

# Efficient Simulation of 3D Electro-optical Waveguides Using the Effective Refractive Index Method

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**Abstract:** 3D FEM simulation of millimeter-scale, complex electro-optically induced waveguide based devices demands the use of grids with more than several million nodes. Hence, the simulation could take substantial time and require large amounts of available memory.

This paper presents a computation algorithm based on the conversion of an initial 3D waveguide structure into an analogous 2D structure, where the wave propagation on the ‘reduced’ dimension is described by an effective refractive index.

It is shown that with the proposed algorithm the computing efficiency could be improved. Moreover the algorithm could be successfully used to simulate large-size passive or electro-optically active waveguide structures. For an electro-optical waveguide coupler, we could demonstrate a good agreement between simulated and calculated coupling length.

**Keywords:** 3D FEM simulation, electro-optical waveguides, effective refractive index method.

## 1. Introduction

Future optical telecommunication networks demand the development of integrated fast optical switches and modulators [1]. Electro-optic (EO) waveguide switching has great potential for integration. Besides lithium niobate, EO polymers are very promising materials for such integrated devices. The EO coefficients of these materials permit refractive index changes at the order of  $10^{-4}$  [2]. Due to this low change in refractive index only a weak guiding is obtained, consequently the corresponding EO waveguide devices (e.g. bending waveguides or couplers) could reach sizes on the millimeter scale. The design of such devices requires simulation of wave propagation in complex structures of large sizes, for which the computing performance is often a limiting factor.

For optimal 3D FEM simulation of wave propagation in EO waveguide structures, we found that the grid spacing on the longitudinal direction must be smaller than  $\lambda/5$ , where  $\lambda$  is

the wavelength of light. The grid spacing on the transversal direction depends on the electromagnetic mode profile; namely, higher order modes need grids with a greater number of nodes. Hence for 3D simulation of millimeter scale device the corresponding grid should have several million nodes, this demanding considerable memory capacity and computation time.

To reduce the memory requirement, several Comsol methods can be applied such as symmetries, piecewise simulation or making use of an indirect solver. In the present paper, we propose a computation algorithm based on the conversion of an initial 3D waveguide structure into an analogous 2D structure by using the effective refractive index method (ERIM).

## 2. Governing equations

The mode field propagation in a waveguide can be described by the wave equation:

$$\nabla \times [\nabla \times \vec{E}(x, y, z)] - k_0^2 n^2(x, y, z) \vec{E}(x, y, z) = 0 \quad (1)$$

where  $k_0$  is the wave vector,  $n$  is the refractive index and  $\mathbf{E}$  is the electric field.

For wave propagation, an analytic approximation can be made using ERIM [3]. For TM modes, the electric field can be approximated by:

$$\vec{E}(x, y, z) = E_x(x, y, z) \quad (2)$$

Thus, Eq. (1) can be re-written as:

$$-\Delta E_x(x, y, z) - k_0^2 n^2(x, y, z) E_x(x, y, z) = 0 \quad (3)$$

With the following dimension reduction assumption:

$$-\Delta E_x(y, z) - k_0^2 n_{eff}^2(y, z) E_x(y, z) = 0, \quad (4)$$

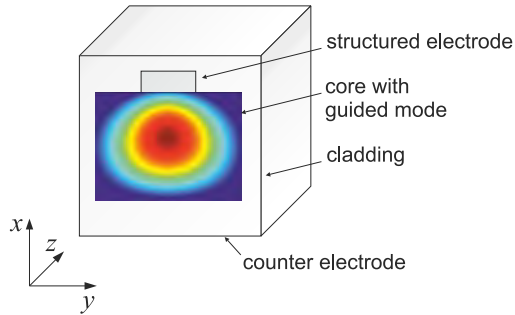
we can extract the effective refractive index  $n_{eff}(y, z)$  from (3), for a plane wave on every point  $(y_i, z_j)$  by using:

$$d^2 E_x(x, y_i, z_j) / dx^2 + [k_0^2 n^2(x, y_i, z_j) - k_0^2 n_{eff}^2(y_i, z_j)] E_x(x, y_i, z_j) = 0 \quad (5)$$

Eq. 5 represents the scalar wave equation for TM modes.

