

**STATISTICS OF  
NUMERICAL  
EXPERIMENTS WITH  
MULTI-FRACTURE  
SYSTEMS**

**Ekkehard  
Holzbecher**

**German Univ.  
of Techn. in  
Oman**

# FRACTURED MEDIA

Fractured Porous Media is a topic of high scientific and technical interest, mainly in

SANDSTONE



<http://pyrite.igs.indiana.edu/indgeol/reference/>

SANDSTONE



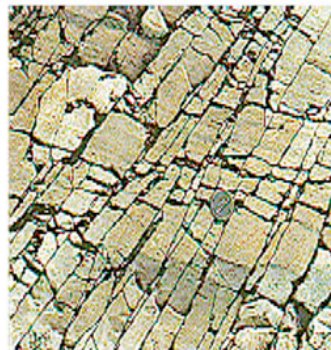
(J.Olson)

SANDSTONE



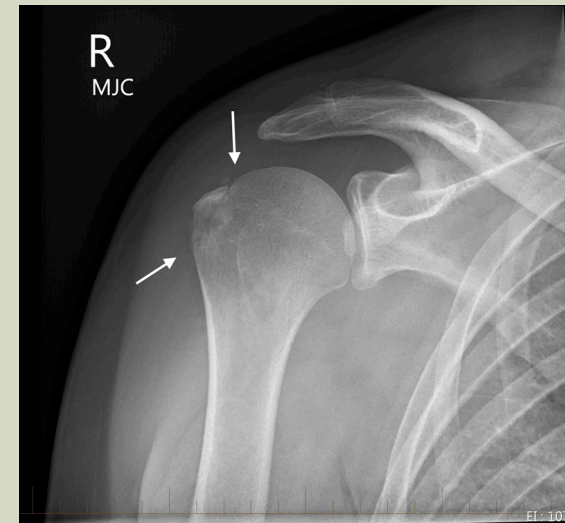
Emanuel J.M. Willemse

LIMESTONE



(Roberts 1989)

- ◆ Material sciences
- ◆ Geology
- ◆ Medical sciences



<https://www.fifamedicalnetwork.com/764937/>

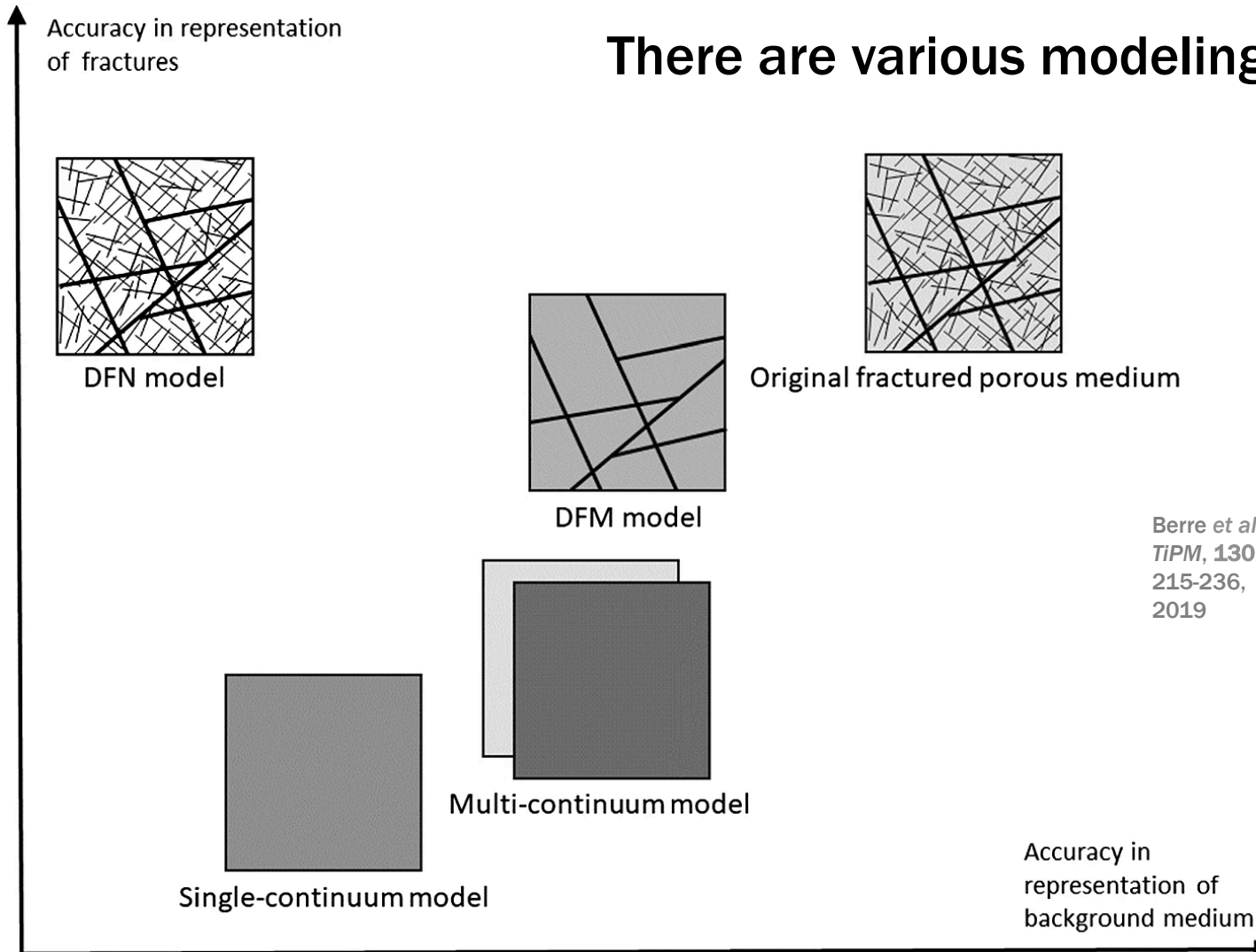
<https://csegrecorder.com/articles/view/seismically-guided-fracture-characterization>

