Scalar expression.

Because this is a discontinuous problem and no diffusion is involved in equation (1), artificial diffusion is needed then. In our problem, we set the diffusion coefficient c to be 10^{-2}.

Initial condition for the saturation of water s_w, i.e. equation (1), should be $s_w(t_0)=s_{wi}$, for pressure P, i.e. equation (2), should be an arbitrary value. Boundary condition at inlet, for s_w should be $s_w=1-s_{or}$, for P should be $-n\cdot\Gamma=Q$ meaning the injection rate is Q. At outlet, $s_w=s_{wi}$ before water breaks through, and P should be arbitrary value smaller than or equal to its initial value. If we inject more water into the reservoir and water breaks through, then we should extend the geometry to make sure at the outlet $s_w=s_{wi}$. We only analyze the domain $X\in[0,1]$. The advantage of using dimensionless form for all equations is that all parameters have no unit and the domain of interest is a unit square. Since the top and bottom of reservoir are assumed to be impermeabale, boundary conditions there should be $-n\cdot\Gamma=0$ for both equations (1) and (2).

After the calculation is finished in COMSOL, we export the structure and data to Matlab and calculate average saturation of water and average fractional flow of water in Matlab, as shown in Figures 3 and 4.

4. Case study

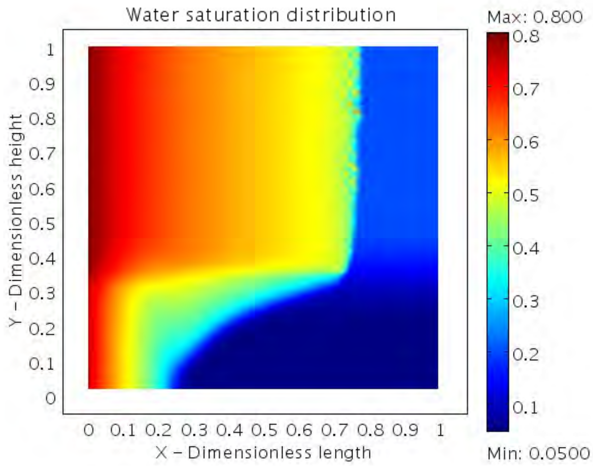
4.1 2-layer reservoir

The dimensionless parameters applied in this case are listed in Table 1.

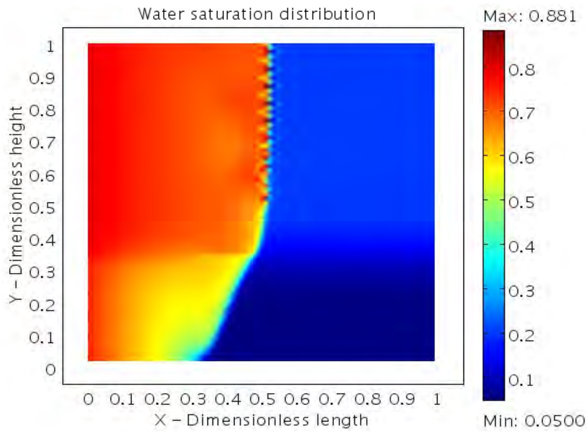
Table 1 Dimensionless parameters for the two-layer model. The values in brackets correspond to the mobility ratio (oil to water) $M=1.33$, other values correspond to $M=0.33$. ($M=\frac{kr_{owi}}{kr_{wor}}\frac{\mu_w}{\mu_o}$)

Dimensionless parameters	Layer 1	Layer 2
Fraction of thickness H	0.33	0.67
Irreducible water saturation s_{wi}	0.05	0.2
Residual oil saturation s_{or}	0.25	0.2
Relative water permeability at residual oil saturation kr_{wor}	0.8	0.8
Relative oil permeability at irreducible water saturation kr_{owi}	0.8 (0.4)	0.8 (0.4)
Dimensionless permeability in X-direction K_X	0.33	0.67
Dimensionless permeability in Y-direction K_Y	0.33	0.67
Dimensionless porosity Φ	1	
Viscosity ratio of water to oil μ_w / μ_o	1:3 (1:1.5)	
Anisotropy ratio E	1000	
Dimensionless injection rate Q	1	

Figure 2 shows the water saturation in the two layers, at the same time but different mobility ratios, $M=0.33$ and $M=1.33$.