### 3. Proposed reduction dimension algorithm

The proposed computation algorithm for wave propagation is based on the conversion of a 3D problem to a 2D problem by implementing a calculated effective refractive index. This algorithm is based on Comsol's solvers in combination with MatLab scripts. A 3D geometry is built using Comsol's electrostatic module (Fig. 1).



**Figure 1.** EO waveguide structure. The optical mode is guided in the core material and shaped by the electrical field.

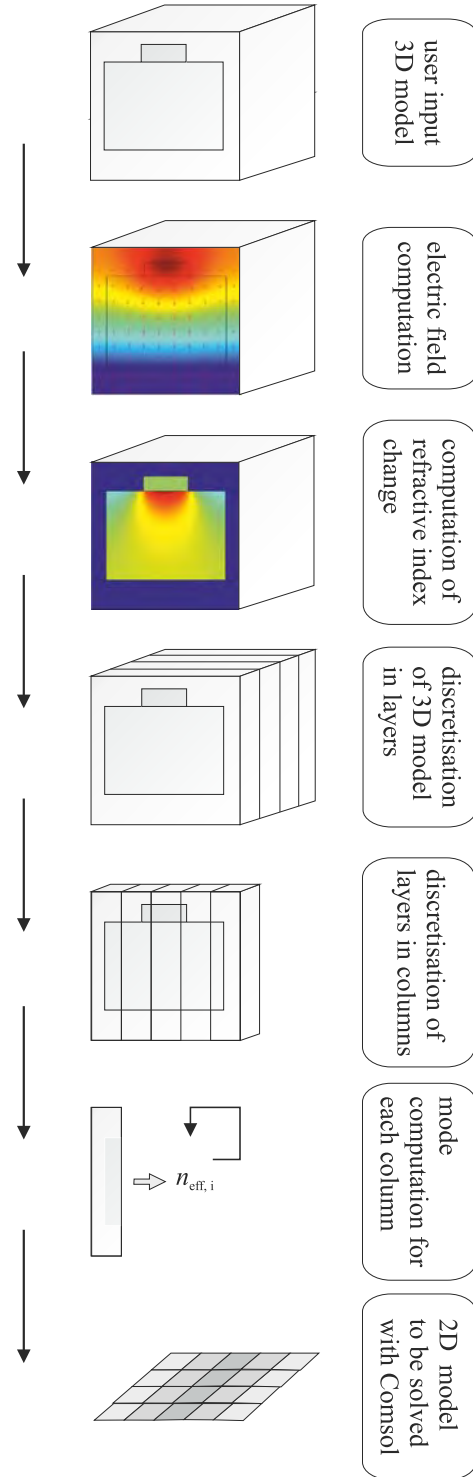
Fig. 2 shows the steps flow of the algorithm.

- i) An input 3D EO waveguide model (from Fig. 1) is built in Comsol.
- ii) For this model, the electric potential distribution is then calculated.
- iii) The electric potential is converted into refractive index change distribution (due to the electro-optic effect).
- iv) The model is discretised, with MatLab, on the propagation direction  $z$  (in layers) and again on the traversal direction  $y$  (in column like elements). Accordingly, the 3D problem is converted to a 1D problem.
- v) An effective refractive index  $n_{eff,i}$  is then calculated for each column using the RF (perpendicular waves) module of Comsol.
- vi) Finally, a 2D model of  $n_{eff}$  is obtained, where the light-wave propagation can be solved in the Comsol's 2D-RF (in-plane waves) module.

The advantage of this algorithm over a 3D direct solver is that, in the calculation process, only one column from a layer is loaded in the main memory and solved with a finite element grid. The result is saved and then the next column is solved.