# MODELING

There are various modeling approaches:

- **Single continuum**
- **Multi continuum**
- **Discrete Fracture Network (DFN)**
- **Discrete Fracture Matrix (DFM)**

Berre et al.,  
*TIPM*, 130,  
215-236,  
2019



# MODELING APPROACH

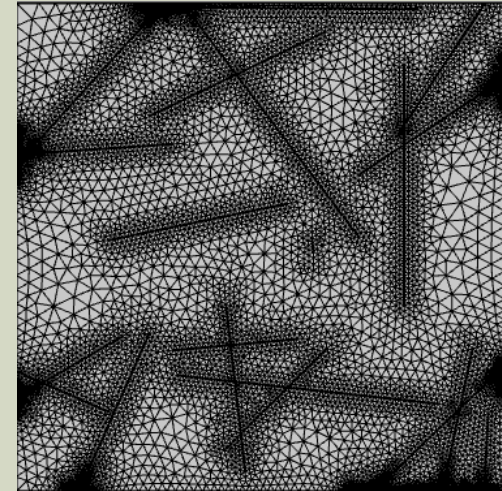
**Discrete Fracture Matrix (DFM) model:**  
1D fractures in 2D porous matrix

$$\nabla K \nabla h = 0 \quad \text{with} \quad \begin{cases} K = K_m & \text{in matrix} \\ K = K_f & \text{in fracture} \end{cases}$$

$$K = \frac{k\rho g}{\mu} \quad \text{and} \quad h = \frac{p}{\rho g}$$

Dirichlet boundary condition

on opposite boundaries with head gradient  $\Delta h$



$h$	hydraulic head
$K$	hydraulic conductivity
$k$	permeability
$\rho$	fluid density
$\mu$	dynamic viscosity
$p$	pressure
$g$	acceleration due to gravity

# GENERATION USING COMSOL METHODS

In order to examine the influence of the fracture network parameters values of the reference set-up (Table) were altered to obtain new constellations. For each of the constellations a set of 40 scenarios with random fractures was run. For each scenario the hydraulic conductivity was evaluated according to formula:

$$K_{eff} = \frac{\int u_x dy}{\Delta h}$$

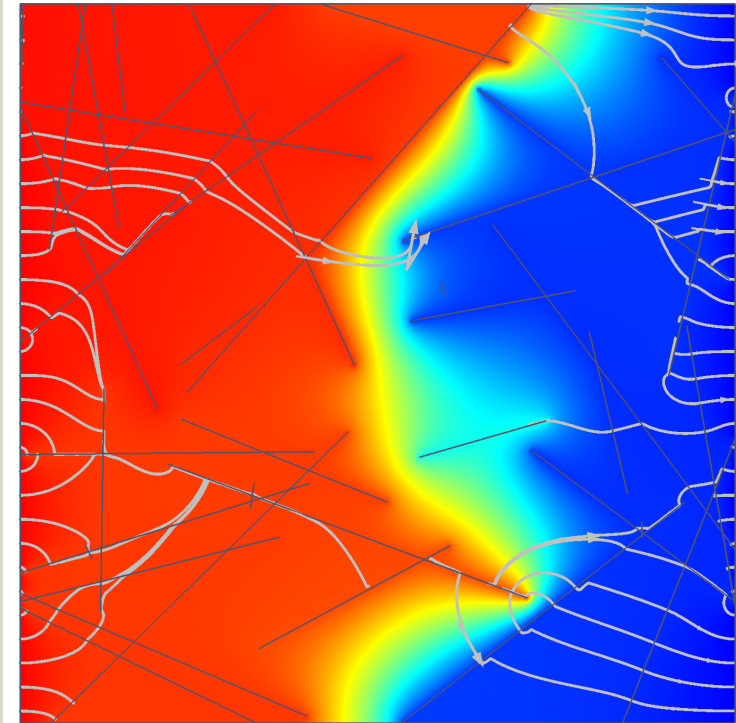
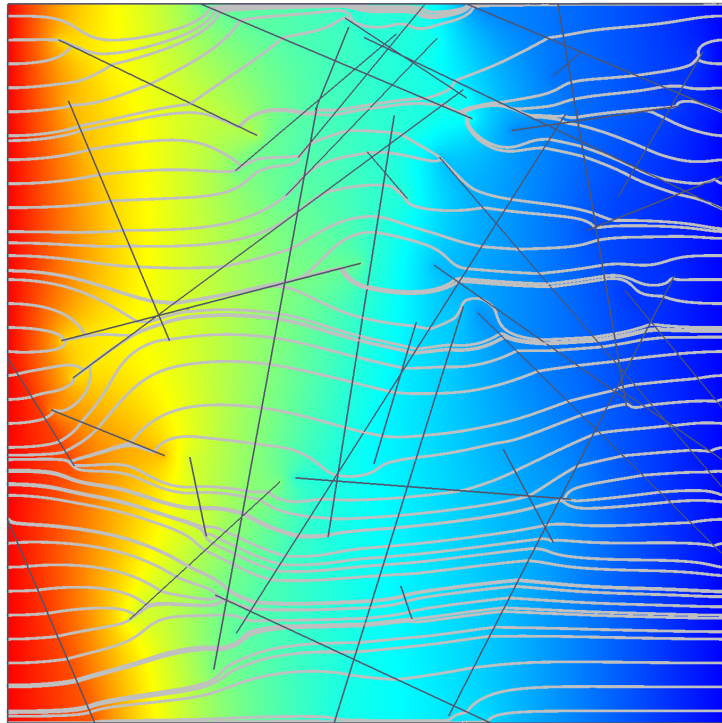
with in- or outflow velocity components  $u_x$

Uniform and power law distributions, using COMSOL methods (see left)

Parameter	Value [Unit]
Domain length	1 [m]
Domain width	1 [m]
Fracture conductivity	0.01 [m/s]
Matrix conductivity	$10^{-6}$ [m/s]
Head gradient	1 [-]
Fracture aperture	5 [mm]
Minimum fracture length	1 [mm]
Maximum fracture length	0.3 [m]
Number of fractures	40