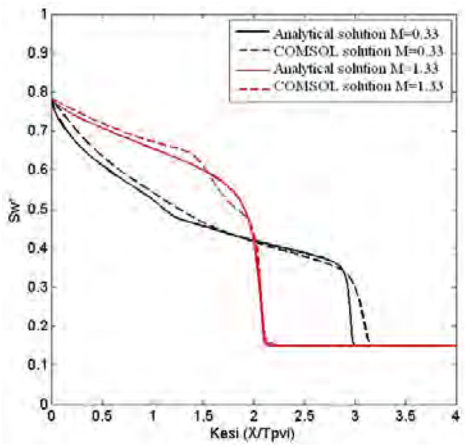
(a)



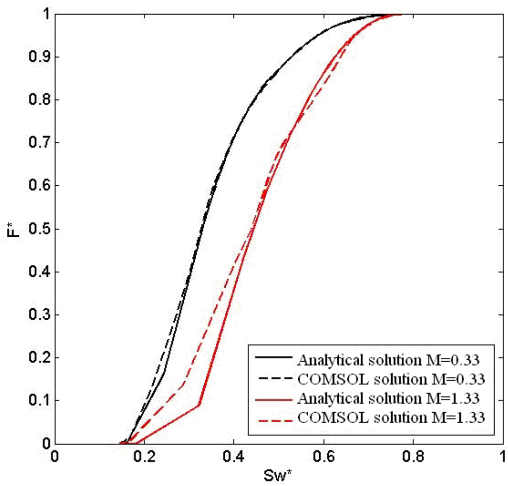
(b)

Figure 2. Water saturation profile at time=0.25 p.v.i. The X-axis is the dimensionless distance along the reservoir, and the Y-axis is the dimensionless height (across the reservoir). (a) $M=0.33$, (b) $M=1.33$.

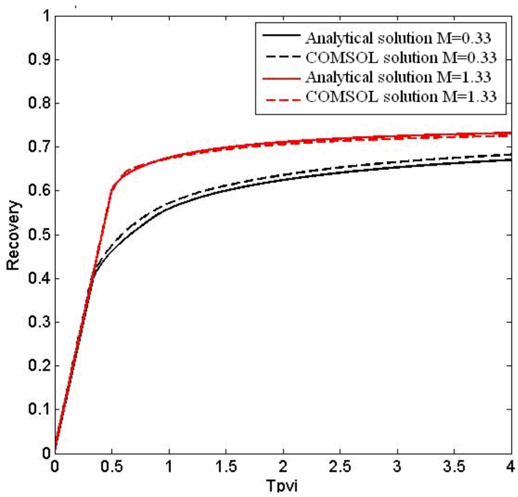
In Figure 2, we see that when mobility ratio of oil to water is larger than 1, the fronts of the two layers tend to merge (Figure 2(b)). When mobility ratio is smaller than 1, the effect is the opposite (Figure 2(a)).



(a)



(b)



(c)

Figure 3. Comparison of the results obtained by COMSOL 2D simulation and analytical derivation for a reservoir consisting of two communicating layers. Solid lines represent the results by analytical derivation; dashed lines the results of COMSOL. Black and red lines represent the results for an unfavorable ($M = 0.33$) and a favorable mobility ratio ($M = 1.33$), respectively. (a) average water saturation, (b) pseudo-fractional flow, (c) oil recovery.