### 4. Simulation of case study 3D structures

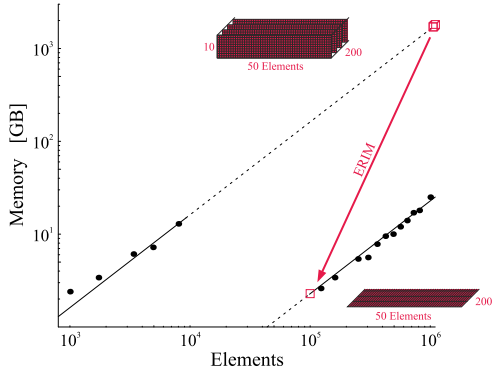
The proposed algorithm was applied to analyze the wave propagation in different case study 3D structures. To evaluate the performance of the proposed algorithm, the results obtained were compared to analytical or numerical solutions.



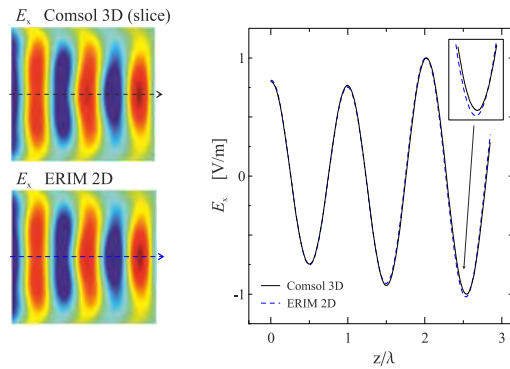
**Figure 2.** Schematic steps in the proposed algorithm.

## 4.1 EO waveguide

The simulation of large size 3D waveguide structures with a Comsol direct solver is limited by the amount of needed memory and long computation times. On the other hand, the memory required to solve a 3D problem with a number of discrete elements is obviously smaller for a 2D problem with the same number of discrete elements, as shown in Fig. 3. Accordingly, the required memory can be considerably reduced by converting the 3D to a 2D problem with the developed algorithm. The data marked in red in Fig. 3 correspond to an example of a 3D model, which, when reduced to a 2D model by making use of ERIM, demands indeed significantly less memory for simulation in Comsol.



**Figure 3.** Memory capacity dependence on the number of discrete elements for 3D and 2D models. The data marked in red correspond to a 3D model, which is converted to a 2D model by making use of ERIM.



**Figure 4.** Comparison of simulation results using the Comsol 3D solver and the proposed algorithm: wave propagation (images - left); electric field amplitude (graph - right).

To test the simulation efficiency, we solved an EO waveguide model of  $(1 \times 1 \times 1) \mu\text{m}^3$  (Fig. 1) for a  $0.5 \mu\text{m}$  wavelength with both the proposed algorithm and a direct Comsol solver.

As shown in Fig. 4, the simulated electric field distribution results obtained with the proposed algorithm and the 3D Comsol solver, are very similar, with only a small difference of less than 1 % for the electric field amplitude.

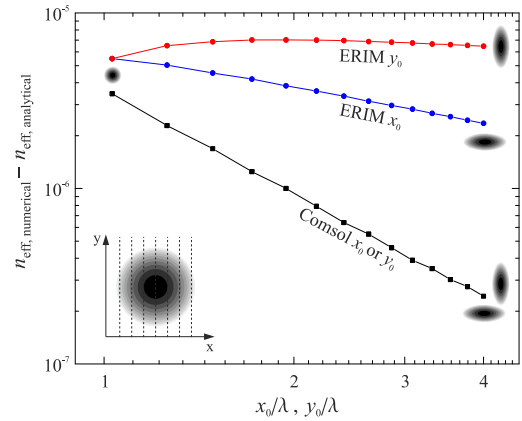
## 4.2. Parabolic refractive index profile

For a 2D parabolic refractive index profile the wave equation can be solved analytically. Thus, we chose to use this model to evaluate the errors of  $n_{\text{eff}}$  made by the algorithm.

The parabolic refractive index profile is given by:

$$n(x, y) = n_0 \sqrt{1 - \frac{x^2}{x_0^2} - \frac{y^2}{y_0^2}}. \quad (6)$$

The constants  $x_0$  and  $y_0$  describe the ‘stretching’ or ‘compressing’ of the parabolic refractive index. The proposed algorithm solves this problem differently on the  $y$ -direction than on the  $x$ -direction. Hence, some differences (errors) may be expected to occur.