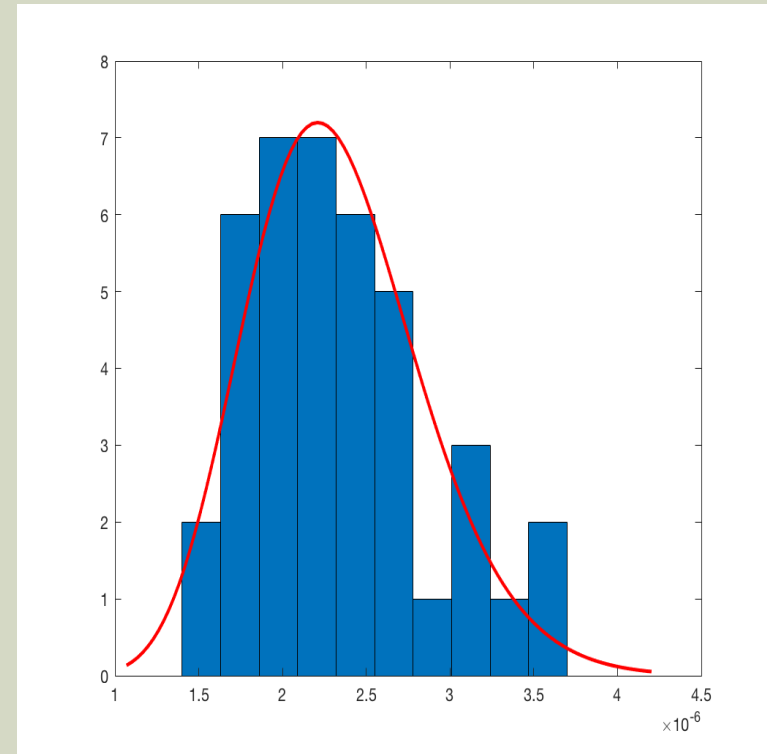
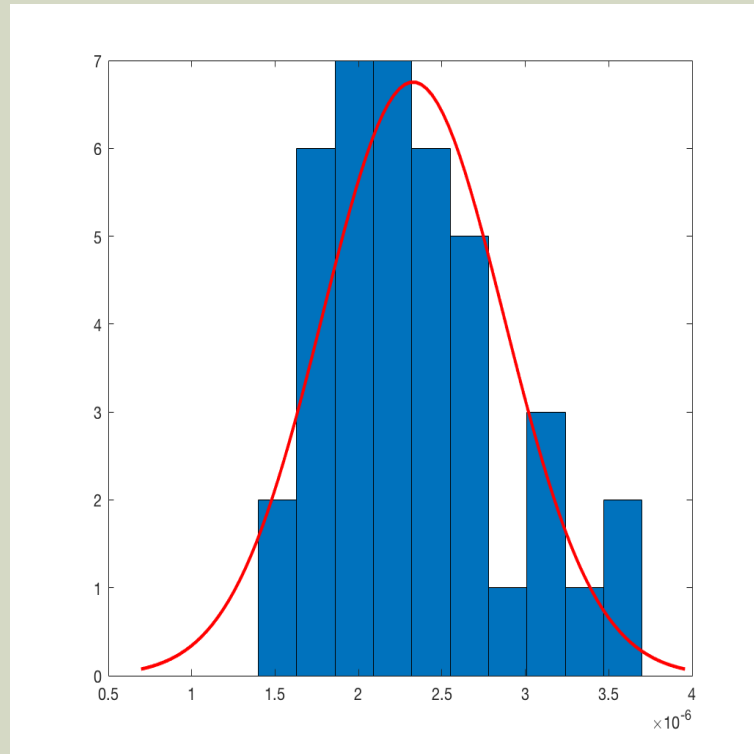
```
model.methodCall("methodcall15").run();
while (ind < NUMBER_OF_FRACTURES) {
  hx = Math.random()*MODEL_LENGTH;
  hy = Math.random()*MODEL_LENGTH;
  ha = Math.random()*Math.PI;
  hl = Math.pow(lmax, alfa) - (Math.pow(lmax, alfa) - Math.pow(lmin, alfa)) * Math.random();
  h1 = Math.pow(hl, 1/alfa);
  model.component("comp1").geom("geom1").create("l1"+ind, "LineSegment");
  with(model.component("comp1").geom("geom1").feature("l1"+ind));
    set("specify1", "coord");
    set("coord1", new double[]{hx+0.5*h1*Math.cos(ha), hy+0.5*h1*Math.sin(ha)});
    set("specify2", "coord");
    set("coord2", new double[]{hx-0.5*h1*Math.cos(ha), hy-0.5*h1*Math.sin(ha)});
  endwhile();
  model.component("comp1").geom("geom1").feature("l1"+ind).set("contributeto", "mf1");
  ind++;
}
```