From Figure 3, we see that the implementation in COMSOL Multiphysics gives very close results to analytical derivations. Both methods proves that at favorable mobility ratio cross flow improves oil recovery, while at unfavorable mobility raio, cross flow decreases oil recovery.

4.2 Log normal distributed permeability

In this section, we consider a special case of continuous distribution of permeability, log-normal distribution. We assume that the permeability increases along the height of the reservoir.

The log-normal probability distribution density of permeability is given by

$$\varphi(\ln k) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{(\ln k - \beta)^2}{2\sigma^2}\right] \tag{8}$$

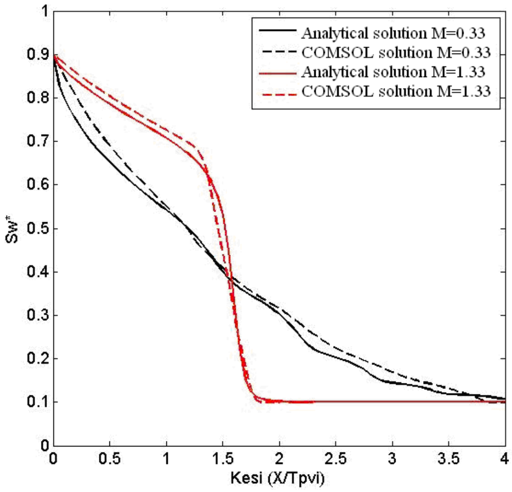
The relation between k and the height of the reservoir Y is

$$Y(k) = \int_{\ln k}^{+\infty} \varphi(\ln k') d \ln k' \tag{9}$$

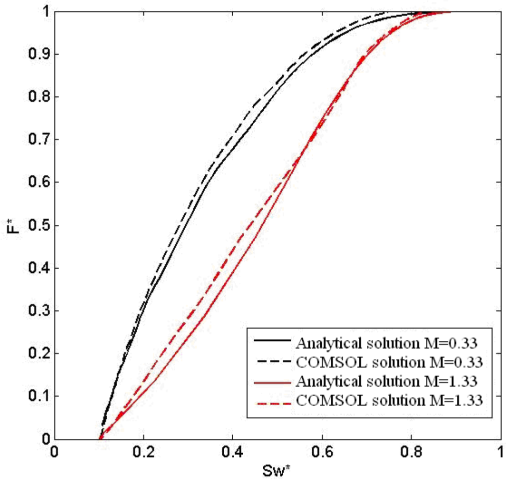
For normal distribution, Equation (8), the integration (Equation (9)) with respect to $\ln k$ from $\beta - 3\sigma$ to $\beta + 3\sigma$ goes up to 0.97. Hence, it is reasonable to set the calculation range of $\ln k$ to be $[\beta - 3\sigma, \beta + 3\sigma]$, and, therefore, the calculation range of k to be $[\exp(\beta - 3\sigma), \exp(\beta + 3\sigma)]$. In our calculation, this range is divided into ten equal intervals. The value of each interpolation point is substituted into Equation (9) to give a $Y(k)$. The distance between two adjacent values of $Y(k)$ is considered as dimensionless height H of certain layer. The parameters of this case are listed in Table 2. Other parameters are the same as in Table 1. Results are given in Figure 4.

Table 2 Dimensionless parameters for the log-normal distributed permeability model. The values in brackets correspond to the mobility ratio $M = 1.33$, the other values correspond to $M = 0.33$.