**Figure 5.** Deviation from the analytical value of the effective refractive index for a wave propagating in a parabolic refractive index profile as a function of  $x_0$  and  $y_0$ . The insert represents a principle sketch of the parabolic refractive index profile.

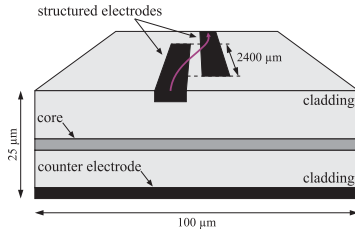
The effective refractive indices for this model were again calculated using both a Comsol direct solver and the dimension reduction algorithm. All obtained results were found to deviate from the analytical solution. These deviations are caused by the number of elements chosen and the finite simulation area. Fig. 5 shows the results: the same values were obtained with the Comsol solver for the effective refractive index regardless of whether  $x_0$  is varied or  $y_0$ . Contrary, but also as expected, the reduction algorithm gives different solutions for  $x_0$  and  $y_0$ .

### 4.3 Waveguide coupler

Large-size 3D structures are not easily solvable with either direct solvers, due to large main memory requirements, or indirect solvers because of the long computation times involved. The dimension reduction algorithm was developed to simulate specifically such structures.

To demonstrate this, the proposed algorithm was used to simulate a large size structure model of a coupler of  $(25 \times 100 \times 2400) \mu\text{m}^3$  size for a propagating light wave of 633 nm wavelength. This model being much too large for directly solving with sufficient accuracy, the reduction algorithm could convert it to a 2D model and then the structure is solved with Comsol.

The coupler structure is presented in Fig. 6. A waveguide is induced in the EO core layer by applying an electric field between structured top electrodes and counter bottom electrode. The input waveguide is coupled via mode interference onto the output waveguide.



**Figure 6.** Principle cross-sectional sketch of a coupler. The core material is EO active.

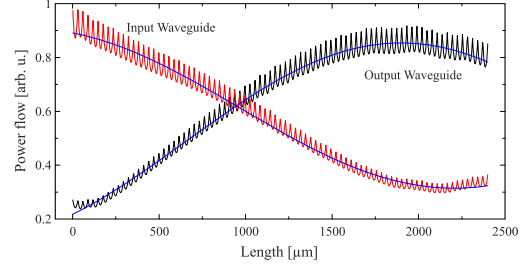
An oscillating energy transfer occurs between the two waveguides. An analytical description for the power transfer is approximately given in [5]:

$$\frac{I(z)}{I_0} = \sin^2\left(\frac{k_0 \Delta n_{eff}}{2} \cdot z\right) \quad (7)$$

where  $I(z)$  is the intensity as a function of propagation length  $I_0$  is the input intensity and  $\Delta n_{eff}$  is the difference of effective refractive indices between symmetric and asymmetric modes.

Fig. 7 shows the obtained simulation results: the input waveguide power decreases with propagation length (coupling length), being transferred onto the output waveguide, and reaches a minimum after a coupling length of about 2100  $\mu\text{m}$ .

A complete transfer of power is expected to occur after a coupling length of 2082  $\mu\text{m}$  (this is calculated using  $\Delta n_{eff}$ ), which is in good agreement with the simulated value.



**Figure 7.** Optical power transfer between two coupled waveguides (input and output) in a coupler. The high frequency oscillation in the power flow is caused by the coarse mesh chosen.

This simulation took less than a day and the main memory usage was below 10 GB.

### 5. Conclusions

We implemented the effective refractive index method (ERIM) in Comsol for solving waveguides. The obtained results show that by using the proposed dimension reduction algorithm the simulation efficiency, i.e. amount of required memory and computation time could be improved. The errors given by the ERIM based algorithm were small.

The reduction algorithm could be used to simulate large-size passive or active (EO) waveguide structures. For a case study of a large scale waveguide 3D coupler structure, we could demonstrate a good agreement between the simulated and expected coupling length.

### 6. References

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### 7. Acknowledgements

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