# RESULTS EXAMPLES



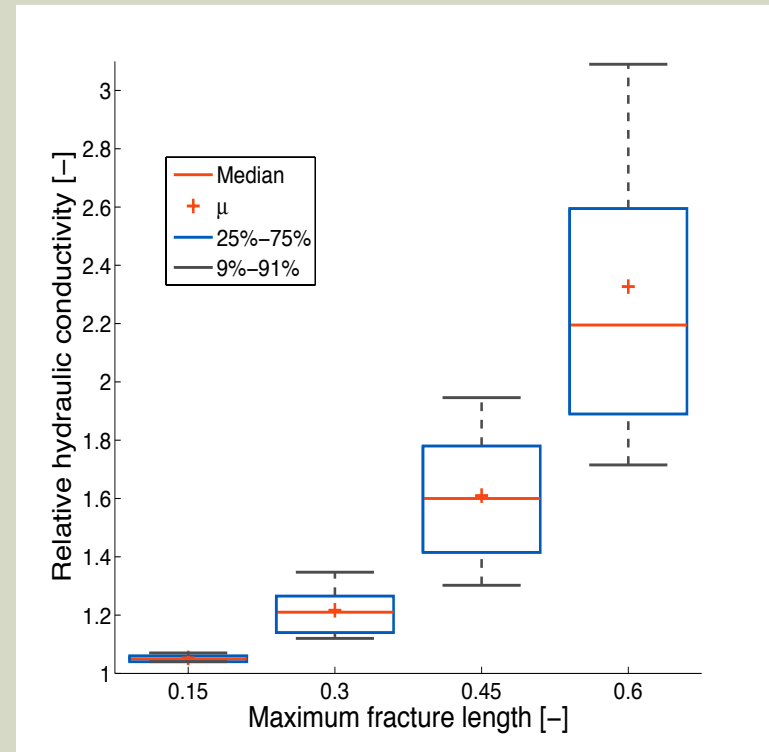
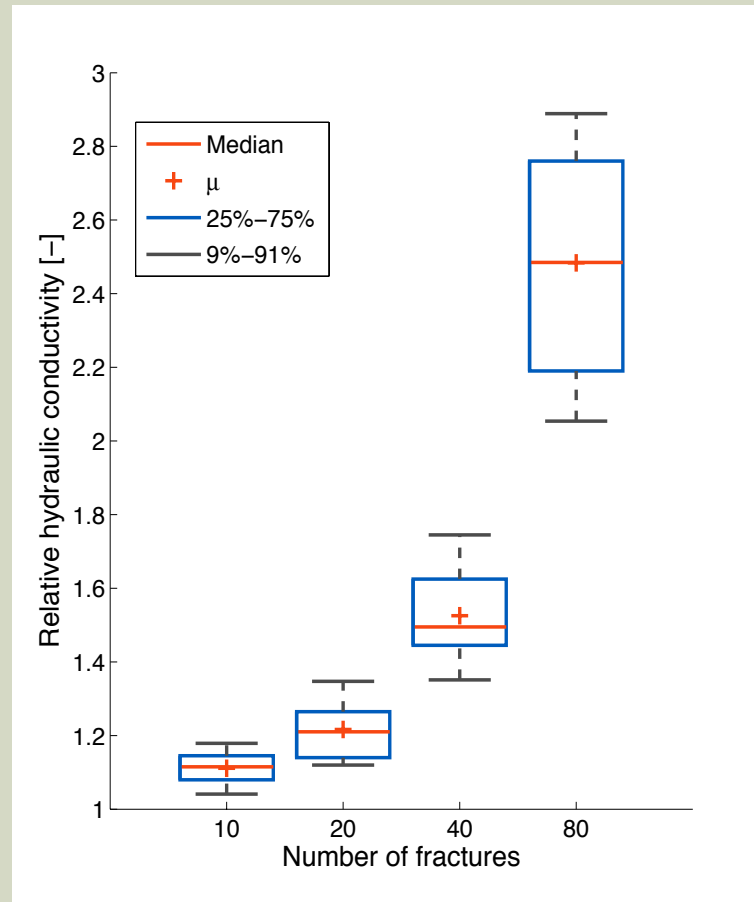
Head gradient from left to right, visualized by colormap  
fractures = black, streamlines = grey

# STATISTICS 1: CONDUCTIVITY DISTRIBUTIONS



**Example of the hydraulic conductivity distribution from 40 scenario runs with fitted statistics; left: fitted normal distribution, right: to gamma distribution**

# STATISTICS 2: BOXPLOTS



**Boxplots showing mean, median and percentiles of relative hydraulic conductivity in dependency of number of fractures and maximum fracture length**



# CONCLUSIONS

- There is a relative increase of the effective hydraulic conductivity with each additional fracture by 0.02 (with regression coefficient  $R^2=0.9756$ )
- There is a quadratic relationship of effective hydraulic conductivity and maximum fracture length ( $R^2=0.9917$ )

$$K_{eff} / K_m = 1.53 - 4.05x + 9.33x^2$$

- The effective conductivity increases with the ratio of hydraulic conductivities  $K_f / K_m$
- The results of the simulations and the statistics provide clues how the hydraulic conductivity of a sample is affected by properties of the fracture system. The presented approach may enable the prediction of conductivity from basic fracture characteristics.

**THANK YOU**

**For details see the conference paper**

or:

**contact me under**

`ekkehard.holzbecher@gutec.edu.om`