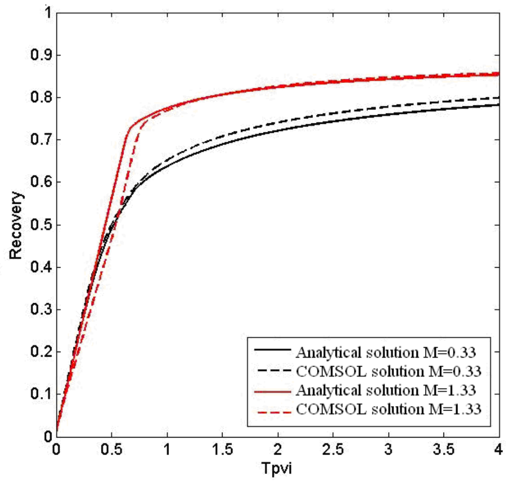
Dimensionless parameters	Value
β	$\ln 5$
σ	0.5
Irreducible water saturation s_{wi}	0.1
Residual oil saturation s_{or}	0.1
Relative water permeability at residual oil saturation kr_{wor}	0.8 (0.4)
Relative oil permeability at irreducible water saturation kr_{owi}	0.8
Dimensionless porosity Φ	1
Viscosity ratio of water to oil μ_w / μ_o	1:3 (1:1.5)



(a)



(b)



(c)

Figure 4. Comparison of the results obtained by COMSOL 2D simulation and analytical derivation for a reservoir where the permeability is of log-normal distribution. Solid lines represent the results by analytical derivation; dashed lines the results of COMSOL. Black and red lines represent the results for an unfavorable ($M = 0.33$) and a favorable mobility ratio ($M = 1.33$), respectively. (a) average water saturation, (b) pseudo-fractional flow, (c) oil recovery.

5. Conclusions

This is a complete 2D simulation for waterflooding in layered reservoir. We involve less assumptions than Hearn's method. From

Figures 3-4, we see that the implementation in COMSOL Multiphysics gives very close results to analytical derivations. Both methods show that at favorable mobility ratio cross flow improves oil recovery, while at unfavorable mobility raio, cross flow decreases oil recovery. This is in agreement with the work of El-Khatib [8].

We can change the value of E to get different levels of inter-layer communication. When E increases, the inter-layer communication increases. Gravity and capillary effect can be involved in this model easily. The work of gravity-dominant regime is going on.

Because this is a discontinuous problem and capillary pressure is not considered here, artificial diffusion is needed. In this problem, quad mesh is better than triangle mesh.

6. References

1. Bedrikovetsky, P. *Mathematical Theory of Oil and Gas Recovery-with Applications to Ex-ussr Oil and Gas Fields*. P3-7, P12, P22, P428-533 (1993)
2. Kanevskaya, R.D.: Asymptotic Analysis of the Effect of Capillary and Gravity Forces on the Two-Dimensional Transport of Two-Phase Systems in a Porous Medium. *Fluid Dynamics*, **4**, 557-563 (1988)
3. Dykstra, H. and Parsons, R.L. The Prediction of Oil Recovery by Waterflooding. *Secondary Oil Recovery of Oil in the United States*, 2nd edition, API, 160-174 (1950)
4. Hearn, C.L. Simulation of Stratified Water Flooding by Pseudo Relative Permeability Curves. *SPE Journal* 2929, (1971)
5. Kurbanov, A.K., On Some Generalization of the Equations of Flow of a Two-Phase Liquid in Porous Media. *Collected Research Papers on Oil Recovery, VNINeft*, **15**, 32-38 (1961)
6. Kurbanov, A.K. and Atanov, G.A., On the Problem of Oil Displacement by Water from Heterogeneous Reservoirs. *Oil and Gas of Tyumen, Collected Research*, **13**, 36-38 (1972)
7. Corey, A.T. and Rathjens, C.H., Effect of Stratification on Relative Permeability. *Journal of Petroleum Technology* (1956)
8. El-Khatib, N., Waterflooding performance of communicating stratified reservoir with log-normal permeability distribution. *SPE Reservoir Eval. & Eng.* **2(6)**, 542-549 (1999)
9. Diaz-Viera, M.A., Lopez-Falcon, D.A., Moctezuma-Berthier, A. and Ortiz-Tapia, A., COMSOL Implementation of a Multiphase Fluid Flow Model in Porous Media. *COMSOL Conference*. Boston, US (2008)